Soszyńska Joanna

Kołowrocki Krzysztof

Blokus-Roszkowska Agnieszka

Guze Sambor
Maritime University, Gdynia, Poland

Testing uniformity of statistical data from complex technical systems operation processes

Keywords

system operation, statistical data, uniformity testing, maritime transport

Abstract

There is presented the contents of the training course addressed to industry. The curriculum of the course includes the methods and procedures for testing the uniformity of the statistical data collected from the operation process of the complex technical system in different experiments and their application in practice. It is based on theoretical backgrounds concerned with the semi-markov modeling of the complex technical systems operation processes and on the statistical methods of uniformity testing data of two samples coming from the complex technical system operation process. The illustration of the proposed methods and procedures practical application in maritime transport is included.

1. Introduction

The training course is concerned with the methods, algorithms and procedures of testing the uniformity of two sets of statistical data coming from the operation process of the complex technical system in two different experiments and their application in practice and it is based on the results given in [5] and [1]. The participants of the course are provided training materials and a disk with the computer program included in [3]. Presented at the training course example of practical application is coming from in [6].

The training course includes the following items:

- Theoretical backgrounds based on [5]: mathematical model of the complex technical system operation process and its basic parameters and characteristics;
- Methodology of description of the complex technical systems: fixing the system designation and operation conditions, fixing the system subsystems and components;
- Methodology of defining parameters of the system operation process based on [1]: fixing the number of disjoin operation states of the system, defining the operation states of the system, fixing the possible transitions between the system operation states, fixing the parameters of the system operation process model under uniformity testing;
- Procedure of the system operation process data collection based on [1]: fixing two different experiments of the system operation process data collection, fixing the experiments durations times, defining the system operation process single realization, fixing the numbers of the observed realizations of the system operation process in each of two experiments, fixing two samples of the realizations of the conditional sojourn times of the system operations process at the particular operation states in all observed realizations of the system operation process in two experiments;
- Procedure of testing the uniformity of two sets of empirical realizations of the conditional sojourn times of the system operation process in the particular operation states coming from two experiments based

on [1]: determining the realizations of the empirical distributions of the system conditional sojourn times at the particular operation states on the basis of two samples collected in two experiments, analyzing the realization of the empirical distributions, formulating the hypothesis on uniformity of two samples of realizations of the conditional sojourn times in the particular operation states collected in two experiments, verifying the hypothesis concerning the uniformity of two sets of empirical realizations of the conditional sojourn times in the particular operation states in collected in two experiments;
- Procedure of applying the computer program for testing the uniformity of statistical data from the complex technical system operation process based on [3];
- Application of the procedures and computer program for testing the uniformity of statistical data from the operation processes of real complex technical systems: testing the uniformity of statistical data sets from the operation process of the ferry technical system based on [6].

2. Theoretical backgrounds

Training material is given in [5].

3. Procedures of uniformity testing statistical data from the complex technical system operation process

3.1. Methodology of description of the complex technical system

The description of the complex technical systems should include at least the following items:
- the system designation,
- the system operation conditions,
- the system subsystem and components.

3.2. Methodology of defining the parameters of the system operation process

To make the uniformity testing statistical data sets coming from the system operations process the experiment delivering the necessary statistical data should be precisely planned.
Firstly, before the experiment, we should perform the following preliminary steps:

i) to analyze the system operation process;

ii) to fix or to define its following general parameters:

- the number of the operation states of the system operation process ν,

- the operation states of the system operation process $z_1, z_2, \ldots, z_\nu$;

iii) to fix the possible transitions between the system operation states;

iv) to fix the parameters of the system operation process which realizations are under uniformity testing.

3.3. Procedure of the system operation process data collection

To test the uniformity of the realizations of the parameters of the system operation process in different experiments, we should collect necessary statistical data performing the following steps:

i) to fix two experiments of the system operation process data collection and their following basic parameters:

- the duration times of the experiments Θ_1, Θ_2,

- the system operation process single realizations,

- the numbers of the investigated (observed) realizations of the system operation process $n_1(0)$, $n_2(0)$;

ii) to fix and to collect the following statistical data concerned with the empirical distributions of the conditional sojourn times θ_{bl}^1 and θ_{bl}^2, $b, l \in \{1,2,...,\nu\}$, $b \neq l$, of the system operation process in the particular operation states in two experiments:

- the number of realizations

$$n_{bl}^1,\; b, l \in \{1,2,...,\nu\},\; b \neq l,$$

of the sojourn times θ_{bl}^1, $b, l \in \{1,2,...,\nu\}$, in the first experiment,

- the independent sample of non-decreasing ordered realizations

$$\theta_{bl}^{1k},\;\; k = 1,2,...,n_{bl}^1,\;\; b \neq l, \tag{1}$$

of the sojourn times θ^1_{bl}, $b,l \in \{1,2,...,\nu\}$, in the first experiment,

- the number of realizations

n^2_{bl}, $b,l \in \{1,2,...,\nu\}$, $b \neq l$,

of the sojourn times θ^2_{bl}, $b,l \in \{1,2,...,\nu\}$, in the second experiment,

- the independent sample of non-decreasing ordered realizations

$$\theta^{2k}_{bl},\ k = 1,2,...,n^2_{bl},\ b \neq l, \tag{2}$$

of the sojourn times θ^{2k}_{bl}, $b,l \in \{1,2,...,\nu\}$, in the second experiment.

3.4. Procedure of testing the uniformity of distributions of the conditional sojourn times of the system operation process in the particular operation states

We consider test λ based on Kolmogorov-Smirnov theorem that can be used for $b,l \in \{1,2,...,\nu\}$, $b \neq l$, testing whether two independent samples of realizations of the conditional sojourn times θ_{bl}, in particular operation states of the system operation process are drawn from the population with the same distribution.

