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Confidence Intervals for the Long-Term Noise Indicators Using the Kernel Density Estimator

Bartłomiej STĘPIEŃ

Department of Mechanics and Vibroacoustics
AGH University of Science and Technology
al. A. Mickiewicza 30, 30-059 Kraków, Poland; e-mail: Bartlomiej.Stepien@agh.edu.pl

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A non-classical model of interval estimation based on the kernel density estimator is presented in this paper. This model has been compared with interval estimation algorithms of the classical (parametric) statistics assuming that the standard deviation of the population is either known or unknown. The non-classical model does not have to assume belonging of random sample to a normal distribution. A theoretical basis of the proposed model is presented as well as an example of calculation process which makes possible determining confidence intervals of the expected value of long-term noise indicators L_{DEN} and L_{N}. The statistical analysis was carried out for 95% interval widths obtained by using each of these models. The inference of their usefulness was performed on the basis of results of non-parametric statistical tests at significance level $\alpha = 0.05$. The data used to illustrate the proposed solutions and carry out the analysis were results of continuous monitoring of traffic noise recorded in 2004 in one of the main arteries of Kraków in Poland.

Keywords: long-term noise indicators; non-classical statistics; interval estimation; kernel density estimator.

1. Introduction

Directive 2002/49/EC of the European Parliament (2002) obligates the European Union countries to implement the common long-term policy of the environment protection against noise. Its realisation is based on the estimation of long-term noise indicators in the areas under protection. The two basic indicators are: the average A-weighted long-term day-evening-night level L_{DEN}, and the average A-weighted long-term night-time level L_{N}.

The basis for creating noise maps for sites under protection are the values of the above-mentioned long-term noise indicators. Any plans to prevent and reduce the harmful effects of noise in the environment are then associated with their values. These indicators characterise the acoustic climate over a long period. Most often it is assumed that this is one full calendar year, so values of the indicators depend on many factors (i.e. traffic intensity, structure of the vehicle stream, average vehicle velocity, type and technical condition of the road surface, distance of the nearest buildings from the road edge, technical condition of the vehicles). Estimation of long-term noise hazard indicators requires access to results of an all-year-long sound level monitoring program. In practice, it is almost impossible to meet such a requirement. Therefore estimations of indicators are usually done on the basis of highly limited random sample. They are obtained as results of environmental sampling inspections. Sample size n is very small and ranges from few to a dozen or so elements.

A point estimation of noise indicators has been already performed (Kephalopoulos *et al.*, 2007; Makarewicz, 2011; Asensio *et al.*, 2011; Makarewicz *et al.*, 2014). It should be noted that the probability of point estimation of a parameter being equal to the actual value of the estimated parameter is close to zero. There is no information about the distance between expected value of the estimated parameter and the true value of the population parameter in the point estimation. Overrating or underestimating values of noise indicators can have notable social and financial consequences.

For this reason, it seems to be necessary to examine the issue of confidence intervals of the expected value of long-term noise indicators. Because the point estimate is unlikely to be exactly correct, a range of values is usually specified in which the population parameter is likely to be. The confidence interval will include the true value of the population parameter with some probability. The interval estimation takes into account the estimation error for a given confidence level, as opposed to the point estimation.

For this reason, the interval data analysis is used in acoustics. This approach has been successfully applied, among other things, to real-time analysis of acoustic signal (Heiss, Krapf, 2007). The interval arithmetic finds application in modelling the railway noise (Batko, Pawlik, 2012a) and in determination of other acoustic parameters such as reverberation time of rooms (Batko, Pawlik, 2012b) and partitions sound insulation (Batko, Pawlik, 2013a) and its uncertainty (Batko, Pawlik, 2013b). However, interval estimation algorithms based on bootstrap resampling method are used in the analysis of long-term noise indicators (Stępień, 2016).

Results of acoustical measurements usually do not meet such assumptions as normality of measurements' results, adequate sample size, lack of correlation among elements of the sample, or observation equivalence. According to Don and Rees (1985), Tang and Au (1999), Batko and Stępień (2010; 2011; 2014), Giménez and González (2009), the assumption on normal distribution of measurement results is in general false. Additionally, Wszołek and Kłaczyński (2006) proved that the road traffic noise probability distributions are not related to any statistical distribution known in the literature. However, the probability density function of the average long-term sound levels shows some asymmetry (Batko, Przysucha, 2011).

In practice, it is necessary to estimate the average long-term noise indicators $L_{\mathrm{Aeq.LT}}$ on the basis of environment sampling inspections (Schomer, DeVor, 1981; Gaja *et al.*, 2003; Romeu, 2006). Moreover, samples from inspections are small and correlated. Extra-statistical information concerning occurrence of certain noise expositions in the environment, especially at night-time (more than one maximum) also discredits this assumption.

