Paper

Rearrangeability of 2×2 W-S-W Elastic Switching Fabrics with Two Connection Rates

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Abstract—The rearreangeable conditions for the 2×2 three-stage switching fabric of a W-S-W architecture for elastic optical switches are considered in this paper. Analogies between the switching fabric considered and the three-stage Clos network are shown. On the other hand, differences are also shown, which presented the modifications required in the control algorithm used in rearrangeable networks. The rearrangeable conditions and the control algorithm are presented and proved. Operation of the proposed control algorithm is shown based on a few examples. The required number of frequency slot units in interstage links of rearrangeable switching fabrics is much lower than in the strict-sense non-blocking switching fabrics characterized by the same parameters.

Keywords—elastic optical networks, elastic optical switching nodes, interconnection networks, rearrangeable non-blocking conditions

1. Introduction

The Elastic Optical Network (EON) architecture has been proposed to utilize the bandwidth available in optical fiber more efficiently. By breaking the fixed-grid spectrum allocation limit of conventional Wavelength Division Multiplexing (WDM) networks, EONs increase flexibility in connection provisioning [1], [2]. To do so, depending on the traffic volume, an appropriately sized optical spectrum is allocated to the connections in EON. This optical spectrum is referred to as the Frequency Slot Unit (FSU).

Furthermore, unlike the rigid optical channels of conventional WDM networks [3], a light-path can expand or contract elastically to meet different bandwidth demands in EON. In this way, the incoming connection request can be served in a spectrum-efficient manner. This technological advance poses additional challenges on the networking level, especially in terms of efficient establishment of the connection.

Similarly to WDM networks, an elastic optical connection must occupy the same spectrum portion between its end nodes, that is, ensuring the so-called spectrum continuity constraint. However, when wavelength conversion (or spectrum conversion) is introduced in WDM (EON) networks, blocking probability is significantly reduced. In addition, in EONs, the entire bandwidth of each connection must be contiguously allocated. Bandwidth assigned to an optical channel depends on the required transmission data rate, distance to be covered, path-quality, wavelength spacing between channels, and/or the modulation scheme used [2], [4]–[6].

Several architectures of elastic optical switching nodes were proposed in literature [7]–[10]. In this paper, we deal with one of these switching fabric architectures, i.e. the W-S-W (wavelength-space-wavelength) switching fabric, called the WSW1 [11]. Strict-sense non-blocking (SSNB) conditions for the WSW1 architecture have been proved in [11] as well. We proposed rearrangeable non-blocking (RNB) conditions for this architecture in [12] for simultaneous routing of connections with a limited number of connection rates. The term *simultaneous connections* means that all connections arrive at the same time at all inputs, and must be served simultaneously.

Simultaneous connections can be routed using the modified matrix decomposition algorithm. Several such algorithms were proposed in literature, for instance the Neiman's algorithm [13], which consists of a relatively simple iteration phase followed by a relatively complex iterative phase. The latter is necessary only if the matrix cannot be decomposed completely after using phase one. One of the modifications to phase one of Neiman's algorithm was proposed for instance in [14]. Neiman's algorithm is used to route connections simultaneously in the three-stage Clos switching fabric [15]. The three-stage Clos network consists of two outer stages of rectangular switches, and of an inner stage of square switches. The WSW1 switching fabric can be modeled by the Clos network, as it will be shown later in this paper. However, we cannot use the same routing algorithms which are used for the three-stage Clos switching networks directly in the WSW1 switching fabric, for reasons mentioned in [12].

In our model, the number of simultaneous connection rates that can be served is limited to z. The upper bound for RNB connections when $r > 2$ was derived in [12]. The aim for using the RNB switching fabric is to reduce the required number of FSUs in the interstage links, i.e. to reduce the cost of this switching fabric. In this paper, we improve the result presented in [16]. The necessary and sufficient RNB conditions have been derived in [12] for the special case when $r = z = 2$ and $\frac{n}{m_1}, \frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers. In [16], we generalized these conditions to the general case, when $r = z = 2$ and for any values of n, m_1, and m_2. We also showed that after applying the decomposition algorithm mentioned in [12], we will get a set of connections that can be set up through the interstage links.

The main idea of this paper is to propose merge operation for matrices, and to show how to calculate the required number of FSUs in the interstage links. We also aim to present how to determine the sequence of matrices that can be merged together, satisfying the condition that each connection must use adjacent FSUs.

The remaining portions of the paper are organized as follows. In the next section, the WSW1 switching fabric is presented and the problem is described in a more detailed way. The connection model and its representation are presented as well. In Section 3, the RNB results for the proposed model are derived and proved. In Section 4 examples of the algorithm's operation are presented. The paper ends with conclusions.