We assume that we have defined in previous section two independent samples of non-decreasing ordered realizations (1) and (2) of the sojourn times θ^1_{bl} and θ^2_{bl}, $b,l \in \{1,2,...,\nu\}$, $b \neq l$, coming from two different experiments, respectively composed of n^1_{bl} and n^2_{bl} realizations and we mark by

$$H^1_{bl}(t) = \frac{1}{n^1_{bl}} \#\{k : \theta^{1k}_{bl} < t, k \in \{1,2,...,n^1_{bl}\}\}, \tag{3}$$

$t \geq 0$, $b,l \in \{1,2,...,\nu\}$, $b \neq l$,

and

$$H^2_{bl}(t) = \frac{1}{n^2_{bl}} \#\{k : \theta^{2k}_{bl} < t, k \in \{1,2,...,n^2_{bl}\}\}, \tag{4}$$

$t \geq 0$, $b,l \in \{1,2,...,\nu\}$, $b \neq l$,

their corresponding empirical distribution functions.

Then, according to Smirnov theorem, the sequences of distribution functions given by the equation

$$Q_{n_1 n_2}(\lambda) = P(D_{n_1 n_2} < \frac{\lambda}{\sqrt{n}}) \tag{5}$$

defined for $\lambda > 0$, where

$$n_1 = n^1_{bl},\ n_2 = n^2_{bl},\ n = \frac{n_1 n_2}{n_1 + n_2}, \tag{6}$$

and

$$D_{n_1 n_2} = \max_{-\infty < t < +\infty} \left| H^1_{bl}(t) - H^2_{bl}(t) \right|, \tag{7}$$

is convergent to the limit distribution function

$$Q(\lambda) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\lambda^2},\ \ \lambda > 0, \tag{8}$$

as $n \to \infty$.

The distribution function $Q(\lambda)$ given by (8) is called λ distribution and its Tables of values are available. It means that for sufficiently large n_1 and n_2 we may use the following approximate formula

$$Q_{n_1 n_2}(\lambda) \cong Q(\lambda),\ \lambda > 0. \tag{9}$$

Hence it follows that if we define the statistic

$$U_n = D_{n_1 n_2} \sqrt{n}, \tag{10}$$

where $D_{n_1 n_2}$ is defined by (7), then by (5) and (9) we have

$$P(U_n < u) = P(D_{n_1 n_2}\sqrt{n} < u) = P(D_{n_1 n_2} < \frac{u}{\sqrt{n}})$$

$$= Q_{n_1 n_2}(\lambda) \cong Q(u) \text{ for } u > 0. \tag{11}$$

This result means that in order to formulate and next to verify the hypothesis that the samples of the realizations the system conditional sojourn times θ^1_{bl} and θ^2_{bl}, $b,l \in \{1,2,...,\nu\}$, $b \neq l$, at the operation state z_b when the next transition is to the operation state z_l are coming from the population with the same distribution, it is necessary to proceed according to the following scheme:

- to fix the numbers of realizations n^1_{bl} and n^2_{bl} in the samples,

- to collect the realizations (1) and (2) of the conditional sojourn times θ_{bl}^{1} and θ_{bl}^{2} of the system operation process in the samples,

- to find the realization of the empirical distribution functions $H_{bl}^{1}(t)$ and $H_{bl}^{2}(t)$, defined by (3) and (4) respectively, in the following forms:

$$H_{bl}^{1}(t) = \begin{cases} \frac{n_{bl}^{11}}{n_{bl}^{1}} = 0, & t \le \theta_{bl}^{11} \\ \frac{n_{bl}^{12}}{n_{bl}^{1}}, & \theta_{bl}^{11} < t \le \theta_{bl}^{12} \\ \frac{n_{bl}^{13}}{n_{bl}^{1}}, & \theta_{bl}^{12} < t \le \theta_{bl}^{13} \\ \cdot \quad \cdot \quad \cdot & \\ \frac{n_{bl}^{1k}}{n_{bl}^{1}}, & \theta_{bl}^{1k-1} < t \le \theta_{bl}^{1k} \\ \cdot \quad \cdot \quad \cdot & \\ \frac{n_{bl}^{1n_{bl}^{1}}}{n_{bl}^{1}}, & \theta_{bl}^{1n_{bl}^{1}-1} < t \le \theta_{bl}^{1n_{bl}^{1}} \\ \frac{n_{bl}^{1n_{bl}^{1}+1}}{n_{bl}^{1}} = 1, & t \ge \theta_{bl}^{1n_{bl}^{1}} \end{cases}, \tag{12}$$

where

$$n_{bl}^{11} = 0 \,, \; n_{bl}^{1n_{bl}^{1}+1} = n_{bl}^{1}, \tag{13}$$

and

$$n_{bl}^{1k} = \#\{i : \theta_{bl}^{1j} < \theta_{bl}^{1k}, \; j \in \{1,2,...,n_{bl}^{1}\}\}, \quad k = 2,3,...,n_{bl}^{1}, \tag{14}$$

is the number of the sojourn time θ_{bl}^{1} realizations less than its realization θ_{bl}^{1k},

$$H_{bl}^{2}(t) = \begin{cases} \frac{n_{bl}^{21}}{n_{bl}^{2}} = 0, & t \le \theta_{bl}^{21} \\ \frac{n_{bl}^{22}}{n_{bl}^{2}}, & \theta_{bl}^{21} < t \le \theta_{bl}^{22} \\ \frac{n_{bl}^{23}}{n_{bl}^{2}}, & \theta_{bl}^{22} < t \le \theta_{bl}^{23} \\ \cdot \quad \cdot \quad \cdot & \\ \frac{n_{bl}^{2k}}{n_{bl}^{2}}, & \theta_{bl}^{2k-1} < t \le \theta_{bl}^{2k} \\ \cdot \quad \cdot \quad \cdot & \\ \frac{n_{bl}^{2n_{bl}^{2}}}{n_{bl}^{2}}, & \theta_{bl}^{2n_{bl}^{2}-1} < t \le \theta_{bl}^{2n_{bl}^{1}} \\ \frac{n_{bl}^{2n_{bl}^{2}+1}}{n_{bl}^{2}} = 1, & t \ge \theta_{bl}^{2n_{bl}^{2}} \end{cases}, \tag{15}$$

where

$$n_{bl}^{21} = 0 \,, \; n_{bl}^{2n_{bl}^{2}+1} = n_{bl}^{2}, \tag{16}$$

and

$$n_{bl}^{2k} = \#\{i : \theta_{bl}^{2j} < \theta_{bl}^{2k}, \; j \in \{1,2,...,n_{bl}^{2}\}\}, \quad k = 2,3,...,n_{bl}^{2}, \tag{17}$$

is the number of the sojourn time θ_{bl}^{2} realizations less than its realization θ_{bl}^{2k},