For this reason, it seems to be necessary to implement solutions of non-classical statistics for solving these problems. This technique is based on nonparametric statistical method, allowing to determine the distribution of a random variable without any information on belonging or not to any specific class of distributions and with a limited sample size. For this reason, in what follows it is proposed to use kernel density estimator for constructing confidence intervals. The kernel density estimator has been successfully applied to point estimation of expected value and uncertainty of noise indicators (Batko, Stępień, 2009; 2014; Batko *et al.*, 2015).

This model has been compared with interval estimation algorithms of the classical (parametric) statistics assuming that the standard deviation of the population is either known or unknown.

Discussion of the algorithms, together with an example illustrating their functioning, will be presented further in this paper. The reference base comprises the results of the constant noise monitoring recorded in 2004 in one of the main arteries of Kraków in Poland.

2. Selected models of interval estimation

While analysing the measurement data we need to remember that estimation of the mean value and standard deviation of the normal distribution or estimation of an exponential distribution parameter is basically equivalent to the estimation of the probability distribution of population from which the random sample is taken. Actually, the estimation of the aforementioned parameters is equivalent to estimation of the density function of population. The fact that to estimate the probability density function it suffices to calculate a finite number of numerical estimators is the result of an assumption of a relatively accurate knowledge of a probabilistic model which governs the examined phenomenon – we have assumed that we know this model with the accuracy to a finite number of numerical parameters. In the cases presented above, the estimation of parameters which define the unknown distribution of population can therefore be called a parametric estimation of probability distribution. The parametric estimation requires an adequate random sample size. In practice $n \geq 30$ is often considered an adequate random sample size (Koronacki, Mielniczuk, 2004).

At the same time, a histogram is another estimation of the unknown population density. Being discrete, a histogram can be replaced with a continuous estimator, e.g., an adequate kernel density estimator or an estimator based on splines. These estimators of unknown probability density function do not need any assumptions about the sought form of a function and therefore are called non-parametric estimators. This family includes also distribution estimators based on the jackknife and bootstrap methods and on the Bayes' theorem widened to include the probability distributions. The basic advantage of non-parametric estimators is possibility to draw inference from a small random sample of a few to a dozen elements which does not have asymptotic properties. It is due to this advantage that non-parametric estimators are increasingly used in the probabilistic analysis of environmental noise.

Below, two generally used parametric models and one non-parametric model based on the kernel probability density estimator (*kernel* model) will be presented in detail. The discussed parametric models are based on the assumption that the analysed sample comes from the normal distribution population with a known ($N\sigma_{\mathrm{k}}$ model) or unknown ($N\sigma_{\mathrm{u}}$ model) standard deviation.

2.1. Parametric models ($N\sigma_{\mathrm{k}}$ and $N\sigma_{\mathrm{u}}$ models)

Consider a random sample $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ from a normal distribution $N(\mu, \sigma)$ with a known standard deviation σ ($N\sigma_{\mathrm{k}}$ model). It is known that the mean from the sample $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ has a normal distribution $N\left(\mu, \sigma/\sqrt{n}\right)$. Therefore, the random variable

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \approx N(0,1) \tag{1}$$

has a standard normal distribution $N(0,1)$ (Koronacki, Mielniczuk, 2004). The interval to which values of random variable Z belong with probability $1-\alpha$, where α is a known number from the interval (0, 1) is given

$$p\left(z_{\alpha/2} \le Z \le z_{1-\alpha/2}\right) = 1-\alpha, \tag{2}$$

where $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are the $(100 \cdot \alpha/2)$th and $100 \cdot (1-\alpha/2)$th percentile points of a standard normal distribution, respectively. These values are given in the standard normal table (e.g. $z_{0.025} = -1.960$). After substituting the right-hand side of the expression (1) in place of Z and rearranging, the confidence interval for the $N\sigma_{\mathrm{k}}$ model is given by (Koronacki, Mielniczuk, 2004)

$$p\left(-z_{1-\alpha/2} \le Z \le z_{1-\alpha/2}\right) = p\left(\overline{x} - z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{x} + z_{1-\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha. \tag{3}$$

Most frequently, the standard deviation of population density is unknown ($N\sigma_{\mathrm{u}}$ model). Therefore, the random variable Z given by the Eq. (1) can be replaced with the random variable (Koronacki, Mielniczuk, 2004)

$$T = \frac{\overline{x} - \mu}{S/\sqrt{n}} \approx t_{n-1}. \tag{4}$$

This idea is not only natural but appropriate as well because the distribution of the random variable T does not depend on the unknown parameter σ and is known. Namely, it is a so-called t-distribution (also called Student's distribution or Student's t-distribution) with $n-1$ degrees of freedom.

Knowing the random variable T and its distribution t_{n-1}, the confidence interval for μ can be written analogously to the previous case. The $N\sigma_{\mathrm{u}}$ confidence interval of intended coverage $1-\alpha$ is defined by (Koronacki, Mielniczuk, 2004) as

$$p\left(\overline{x} - t_{1-\alpha/2,n-1}\frac{\widehat{s}}{\sqrt{n}} \le \mu \le \overline{x} + t_{1-\alpha/2,n-1}\frac{\widehat{s}}{\sqrt{n}}\right) = 1-\alpha, \tag{5}$$

where $t_{1-\alpha/2,n-1}$ indicates the $100 \cdot (1-\alpha/2)$th percentile point of a t_{n-1} distribution and $p\left(T \le t_{1-\alpha/2,n-1}\right) = 1-\alpha/2$, whereas $\widehat{s}$ is an unbiased estimator of standard deviation which value is defined as

$$\widehat{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}. \tag{6}$$

2.2. Non-parametric model (kernel model)

The idea of the density estimator is to spread out the weight of a single observation in plot of the empirical density function. The histogram, then, is the picture of a density estimator that spreads the probability mass of each sample item uniformly throughout the interval (i.e., bin) it is observed in. Note that the observations are in no way expected to be uniformly spread out within any particular interval, so the mass is not spread equally around the observation unless it happens to fall exactly in the centre of the interval.

This subsection describes the kernel density estimator (Rosenblatt, 1956; Parzen, 1962) that spreads out the probability mass of each observation more fairly, not arbitrarily in a fixed interval, but smoothly around the observation, typically in a symmetric way.

Consider an observed random sample $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ from an unknown probability distribution. The kernel density estimator is defined by (Kulczycki, 2005)

$$\widehat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right), \tag{7}$$

for $\mathbf{x} = x_i, i = 1, \ldots, n$. The kernel function $K(\bullet)$ represents how the probability mass is assigned, so for the histogram, it is just a constant in any particular interval. This function controls the shape. The kernel normal function is defined as (Kulczycki, 2005) $K(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$, and was used in this experiment. The smoothing function h is a positive sequence of bandwidths analogous to the bin width in a histogram. The h controls the spread of the kernel. This parameter was calculated on the basis of the algorithm (Bowman, Azzalini, 1997) which was implemented in Matlab package.

With the probability density function, it is possible to determine the interval C in which the sought value $\widehat{x}$ occurs at the set probability $1-\alpha$. For a given value of α there are many such intervals, so we can look for the least of them C^* which satisfies the following conditions:

1) $p(x \in C^*) = 1-\alpha$,
2) for all $x_1 \in C^*$ and $x_2 \notin C^*$ we have $p(x_1) \ge p(x_2)$.

Therefore, from among all intervals C can discriminate one interval for which all points from inside the interval have a greater value of the probability density function than the points from outside of it. This means that any point from this region is more probable that any point from outside of this region.

Using the defined above regions, the kernel confidence interval can be defined as follows:

$$p\left(x_d \leq \widehat{x} \leq x_g\right) = \int_{x_d}^{x_g} \widehat{f}(x)\,\mathrm{d}x = 1-\alpha, \tag{8}$$

where x_d is the lower limit, and x_g is the upper limit of the confidence interval. The confidence interval thus determined satisfies one more condition $p(x_1) \geq p(x_2)$ which results from item 2.

3. The research material

With the intention to solve increasing environmental noise problems and broaden the knowledge about acoustic phenomena observed in the Kraków urban area, a system of continuous noise monitoring has been put into operation as early as in the year 1996. The solution was implemented by the Małopolskie Voivodeship Environment Protection Inspectorate in co-operation with academics from AGH-UST's Department of Mechanics and Vibroacoustics and Department of Robotics and Mechatronics. Location for the measuring station was selected bearing in mind the necessity to diagnose the acoustic climate in the vicinity of the most crowded traffic arteries in Kraków. The selected street crosses a dense urban development area. The measuring probe is situated in the middle of the green median belt separating two carriageways of the road with three lanes in each direction, with the traffic density exceeding 4000 vehicles per hour.

The study on usefulness and effectiveness of the above-presented algorithms in real-life situation was carried out with the use of data representing actual measurement results. To this end, A-weighted sound levels recorded by the above-mentioned noise monitoring system throughout the year 2004 were used. The analysis covered a total of 331 days of the year for which complete 24-hour-long records of the A-weighted equivalent sound levels were available. For the remaining days, the daily records were incomplete or did not exist at all.

On the ground of the recorded A-weighted sound levels, values of 24-hour day-evening-night sound levels $L_{\mathrm{DEN},i}$ and night-time sound levels $L_{\mathrm{N},i}$ were calculated which constituted the examined populations with the size 331. The values were used to determine long-term (annual) noise indicators. The obtained value of the long-term average day-evening-night sound level is $L_{\mathrm{DEN}} = 77.2$ dB with the standard deviation $\sigma(L_{\mathrm{DEN},i}) = 0.9$ dB. The long-term night-time sound level was also determined as equalling $L_{\mathrm{N}} = 69.5$ dB together with its standard deviation $\sigma(L_{\mathrm{N},i}) = 1.0$ dB.