- to formulate the null hypothesis H_0 and the alternative hypothesis H_A the following form:

H_0: The samples of realizations (1) and (2) are coming from the populations with the same distributions,

H_A: The samples of realizations (1) and (2) are coming from the populations with different distributions,

- to fix the significance level α,

- to read from the Tables of λ distribution the value $u = \lambda_0$ such that the following equality holds

$$P(U_n < u) = Q(u) = Q(\lambda_0) = 1 - \alpha, \tag{18}$$

- to determine the critical domain in the form of the interval $(u, +\infty)$ and the acceptance domain in the form of the interval $< 0, u >$,

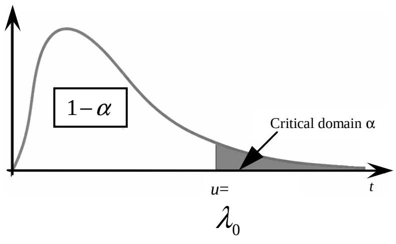

Figure 1. The graphical interpretation of the critical domain and the acceptance domain for the two-sample Smirnov-Kolmogorov test

- to calculate the realization of the statistic U_n defined by (10) according to the formula

$$u_n = d_{n_{bl}^1 n_{bl}^2} \sqrt{n_{bl}}, \tag{19}$$

where

$$d_{n_{bl}^1 n_{bl}^2} = \max\{ d^1_{n_{bl}^1 n_{bl}^2}, d^2_{n_{bl}^1 n_{bl}^2} \}, \tag{20}$$

$$d^1_{n_{bl}^1 n_{bl}^2} = \max\{ |H_{bl}^1(\theta_{bl}^{1k}) - H_{bl}^2(\theta_{bl}^{1k})|, k \in \{1,2,...,n_{bl}^1\}\} \tag{21}$$

$$d^2_{n_{bl}^1 n_{bl}^2} = \max\{ |H_{bl}^1(\theta_{bl}^{2k}) - H_{bl}^2(\theta_{bl}^{2k})|, k \in \{1,2,...,n_{bl}^2\}\}, \tag{22}$$

$$n_{bl} = \frac{n_{bl}^1 n_{bl}^2}{n_{bl}^1 + n_{bl}^2}, \tag{23}$$

- to compare the obtained value u_n of the realization of the statistics U_n with the read from the Tables critical value $u = \lambda_0$,

- to verify previously formulated null hypothesis H_0 in the following way:
if the value u_n does not belong to the critical domain, i.e. when $u_n \le u$, then we do not reject the hypothesis H_0, otherwise if the value u_n belongs to the critical domain, i.e. when $u_n > u$, then we reject the hypothesis H_0.

4. Procedure of applying the computer program for testing the uniformity of statistical data of the system operation process

Training material is given in [3]

5. Testing the uniformity of statistical data from the operation processes of real complex technical systems – using procedures

5.1. Testing the uniformity of statistical data from the ferry technical system operation process

5.1.1. The Stena Baltica ferry description

The m/v Stena Baltica is a passenger Ro-Ro ship operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line.
We assume that the ship is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in *Figure 2*. On the scheme of the ship presented in *Figure 2,* there are distinguished her following subsystems:
S_1 - a navigational subsystem,
S_2 - a propulsion and controlling subsystem,
S_3 - a loading and unloading subsystem,
S_4 - a hull subsystem,
S_5 - an anchoring and mooring subsystem,
S_6 - a protection and rescue subsystem,
S_7 - a social subsystem.

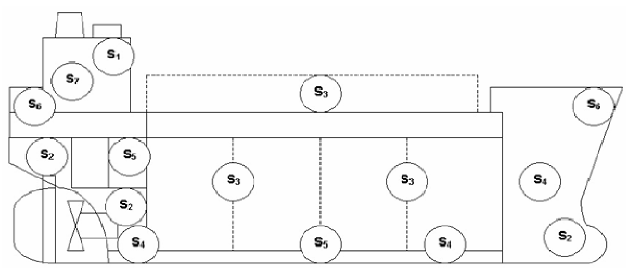

Figure 2. Subsystems having an essential influence on ship safety

5.4.2. Defining the parameters of the Stena Baltica ferry operation process

Taking into account the expert opinion on the operation process of the considered Stena Baltica ferry we fix:
- the number of the ship operation process states $\nu = 18$
and we distinguish the following as its eighteen operation states:

- an operation state z_1 – loading at Gdynia Port,

- an operation state z_2 – unmooring operations at Gdynia Port,
- an operation state z_3 – leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z_4 – navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z_5 – navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z_6 – navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z_7 – mooring operations at Karlskrona Port,
- an operation state z_8 – unloading at Karlskrona Port,
- an operation state z_9 – loading at Karlskrona Port,
- an operation state z_{10} – unmooring operations at Karlskrona Port,
- an operation state z_{11} – ship turning at Karlskrona Port,
- an operation state z_{12} – leaving Karlskrona Port and navigation at restricted waters to "Angoring" buoy,
- an operation state z_{13} – navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state z_{14} – navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state z_{15} – navigation from "GD" buoy to turning area,
- an operation state z_{16} – ship turning at Gdynia Port,
- an operation state z_{17} – mooring operations at Gdynia Port,
- an operation state z_{18} – unloading at Gdynia Port.

Moreover, we fix that there are possible only the transitions between the neighboring system operation states, i.e., from the operation states z_b to the operation states z_{b+1}, $b = 1,2,...,17$, and from the operation state z_{18} to the operation state z_1.

Thus, the parameters of the system operation process under the uniformity testing are the realizations of the conditional sojourn times in the particular operation states $\theta_{b\,b+1}$, $b = 1,2,...,17$, and $\theta_{18\,1}$. To test the uniformity of these parameters of the Stena Baltica ferry operation process the statistical data about this process is needed. The statistical data collected during spring and winter and are given in the Appendix 4A [6]. These data coming from the ferry operation process and concerned with the realizations of the conditional sojourn times in particular operation states $\theta_{b\,b+1}$, $b = 1,2,...,17$, and $\theta_{18\,1}$ are presented in these *Tables 1-14* in the next section.