Time plots of these quantities throughout the year are presented in Fig. 1. On the other hand, Figs. 2a and 2c show histograms of the indicators. Analysing the graphs it can be noted that they reveal features characteristic for negatively skew distributions, the fact being confirmed by calculated skewness values, which are -0.81 for $L_{\mathrm{DEN},i}$ and -0.47 for $L_{\mathrm{N},i}$. Kurtosis (excess kurtosis) for these populations is 6.38 (3.38) and 5.20 (2.20) for $L_{\mathrm{DEN},i}$ and $L_{\mathrm{N},i}$, respectively.

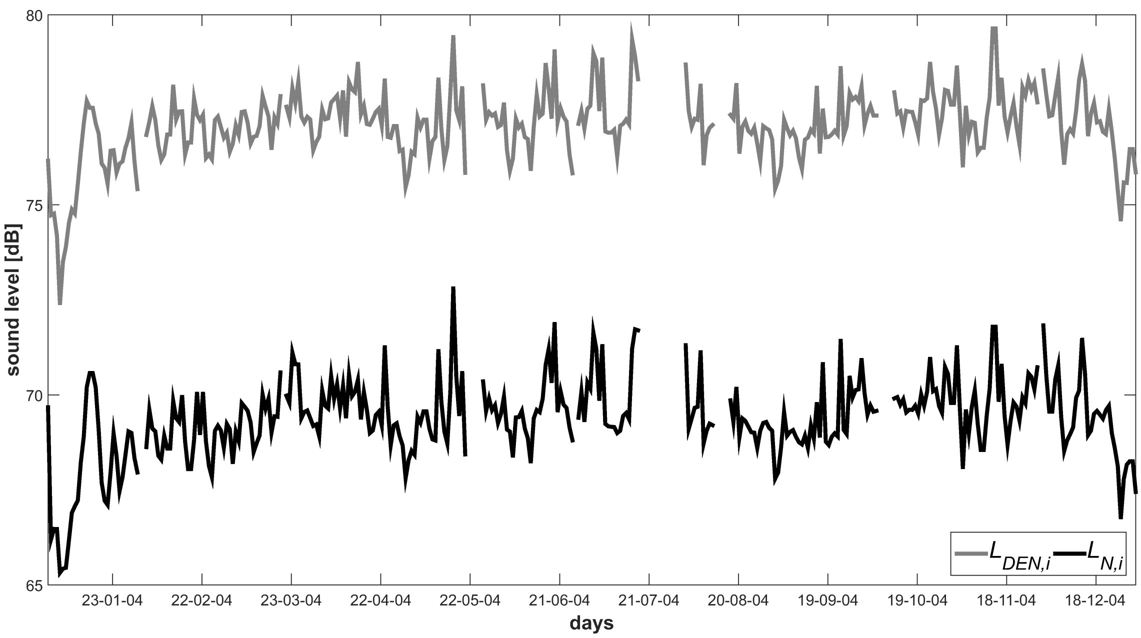

Fig. 1. Noise annoyance indicators time history during the year 2004.

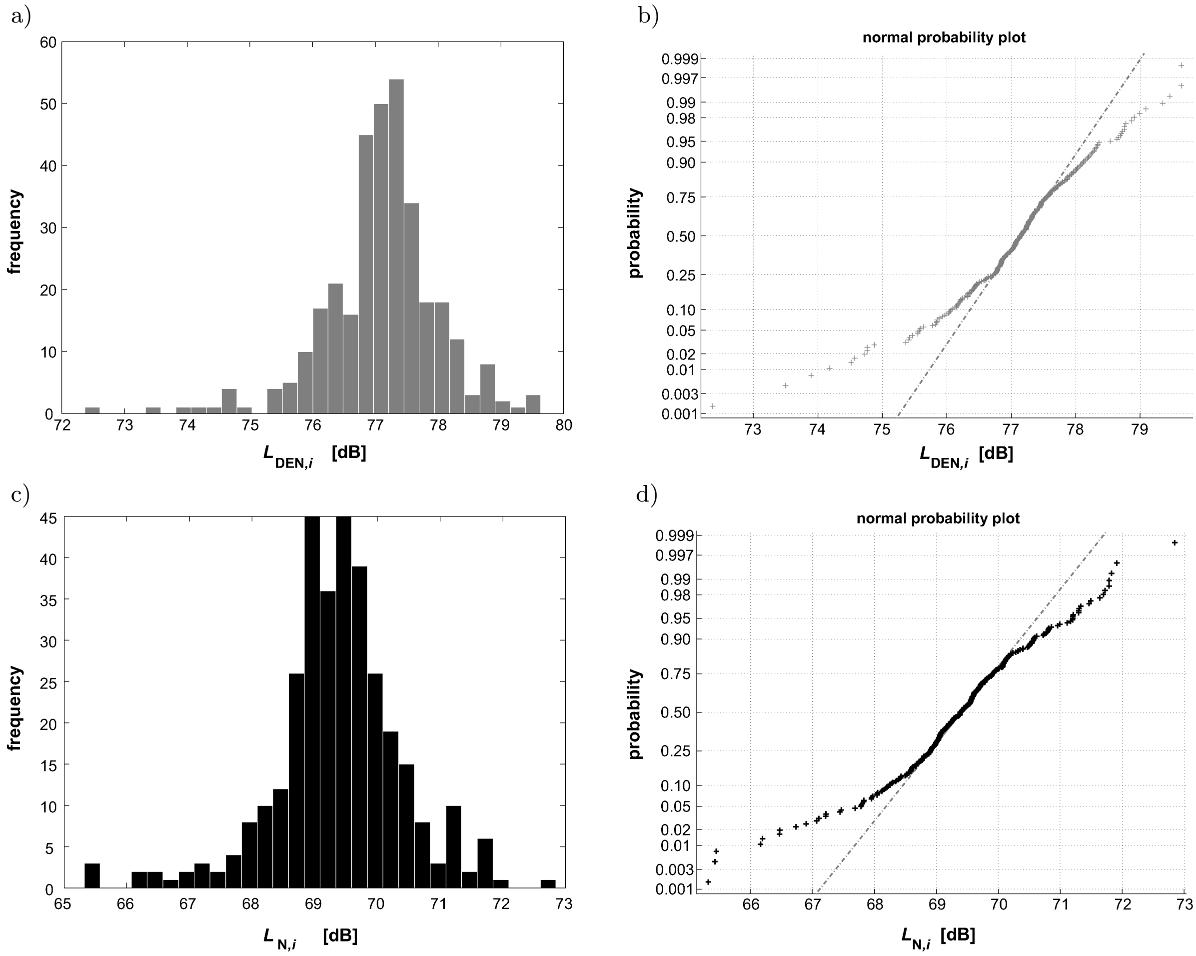

Fig. 2. Noise annoyance indicators during the year 2004: a) histogram of $L_{\mathrm{DEN},i}$; b) normal probability plot of $L_{\mathrm{DEN},i}$; c) histogram of $L_{\mathrm{N},i}$; d) normal probability plot of $L_{\mathrm{N},i}$.

The obtained values show that distributions of the examined populations are not normal. The same conclusion can be drawn from normality plots shown in Figs. 2b and 2d, as observations of the noise annoyance indicators do not follow the grey dashed lines drawn in these figures.

A number of normality tests have been carried out with the objective to confirm that the analysed population did not come from any normal distribution. The analysis included performing the Shapiro-Wilk test, the Jarque-Bera test, the Lilliefors test, and the Kolmogorov-Smirnov test at the significance level $\alpha = 0.05$. Probability values for the performed tests are summarised in Table 1. The obtained values are much lower than the assumed significance level. This is an evidence of significant "distance" between the distribution of probability of the variable describing the 24-hour day-evening-night sound levels and the normal distribution.

Table 1. p-values of normality tests.

Normality test	p-values	
	L_{DEN}	L_{N}
Shapiro-Wilk	8.49e-9	1.64e-7
Jarque-Bera	1.0e-3	1.0e-3
Lilliefors	1.0e-3	1.0e-3
Kolmogorov-Smirnov	3.64e-291	3.64e-291

4. Results of comparing the models

Results of multiple comparison of 95% confidence intervals obtained by three different techniques for constructing confidence intervals using the methods were described earlier in this paper.

One thousand simple random samples of size $n = 10$ were sampled from the populations mentioned above. The sample sizes were chosen with the intention to simulate the number of controlled days on the basis of which the levels L_{DEN} and L_{N} are to be estimated. The reconstruction of the probability density function of long-term noise indicators was done separately on the basis of each sample. This way, 1000 distributions were obtained. From these distributions, 95% confidence intervals were calculated for each of the presented models. Widths of confidence intervals were calculated as the difference between the upper and the lower confidence limits.

This way, 1000-element distributions of 95% confidence interval widths of long-term noise indicators were obtained for each of the models described in Sec. 2.

All the obtained distributions are presented in the form of histograms in Fig. 3. Panels (a) through (b) show histograms of confidence interval widths obtained for long-term day-evening-night average sound level L_{DEN}, while panels (c) through (d) correspond to long-term night-time sound level L_{N}. To make direct visual comparison easier, histograms for L_{DEN} and L_{N} were plotted in the range from 0 dB to 9 dB. All histograms were drawn with the abscissas divided into 30 intervals.