5.1.3. The Stena Baltica ferry operation process data collection

It is assumed that one voyage from Gdynia to Karlskrone and back to Gdynia of the ferry is a single realization of its operation process. The operation process is very regular in the sense that the operation state changes are from the particular operation state z_b, $b = 1,2,...,17$, to the neighbouring operation state z_{b+1}, $b = 1,2,...,17$, only and from the operation state z_{18} to the operation state z_1.

There are considered two experiments, one in spring and one in winter and for these experiments there are collected two sets of the realizations of the conditional sojourn times in particular operation states $\theta_{b\,b+1}$, $b = 1,2,...,17$, and $\theta_{18\,1}$. The realizations $\theta^{1k}_{b\,b+1}$, $b = 1,2,...,17$, and θ^{1k}_{181} of the conditional sojourn times in particular operation states collected during the spring experiment are presented in the *Tables 1-7.* The realizations $\theta^{2k}_{b\,b+1}$, $b = 1,2,...,17$, and θ^{2k}_{181} of the conditional sojourn times in particular operation states collected during the winter experiment are presented in the *Tables 8-14.*

5.1.3.1. Spring data collection of the Stena Baltica ferry operation process

The spring experiment on the ferry operation process is characterized by the following parameters:

- the ferry operation process experiment time is Θ_1 = 42 days,

- the number of the ferry operation process realizations is $n_1(0) =$ 42,

- the numbers the realizations $\theta^{1k}_{b\,b+1}$, $b = 1,2,...,17$, and θ^{1k}_{181} of the ferry conditional sojourn times in particular operation states are n^1_{bl} = 42.

The realizations $\theta^{1k}_{b\,b+1}$, $b = 1,2,...,17$, $k = 1,2,...,42$, of the conditional sojourn time θ^1_{bb+1} for $b = 1,2,...,17$, are given in *Tables 1-7* in the *b*-th rows and the realizations and θ_{181}^{1k}, $k = 1,2,...,42$, of the conditional sojourn time $\theta_{18\,1}^{1}$, are given in *Tables 1-7* in the 18-th rows.

Table 1. Spring realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{11}	θ_{bb+1}^{12}	θ_{bb+1}^{13}	θ_{bb+1}^{14}	θ_{bb+1}^{15}	θ_{bb+1}^{16}
z_1	55	52	47	75	60	60
z_2	4	3	3	2	2	2
z_3	28	31	32	35	37	48
z_4	52	46	48	65	53	47
z_5	598	635	539	572	499	507
z_6	35	42	42	44	35	37
z_7	7	9	8	7	7	5
z_8	25	20	23	27	20	31
z_9	75	59	56	40	66	47
z_{10}	5	3	2	3	2	3
z_{11}	6	5	4	5	4	5
z_{12}	25	22	25	25	23	25
z_{13}	574	427	461	501	498	490
z_{14}	61	43	43	46	49	52
z_{15}	33	32	33	36	35	33
z_{16}	4	4	5	4	4	4
z_{17}	8	10	6	5	5	6
z_{18}	26	26	30	20	16	17

Table 2. Spring realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{17}	θ_{bb+1}^{18}	θ_{bb+1}^{19}	θ_{bb+1}^{110}	θ_{bb+1}^{111}	θ_{bb+1}^{112}
z_1	62	43	50	61	65	63
z_2	2	3	3	4	3	2
z_3	33	38	39	43	40	42
z_4	49	62	45	46	51	47
z_5	621	580	507	511	497	496
z_6	34	40	36	33	38	38
z_7	5	5	5	5	8	7
z_8	15	17	16	21	33	34
z_9	26	60	65	25	55	40
z_{10}	5	6	3	4	4	2
z_{11}	4	4	4	6	4	5
z_{12}	20	33	24	24	22	22
z_{13}	438	561	491	513	496	500
z_{14}	42	63	46	60	50	50
z_{15}	35	34	31	33	34	36
z_{16}	3	4	4	4	4	4
z_{17}	4	5	8	7	6	7
z_{18}	16	22	17	8	17	17

Table 3. Spring realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{113}	θ_{bb+1}^{114}	θ_{bb+1}^{115}	θ_{bb+1}^{116}	θ_{bb+1}^{117}	θ_{bb+1}^{118}
z_1	45	45	40	20	33	50
z_2	2	2	2	2	2	3
z_3	35	36	36	36	37	35
z_4	51	51	51	49	53	44
z_5	595	495	504	507	498	483
z_6	34	39	38	39	38	35
z_7	7	8	7	10	8	8
z_8	18	16	13	3	15	6
z_9	75	77	60	73	82	118
z_{10}	5	2	2	2	3	4
z_{11}	4	4	4	4	4	4
z_{12}	24	24	25	24	23	22
z_{13}	582	491	499	488	464	484
z_{14}	72	50	48	50	48	52
z_{15}	34	35	35	34	35	34
z_{16}	5	5	5	4	4	4
z_{17}	7	7	6	4	4	7
z_{18}	26	40	21	34	40	35

Table 4. Spring realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{119}	θ_{bb+1}^{120}	θ_{bb+1}^{121}	θ_{bb+1}^{122}	θ_{bb+1}^{123}	θ_{bb+1}^{124}
z_1	43	15	45	57	97	68
z_2	2	2	3	2	2	3
z_3	34	34	36	36	39	36

z_4	51	52	50	53	53	54
z_5	497	504	507	503	500	492
z_6	37	36	37	34	38	40
z_7	7	8	8	8	7	9
z_8	9	25	19	31	30	35
z_9	71	55	30	24	34	41
z_{10}	2	2	3	3	2	5
z_{11}	4	4	4	4	4	4
z_{12}	23	22	22	22	26	22
z_{13}	498	496	505	595	493	499
z_{14}	47	53	51	61	61	48
z_{15}	31	32	33	46	34	34
z_{16}	5	5	3	4	6	6
z_{17}	5	5	7	5	4	5
z_{18}	28	22	8	2	12	13