Figure 3 does not present of histograms for $N\sigma_{\mathrm{k}}$ model, because the confidence intervals obtained using this model have a constant width (lack of variation). The obtained value of the confidence interval width for the analysed populations is 1.13 dB and 1.29 dB for L_{DEN} and L_{N}, respectively.

The basic statistical parameters (minimum, maximum, mean, median, variance, kurtosis, and skewness) for the analysed populations are summarised in Table 2.

Shapes of all histograms are similar. They comprise a large group of observations clustered around the mean value and a small tail extending towards higher values. This is a feature typical for distributions showing positive skewness, which is confirmed by skewness values listed in Table 2. The tail is rather short for the all models.

Kurtosis (excess kurtosis) and skewness values for the examined populations differ significantly from those characterising the normal distribution (3 (0) and 0, respectively). The obtained values of these parameters allow to state that the examined populations have distributions differing from normal.

Model $N\sigma_{\mathrm{u}}$ is characterised with most stable results, i.e., showing the smallest variance. This means that the model is resistant to occurrence of outlying observations in the sample based on which the confidence interval for the expected value of long-term noise indicators is determined. This property is also confirmed by confidence interval widths obtained when this model is used. They fall into the ranges from

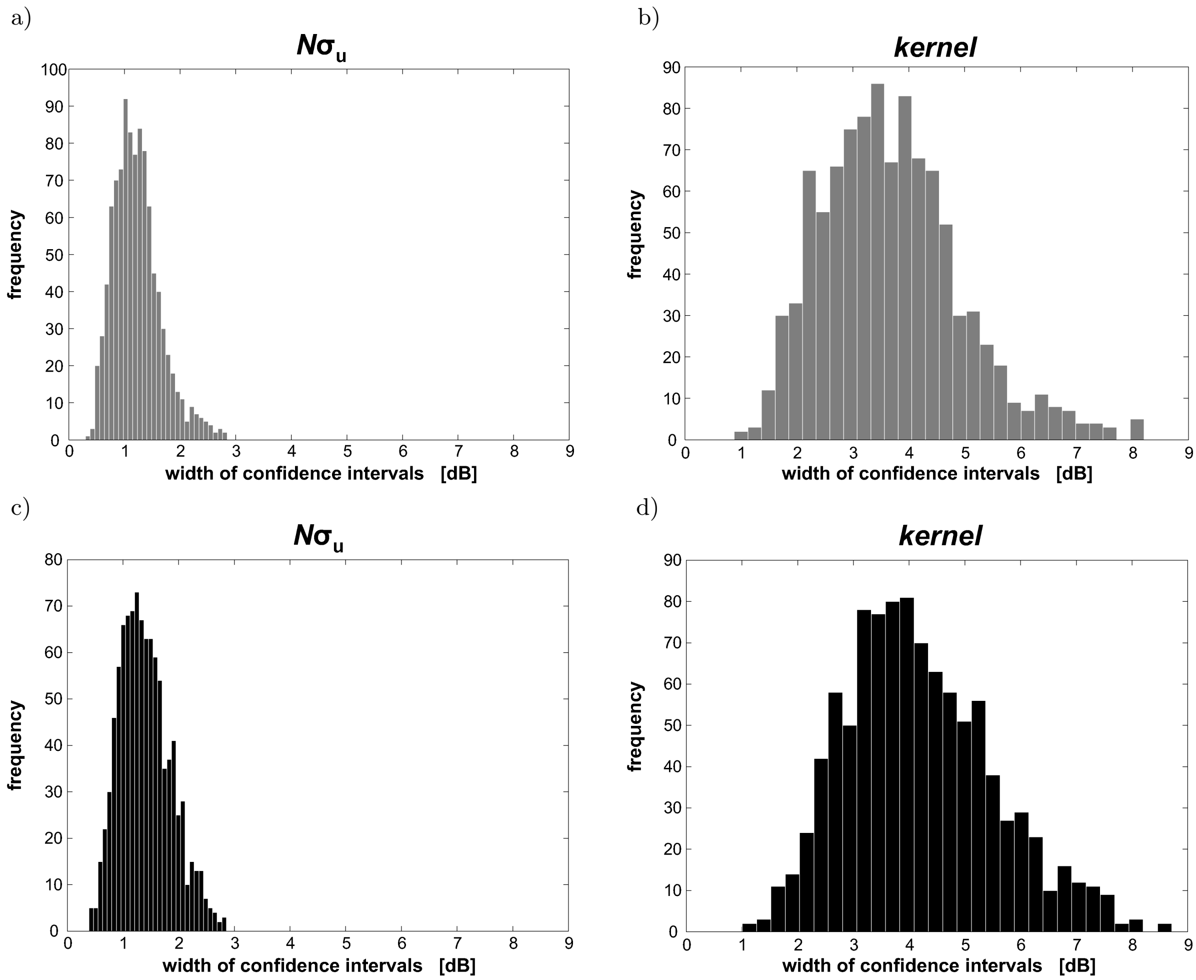

Fig. 3. Histograms of 95% confidence interval widths: a), b) for L_{DEN}; c), d) for L_{N}. These histograms were obtained using: a), c) $N\sigma_{\mathrm{u}}$ model; b), d) *kernel* model.