Table 5. Realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{125}	θ_{bb+1}^{126}	θ_{bb+1}^{127}	θ_{bb+1}^{128}	θ_{bb+1}^{129}	θ_{bb+1}^{130}
z_1	58	35	45	75	72	62
z_2	3	4	3	3	2	3
z_3	37	36	35	39	37	36
z_4	67	51	50	62	49	48
z_5	573	498	506	576	494	505
z_6	36	37	35	38	38	36
z_7	8	7	5	7	10	9
z_8	25	11	17	31	23	25
z_9	55	55	43	45	52	48
z_{10}	3	3	3	3	2	3
z_{11}	4	4	5	5	4	5
z_{12}	23	22	23	26	23	23
z_{13}	573	497	531	500	492	496
z_{14}	58	51	54	47	40	51
z_{15}	34	35	33	35	35	34
z_{16}	5	5	6	5	4	6
z_{17}	4	5	5	5	7	6
z_{18}	18	20	11	10	16	18

Table 6. Realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{131}	θ_{bb+1}^{132}	θ_{bb+1}^{133}	θ_{bb+1}^{154}	θ_{bb+1}^{135}	θ_{bb+1}^{136}
z_1	37	44	46	78	59	65
z_2	6	3	2	2	2	2
z_3	37	36	36	37	36	36
z_4	64	51	53	63	55	53
z_5	576	495	502	574	492	497
z_6	35	39	37	36	38	37
z_7	10	6	7	7	6	6
z_8	23	15	18	19	18	24
z_9	50	58	53	30	30	45
z_{10}	2	2	3	3	2	2
z_{11}	4	5	4	5	4	4
z_{12}	24	23	24	23	28	24
z_{13}	590	508	520	502	508	508
z_{14}	47	47	56	47	46	42
z_{15}	33	34	35	36	35	35
z_{16}	5	5	4	4	5	4
z_{17}	5	6	6	10	5	4
z_{18}	25	18	12	12	17	14

Table 7. Realizations of conditional sojourn times θ_{bl}^{1} at operations states

Operation state z_b	θ_{bb+1}^{137}	θ_{bb+1}^{138}	θ_{bb+1}^{139}	θ_{bb+1}^{140}	θ_{bb+1}^{141}	θ_{bb+1}^{142}
z_1	53	25	55	84	71	67
z_2	2	2	3	2	2	2
z_3	38	37	40	36	37	34
z_4	60	49	46	57	53	51
z_5	584	504	505	573	494	495
z_6	38	35	36	39	36	36
z_7	5	7	5	5	6	6
z_8	15	6	40	28	32	28
z_9	70	35	35	47	40	50
z_{10}	2	2	3	3	3	2
z_{11}	5	4	5	5	4	4
z_{12}	25	25	24	23	26	24
z_{13}	595	506	535	506	503	503

z_{14}	42	45	47	46	51	43
z_{15}	34	35	34	34	33	33
z_{16}	6	4	4	5	5	4
z_{17}	5	3	4	5	3	5
z_{18}	20	11	11	10	13	18

5.1.3.2. Winter data collection of the Stena Baltica ferry operation process

The winter experiment on the ferry operation process is characterized by the following parameters:

- the ferry operation process experiment time is $\Theta_2 =$ 40 days,

- the number of the ferry operation process realizations is $n_2(0) =$ 40,

- the numbers the realizations θ^{2k}_{bb+1}, $b = 1,2,...,17,$ and θ^{2k}_{181} of the ferry conditional sojourn times in particular operation states are $n^2_{bl} =$ 40.

The realizations θ^{2k}_{bb+1}, $b = 1,2,...,17,$ $k = 1,2,...,40,$ of the conditional sojourn time θ^2_{bb+1} for $b = 1,2,...,17,$ are given in *Tables 8-14* in the *b*-th rows and the realizations and θ^{2k}_{181}, $k = 1,2,...,40,$ of the conditional sojourn time $\theta^2_{18\,1}$, are given in *Tables 8-14* in the 18-th rows.

Table 8. Winter realizations of the ferry conditional sojourn times θ^2_{bl} at operations states

Operation state z_b	θ^{21}_{bb+1}	θ^{22}_{bb+1}	θ^{23}_{bb+1}	θ^{24}_{bb+1}	θ^{25}_{bb+1}	θ^{26}_{bb+1}
z_1	65	60	15	36	63	65
z_2	2	2	2	2	3	2
z_3	35	36	39	35	39	3
z_4	49	46	50	51	52	53
z_5	516	690	570	514	539	590
z_6	39	34	38	36	34	40
z_7	5	4	6	5	4	8
z_8	21	20	11	22	17	28
z_9	30	27	80	47	22	37
z_{10}	3	3	2	2	2	3
z_{11}	4	5	5	4	4	4
z_{12}	26	26	28	28	27	27
z_{13}	497	436	595	506	520	493
z_{14}	54	44	45	46	60	60
z_{15}	31	31	33	30	29	39
z_{16}	4	4	5	5	5	4
z_{17}	5	5	5	4	5	4
z_{18}	10	20	22	18	12	20

Table 9. Winter realizations of the ferry conditional sojourn times θ^2_{bl} at operations states

Operation state z_b	θ^{27}_{bb+1}	θ^{28}_{bb+1}	θ^{29}_{bb+1}	θ^{210}_{bb+1}	θ^{211}_{bb+1}	θ^{212}_{bb+1}
z_1	48	55	60	37	62	20
z_2	2	2	2	2	2	2
z_3	38	37	39	39	40	35
z_4	54	46	51	52	51	54
z_5	505	536	601	508	507	509
z_6	35	34	40	32	34	35
z_7	5	4	9	6	5	10
z_8	37	29	22	12	19	9
z_9	28	30	80	37	40	65
z_{10}	3	2	2	2	2	2
z_{11}	4	4	5	5	5	4
z_{12}	25	26	28	28	27	25
z_{13}	504	493	565	498	534	505
z_{14}	48	61	70	52	55	46
z_{15}	31	30	29	29	32	32
z_{16}	5	4	4	4	5	5
z_{17}	4	4	4	5	5	5
z_{18}	25	23	45	10	25	18