Table 2. Basic statistical parameters of 95% confidence interval widths for the presented models. The last column presents the coverage probability of confidence intervals. The desired coverage probability value is 95%.

Indicator	Model	Min [dB]	Max [dB]	Mean [dB]	Median [dB]	Variance [dB^2]	Skewness	Kurtosis (excess kurtosis)	Coverage probability
L_{DEN}	$N\sigma_k$	1.13	1.13	1.13	1.13	0.00	–	– (–)	96.6%
	$N\sigma_u$	0.30	2.85	1.22	1.18	0.18	0.83	3.94 (0.94)	97.6%
	kernel	0.88	8.21	3.68	3.56	1.55	0.70	3.66 (0.66)	100.0%
L_N	$N\sigma_k$	1.29	1.29	1.29	1.29	0.00	–	– (–)	95.8%
	$N\sigma_u$	0.39	2.86	1.40	1.34	0.21	0.49	2.87 (−0.13)	95.9%
	kernel	1.01	8.72	4.19	4.03	1.80	0.49	2.96 (−0.04)	100.0%

0.30 dB to 2.85 dB for L_{DEN} and from 0.39 dB to 2.86 dB for L_N. Variance of the distribution obtained with the use of *kernel* model is the largest from among those presented in this study. This is a result of poor resistance of the algorithm to outlying values occurring in the sample. The outliers have a very substantial effect on arguments of the kernel probability density function and consequently on the values of the lower and upper confidence interval limits.

When analysing the results presented in Table 2, it can be seen that the analysed populations determined with the use of presented models ($N\sigma_k$, $N\sigma_u$, *kernel*) have different statistical parameters. To determine whether the revealed differences are statistically significant, the distributions were subjected to further statistical analysis.

First, the Kruskal-Wallis non-parametric test has been performed at confidence level $\alpha = 0.05$. The test is one of the most popular alternatives for single-factor ANOVA and can be considered an extension of the Mann-Whitney U test for larger numbers of compared groups ($i > 2$). The Kruskal-Wallis test, contrary to ANOVA, fails to meet a number of restrictive assumptions.

The probability values obtained from this test were: 0 for L_{DEN} and L_N. The results are much lower than the adopted significance level and indicate that there are statistically significant differences among the compared models. To find out between which groups the differences actually occur, it was necessary to perform multiple comparisons. For this purpose, the Tukey-Kramer non-parametric test has been performed at the significance level $\alpha = 0.05$. The probability values obtained from the test are presented in Table 3, whereas a graphical representation of the multiple comparisons among analysed populations is presented in Fig. 4. The graphs show the average value of rank (symbol) together with the confidence level (horizontal line) for each of the models. Any two compared group averages are statistically different when their intervals are disjoint. Overlapping intervals mean that there are no statistically significant differences among the compared group averages.

Table 3. p-values of the Tukey-Kramer test. Values for L_N are marked in bold.

p-values			
L_{DEN}			
	$N\sigma_k$	$N\sigma_u$	*kernel*
$N\sigma_k$		0.0131	9.5606e-10
$N\sigma_u$	**0.0160**		9.5606e-10
kernel	**9.5606e-10**	**9.5606e-10**	
L_N			

The results of post-hoc tests clearly indicate that widths of confidence interval calculated with the use of presented algorithms are statistically significantly different from each other.

The confidence intervals obtained using the $N\sigma_k$ model have a constant width regardless of the structure of random sample on the basis of which they were determined. The width is 1.13 dB for L_{DEN} and 1.29 dB for L_N. This is a result of applying constant values of parameters used to determine the confidence interval on the basis of expression (3). These parameters comprise standard deviation σ, $100 \cdot (1-\alpha/2)$th percentile point of a standard normal distribution $N(0,1)$, and size of random sample n. As a result, the variance of the width distributions of confidence intervals is 0 and it is impossible to determine the skewness and kurtosis of such distributions.