Table 10. Winter realizations of the ferry conditional sojourn times θ^2_{bl} at operations states

Operation state z_b	θ^{213}_{bb+1}	θ^{214}_{bb+1}	θ^{215}_{bb+1}	θ^{216}_{bb+1}	θ^{217}_{bb+1}	θ^{218}_{bb+1}
z_1	40	37	41	12	33	37
z_2	2	3	2	2	2	3
z_3	35	33	33	31	32	31
z_4	51	49	51	50	50	49
z_5	512	510	511	517	510	507
z_6	37	39	38	36	35	33

z_7	7	5	7	5	6	6
z_8	15	10	11	2	5	12
z_9	77	62	62	76	64	33
z_{10}	2	2	2	2	2	2
z_{11}	4	4	5	5	4	5
z_{12}	24	24	25	23	24	22
z_{13}	509	514	506	513	522	529
z_{14}	45	51	47	43	51	45
z_{15}	33	34	34	38	32	33
z_{16}	5	4	4	5	5	4
z_{17}	4	3	5	4	4	3
z_{18}	42	30	33	33	29	23

Table 11. Winter realizations of the ferry conditional sojourn times θ_{bl}^{2} at operations states

Operation state z_b	θ_{bb+1}^{219}	θ_{bb+1}^{220}	θ_{bb+1}^{221}	θ_{bb+1}^{222}	θ_{bb+1}^{223}	θ_{bb+1}^{224}
z_1	25	19	75	44	67	75
z_2	2	2	5	2	3	2
z_3	31	27	38	33	37	34
z_4	45	71	60	50	51	47
z_5	524	537	518	531	520	630
z_6	34	34	41	41	35	37
z_7	7	6	7	3	6	7
z_8	3	2	8	13	22	36
z_9	54	40	31	20	20	19
z_{10}	3	3	2	2	2	7
z_{11}	5	4	4	5	3	5
z_{12}	22	21	24	30	24	24
z_{13}	484	521	527	523	508	454
z_{14}	52	57	46	53	53	45
z_{15}	33	34	35	33	33	34
z_{16}	4	4	4	4	4	3
z_{17}	7	5	5	4	4	5
z_{18}	32	25	7	5	5	9

Table 12. Winter realizations of the ferry conditional sojourn times θ_{bl}^{2} at operations states

Operation state z_b	θ_{bb+1}^{225}	θ_{bb+1}^{226}	θ_{bb+1}^{227}	θ_{bb+1}^{228}	θ_{bb+1}^{229}	θ_{bb+1}^{230}
z_1	59	90	69	65	65	50
z_2	3	2	2	2	3	3
z_3	34	39	38	35	40	35
z_4	76	46	54	47	64	53
z_5	537	528	505	529	569	516
z_6	36	41	36	39	38	41
z_7	7	6	10	14	7	6
z_8	24	25	26	17	17	24
z_9	24	23	19	10	33	20
z_{10}	2	7	3	3	2	2
z_{11}	4	3	3	3	5	4
z_{12}	27	27	25	26	27	29
z_{13}	444	498	505	497	497	494
z_{14}	47	52	54	54	58	58
z_{15}	34	35	34	34	33	34
z_{16}	4	4	4	4	5	5
z_{17}	7	6	7	7	7	6
z_{18}	8	12	2	4	12	6

Table 13. Winter realizations of the ferry conditional sojourn times θ_{bl}^{2} at operations states

Operation state z_b	θ_{bb+1}^{231}	θ_{bb+1}^{232}	θ_{bb+1}^{233}	θ_{bb+1}^{234}	θ_{bb+1}^{235}	θ_{bb+1}^{236}
z_1	48	34	18	61	80	57
z_2	2	2	2	2	3	2
z_3	39	39	35	37	42	35
z_4	53	06	48	54	52	53
z_5	515	589	514	506	529	507
z_6	39	38	40	39	39	39
z_7	7	6	5	7	9	7
z_8	17	16	7	12	15	23
z_9	32	71	30	30	33	42
z_{10}	2	3	4	3	5	3
z_{11}	4	4	4	5	4	4
z_{12}	27	26	27	36	25	29
z_{13}	506	595	535	574	509	511

z_{14}	55	49	53	60	53	46
z_{15}	36	34	35	36	35	34
z_{16}	4	4	4	4	4	5
z_{17}	5	5	6	8	10	8
z_{18}	6	35	7	2	12	20

Table 14. Winter realizations of the ferry conditional sojourn times θ_{bl}^{2} at operations states

Operation state z_b	θ_{bb+1}^{237}	θ_{bb+1}^{237}	θ_{bb+1}^{239}	θ_{bb+1}^{240}
z_1	46	33	15	53
z_2	2	3	3	2
z_3	37	38	38	38
z_4	51	62	51	48
z_5	512	582	515	512
z_6	39	43	42	35
z_7	7	8	7	8
z_8	15	15	5	14
z_9	48	72	25	26
z_{10}	3	3	2	3
z_{11}	4	4	4	4
z_{12}	27	31	29	29
z_{13}	506	586	833	580
z_{14}	58	51	49	62
z_{15}	35	34	36	35
z_{16}	4	4	4	5
z_{17}	6	6	6	5
z_{18}	10	23	8	4

5.1.4. Stena Baltica ferry operation process uniformity analysis

We use the two-sample Smirnov-Kolmogorov test described in Section 3.4 to verify the hypotheses that spring and winter data sets consisted of the ferry conditional sojourn times in particular operation states are from the population with the same distribution.

The procedure of testing the uniformity of data sets given in *Tables 1-7* for spring and in *Tables 8-14* for winter is illustrate on the example of the spring realizations θ_{12}^{1k}, $k = 1,2,...,42$, and the winter realizations θ_{12}^{2k}, $k = 1,2,...,40$, of the spring conditional sojourn times θ_{12}^{1} and the winter conditional sojourn time θ_{12}^{2} in the operation state z_1 while the next operation state is z_2.

For spring data, the ordered sample of realizations θ_{12}^{1k} taken from the first rows of *Tables 1-7* is

15, 20, 25, 33, 35, 37, 40, 43, 43, 44, 45, 45, 45, 45, 46, 47, 50, 50, 52, 53, 55, 55, 57, 58, 59, 60, 60, 61, 62, 62, 63, 65, 65, 67, 68, 71, 72, 75, 75, 78, 84, 97

and after applying (12)-(14), the conditional sojourn time θ_{12}^{1} has the empirical distribution function of the form

$$H_{12}^{1}(t) = \begin{cases} 0, & t \le 15, \\ 1/42, & 15 < t \le 20, \\ 2/42, & 20 < t \le 25, \\ 3/42, & 25 < t \le 33, \\ 4/42, & 33 < t \le 35, \\ 5/42, & 35 < t \le 37, \\ 6/42, & 37 < t \le 40, \\ 8/42, & 40 < t \le 43, \\ 9/42, & 43 < t \le 44, \\ 13/42, & 44 < t \le 45, \\ 14/42, & 45 < t \le 46, \\ 15/42, & 46 < t \le 47, \\ 17/42, & 47 < t \le 50, \\ 18/42, & 50 < t \le 52, \\ 19/42, & 52 < t \le 53, \\ 21/42, & 53 < t \le 55, \\ 22/42, & 55 < t \le 57, \\ 23/42, & 57 < t \le 58, \\ 24/42, & 58 < t \le 59, \\ 26/42, & 59 < t \le 60, \\ 27/42, & 60 < t \le 61, \\ 29/42, & 61 < t \le 62, \\ 30/42, & 62 < t \le 63, \\ 32/42, & 63 < t \le 65, \\ 33/42, & 65 < t \le 67, \\ 34/42, & 67 < t \le 68, \\ 35/42, & 68 < t \le 71, \\ 36/42, & 71 < t \le 72, \\ 38/42, & 72 < t \le 75, \\ 39/42, & 75 < t \le 78, \\ 40/42, & 78 < t \le 84, \\ 41/42, & 84 < t \le 97, \\ 1, & t > 97; \end{cases}$$

For winter data, the ordered sample of realizations θ_{12}^{2k} taken from the first rows of *Tables 8-14* is

12, 15, 15, 18, 19, 20, 25, 33, 33, 34, 36, 37, 37, 37, 40, 41, 44, 46, 48, 48, 50, 53, 55, 57, 59, 60, 60, 61, 62, 63, 65, 65, 65, 67, 69, 75, 75, 75, 80, 90

and after applying (15)-(17), the conditional sojourn time θ_{12}^{2} has the empirical distribution function of the form

$$H_{12}^{2}(t)=\begin{cases} 0, & t \le 12, \\ 1/40, & 12 < t \le 15, \\ 3/40, & 15 < t \le 18, \\ 4/40, & 18 < t \le 19, \\ 5/40, & 19 < t \le 20, \\ 6/40, & 20 < t \le 25, \\ 7/40, & 25 < t \le 33, \\ 9/40, & 33 < t \le 34, \\ 10/40, & 34 < t \le 36, \\ 11/40, & 36 < t \le 37, \\ 14/40, & 37 < t \le 40, \\ 15/40, & 40 < t \le 41, \\ 16/40, & 41 < t \le 44, \\ 17/40, & 44 < t \le 46, \\ 18/40, & 46 < t \le 48, \\ 20/40, & 48 < t \le 50, \\ 21/40, & 50 < t \le 53, \\ 22/40, & 53 < t \le 55, \\ 23/40, & 55 < t \le 57, \\ 24/40, & 57 < t \le 59, \\ 25/40, & 59 < t \le 60, \\ 27/40, & 60 < t \le 61, \\ 28/40, & 61 < t \le 62, \\ 29/40, & 62 < t \le 63, \\ 30/40, & 63 < t \le 65, \\ 34/40, & 65 < t \le 67, \\ 35/40, & 67 < t \le 69, \\ 36/40, & 69 < t \le 75, \\ 38/40, & 75 < t \le 80, \\ 39/40, & 80 < t \le 90, \\ 1, & t > 90. \end{cases}$$

Consequently, the null hypothesis is

H_0 : The winter and spring sets of the realizations of the conditional sojourn times θ_{12}^{1} and θ_{12}^{2} are coming from the populations with the same distribution.

To verify this hypothesis we use the two-sample Smirnov-Kolmogorov test λ at the significance level $\alpha = 0.05$.

From the table of the λ distribution for the significance level $\alpha = 0.05$ we get the critical value $\lambda_0 = u \cong 1.36$.

Using the above empirical distributions we form a common Table composed of all their values. In the Table 12, the values t_k are assuming all realizations θ_{12}^{1k}, $k = 1,2,...,42$, and θ_{12}^{2k}, $k = 1,2,...,40$, of the conditional sojourn times θ_{12}^{1} and θ_{12}^{2} i.e. they represent all discontinuity points of the empirical distribution functions $H_{12}^{1}(t)$ and $H_{12}^{2}(t)$ were they have jump in their values $H_{12}^{1}(t_k)$ and $H_{12}^{2}(t_k)$ respectively.

Table 15. The values and differences of distribution functions $H_{12}^{1}(t_k)$ and $H_{12}^{2}(t_k)$ in all of their discontinuity points

$t_k = \theta_{12}^{1k} \vee \theta_{12}^{2k}$	$H_{12}^{1}(t_k)$	$H_{12}^{2}(t_k)$	$\lvert H_{12}^{1}(t_k) - H_{12}^{2}(t_k)\rvert$
12	0	0	0
15	0	1/40	0.025
18	1/42	3/40	0.051
19	1/42	4/40	0.076
20	1/42	5/40	0.101
25	2/42	6/40	0.102
33	3/42	7/40	0.104
34	4/42	9/40	0.129
35	4/42	10/40	0.156
36	5/42	10/40	0.131
37	5/42	11/40	0.156
40	6/42	14/40	0.207
41	8/42	15/40	0.185
43	8/42	16/40	0.209
44	9/42	16/40	0.186
45	13/42	17/40	0.115