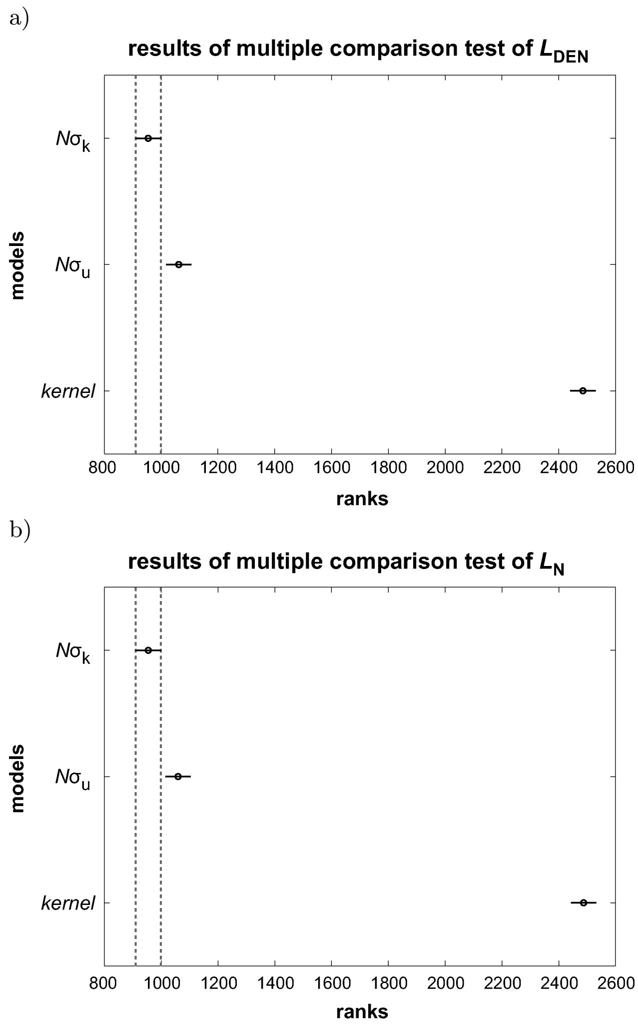

Fig. 4. Results of multiple comparisons made with the use of the Tukey-Kramer test: a) for L_{DEN}; b) for L_{N}. The model significantly different from the others is marked in bold. The upper and lower limits of the confidence interval for $N\sigma_{\text{k}}$ model used to Tukey-Kramer test are marked with dashed lines. This demonstrates that intervals for $N\sigma_{\text{k}}$ and $N\sigma_{\text{u}}$ models are disjoint.

One of the parameters used to describe properties of confidence intervals is the degree of coverage of the measured (actual) value with the determined confidence intervals. Values of this parameter fall into ranges from 96.6% to 100.0% for L_{DEN} and from 95.8% to 100.0% for L_{N}. From the coverage probability values listed in Table 2 it can be seen that they are higher from the theoretical value of 95%. In the case of the long-term average sound level L_{DEN} and long-term night-time sound level L_{N}, the highest degree of coverage at the level of 100.0% is obtained from the *kernel* model. This is a result of relatively large confidence interval widths obtained with the use of this model.

Analysing further properties of the obtained confidence intervals it should be also mentioned that percentiles of distribution $N(0,1)$ used to determine the confidence interval limits in the $N\sigma_{\text{k}}$ and $N\sigma_{\text{u}}$ methods are symmetrical with respect to 0 (zero). As a result, the obtained confidence intervals are symmetrical with respect to point estimate of long-term noise level indicators. In the *kernel* model the obtained confidence intervals are asymmetrical with respect to 0 (zero). This is the reason for which intervals shifted more or less to the left or right with respect to the point estimate are obtained. This algorithm is characterised by a higher degree of coverage with the measured (actual) value than the $N\sigma_{\text{k}}$ and $N\sigma_{\text{u}}$ methods for which the intervals are symmetrical with respect to the expected value. This asymmetry reflects probabilistic properties of the examined noise indicators and results in a better coverage of the measured (actual) value with the determined confidence level which is confirmed by results quoted in the last column of Table 2.

5. Conclusions

The non-classical model and two classical models of interval estimation were compared in this paper on the basis of widths of 95% confidence intervals. The statistical analysis was carried out on the basis of Kruskal-Wallis test. Next, multiple comparison procedures were used for pairwise comparisons between the means using non-parametric Tukey-Kramer test at significance level $\alpha = 0.05$.

Based on the simulation experiment described above it can be concluded that width differences are statistically significant for all three models, i.e., $N\sigma_{\text{k}}$, $N\sigma_{\text{u}}$, *kernel*, presented in Sec. 2, at significance level $\alpha = 0.05$.

The narrowest confidence intervals for the expected value of long-term noise indicators were obtained using the $N\sigma_{\text{k}}$ model. Interval widths calculated on the basis of *kernel* model turned out to be the largest.

The confidence intervals obtained using the $N\sigma_{\text{k}}$ model have a constant width. This is a result of applying constant values of parameters (σ, $z_{1-\alpha/2}$, n) used to determine the confidence interval in this model.

A major advantage of the *kernel* model is the lack of assumption about the probability distribution of statistics in population. Additionally, it must be emphasised that the *kernel* method can be successfully applied to a small random sample without any asymptotic properties.

The confidence intervals of the *kernel* model is asymmetric with respect to the point estimate $\widehat{x}$. These asymmetric intervals show probabilistic properties of the examined populations.

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