46	14/42	17/40	0.092
47	15/42	18/40	0.093
48	17/42	18/40	0.045
50	17/42	20/40	0.095
52	18/42	21/40	0.096
53	19/42	21/40	0.073
55	21/42	22/40	0.05
57	22/42	23/40	0.051
58	23/42	24/40	0.052
59	24/42	24/40	0.029
60	26/42	25/40	0.006
61	27/42	24/40	0.032
62	29/42	28/40	0.009
63	30/42	29/40	0.011
65	32/42	30/40	0.012
67	33/42	34/40	0.064
68	34/42	35/40	0.065
69	35/42	35/40	0.042
71	35/42	36/40	0.067
72	36/42	36/40	0.043
75	38/42	36/40	0.005
78	39/42	38/40	0.021
80	40/42	38/40	0.002
84	40/42	39/40	0.023
90	41/42	39/40	0.001
97	41/42	1	0.024
>97	1	1	0

Next, according to (20)-(23), from *Table 15,* we get

$$d_{42\,40} = \max_{t_k} \left| H^1{}_{12}(t_k) - H^2{}_{12}(t_k) \right| \cong 0.209,$$

and according to (23)

$$n_{12} = \frac{42 \cdot 40}{42 + 40} = 20.48.$$

Thus, the realization u_n of the statistics (10), according to (19), is

$$u_n = d_{42\,40}\sqrt{n_{12}} = 0.209\sqrt{20.48} \cong 0.946.$$

Since

$$u_n \cong 0.946 < u = 1.36,$$

then we do not have arguments to reject the null hypothesis H_0.

After proceeding in an analogous way with data in the remaining operation states we tested positively the uniformity of the spring sets of the realizations of the conditional sojourn times $\theta^1_{b\,b+1}$, $b = 2,3,...,17,$ and θ^1_{181} and the winter sets of the realizations of the conditional sojourn times $\theta^2_{b\,b+1}$, $b = 2,3,...,17,$ and θ^2_{181}. Thus, we may join the statistical data collected in spring and winter and create new statistical data sets of realizations of the conditional sojourn times $\theta_{b\,b+1}$, $b = 1,2,...,17,$ and θ_{181} with the following their operation process statistical data:

- the ferry operation process experiment time $\Theta = 82$ days,

- the number of the ferry operation process realizations $n(0) = 82$,

- the numbers the realizations $\theta^k_{b\,b+1}$, $b = 1,2,...,17,$ and θ^k_{181} of the ferry conditional sojourn times $\theta_{b\,b+1}$, $b = 1,2,...,17,$ and θ_{181} in particular operation states $n_{bl} = 82$,

- the realizations $\theta^k_{b\,b+1}$, $b = 1,2,...,17,$ and θ^{1k}_{181}, $k = 1,2,...,82,$ of the conditional sojourn times θ_{bb+1}, $b = 1,2,...,17,$ and $\theta_{18\,1}$, given in *Tables 1-14*.

After these joining the statistical data of two experiments we may go to the operation process identification proceeding accordingly to the procedures proposed in [7].

6. Testing the uniformity of statistical data from the operation processes of real complex technical systems – using computer program

The computer program is based on the methods and algorithms presented in [1] that use the Kolmogorov-Smirnov test for testing the uniformity of statistical data. The computer program allows to test the uniformity of the two sets of statistical data containing the realizations of the conditional sojourn times of the complex technical system operation process in the fixed operation state coming from two independent experiments. In addition, if the uniformity of the data is confirmed, the computer program enables joining these two data sets into one set of statistical data that can be used to carry out the identification of the distribution of conditional sojourn time of the system operation process in this fixed operation state using the computer program prepared in [4]. The computer program may be used for testing the uniformity of empirical data coming from of the operation processes of real technical systems, particularly, from the operation process of maritime transportation systems [6]. It may also be used to construct the integrated safety and reliability decision support systems for various maritime and coastal transport sectors. This program together with the description may also be included into this training course addressed to industry.

Acknowledgements

The paper describes the part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A*STAR SERC grant No. 072 1340050).

References

[1] Blokus-Roszkowska A., Guze S., Kołowrocki K., Jurdziński M., Kwiatuszewska-Sarnecka B., Milczek B., Soszyńska J., Salahuddin Habibullah M. & Fu X. (2009). Integrated package of solutions for complex industrial systems and processes safety and reliability optimization. WP 7 – Task 7.3. Methods of unknown parameters of complex technical systems operation, reliability, availability, safety models evaluation. Report. Poland-Singapore Joint Project.

[2] Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B. & Soszyńska, J. (2008). Models of safety, reliability and availability evaluation of complex technical systems related to their operation processes. WP 4 – Task 4.1. Report. Poland-Singapore Joint Project.

[3] Blokus-Roszkowska, A., Guze, S., Kołowrocki, K. & Soszyńska, J. (2010). Packages of Tools for Complex Industrial Systems and Processes Safety and Reliability Optimization. WP8 - Task 8.2. The computer program for testing uniformity of statistical data from operation processes of complex technical systems. Report. Poland-Singapore Joint Project.

[4] Blokus-Roszkowska, A., Guze, S., Kołowrocki, K. & Soszyńska, J. (2010). Packages of Tools for Complex Industrial Systems and Processes Safety and Reliability Optimization. WP8 - Task 8.1. The computer program for identification of the operation processes of complex technical systems. Report. Poland-Singapore Joint Project.

[5] Kołowrocki K. & Soszyńska J. (2009). Integrated package of solutions for complex industrial systems and processes safety and reliability optimization. WP 7 – Task 7.1. Methods of complex technical systems operation processes modeling. Report. Poland-Singapore Joint Project.

[6] Kołowrocki, K., Soszyńska, J. & Kamiński, P. (2010). Applications and Testing of Packages of Tools in Complex Maritime Transportation Systems and Processes Safety and Reliability Optimization. WP9 – Task 9.4. Stena Baltica ferry technical system operation, safety, risk and cost identification, prediction and optimization. Report. Poland-Singapore Joint Project.

[7] Soszyńska J., Kołowrocki K., Blokus-Roszkowska, A. & Guze, S. (2010). Identification of complex technical systems operation processes. Summer Safety and Reliability Seminars, Issue 4, Vol. 2, 51-66.