2. Switching Fabrics and the Model Used

The WSW1 switching fabric considered in this paper was described in more detail in [11]. Here, we will only provide a short description which will make the paper easier to follow. This architecture is presented in Fig. 1. In the first and third stages, there are r Bandwidth-Variable Wavelength converting Switches (BV-WSs), and one Bandwidth-Variable wavelength selective Space Switch (BV-SS) of capacity $r \times r$ is in the second stage. Each BV-WS in the first stage has one input fiber with n FSUs and one output fiber with k FSUs, while each BV-WS in the third stage has one input fiber with k FSUs and one output fiber with n FSUs. The internal architecture of BV-WSs and BV-SS can be found in [11]. The switching fabric serves m-slot connections, FSUs in input/output fibers are numbered from 1 to n, BV-WSs in both input and output stages are numbered from 1 to r, and FSUs in interstage fibers are numbered from 1 to k (see Fig. 1).

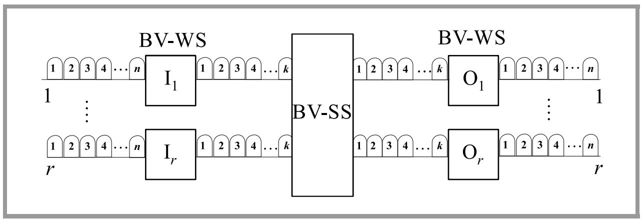

Fig. 1. The WSW1 switching fabric architecture.

In the presented considerations we assumed that BV-WSs have full range conversion capability, i.e. an m-slot connection which uses a set of m adjacent FSUs in the input fiber can be switched to a set of any other m adjacent FSUs in the output fiber. A new m-slot connection from input switch I_i to output switch O_j will be denoted by (I_i, O_j, m). When the numbers of FSUs occupied by this connection are important, the number of the first FSU will be also provided. Thus, $(I_i[x], O_j[y], m)$ denotes the m-slot connection in the input fiber of switch I_i which occupies FSUs from x to $x+m-1$, and FSUs from y to $y+m-1$ of output fiber of switch O_j. In the switching fabric, when a new connection (I_i, O_j, m) arrives, a control algorithm must find a set of m adjacent FSUs in interstage links, which can be used for this connection, and these must be FSUs with the same numbers in the interstage links from I_i and to O_j, since BV-SS has no spectrum conversion capability. In the case of the simultaneous connection model, we have a set of compatible connection requests which occupy most of FSUs in the input and output fibers, i.e. the number of free FSUs in each input/output fiber is less than m_1. This set of connections is denoted by $\mathbb{C}$ and is divided into two different types of connections: m_1 and m_2.

Example 1. Let us introduce a simple example. The set of connection requests $\mathbb{C}$ for the switching fabric of capacity 2×2 with $n = 13$ consists of eight connections (see Fig. 2). These connections are divided into two types: with $m_1 = 2$ and $m_2 = 5$. There are three m_2-slot connections and five m_1-slot connections. Additionally, one FSU remains free in input fiber no. 1.

The exact mechanism of routing these connections in the WSW1 switching fabric will be explained in detail later, in Section 3. The problem now is which FSUs in interstage

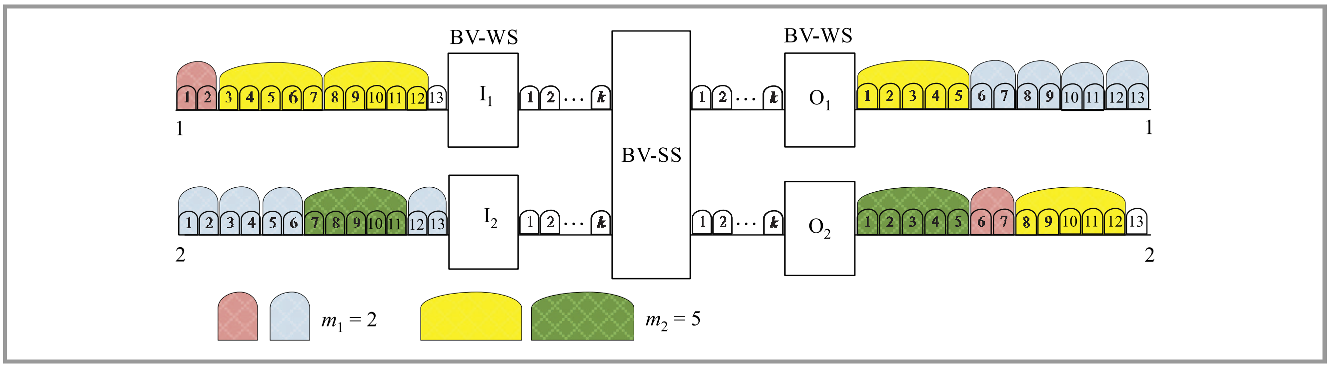

Fig. 2. The 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_2[6], 2); (I_1[3], O_1[1], 5); (I_1[8], O_2[8], 5); (I_2[1], O_1[6], 2); (I_2[3], O_1[8], 2); (I_2[5], O_1[10], 2); (I_2[7], O_1[1], 5); (I_2[12], O_1[12], 2)\}$.

links should be used by these connections, and how many FSUs are needed to set up all these connections, i.e. when the switching fabric is RNB. In [12] we proposed a control algorithm to assign FSUs to particular connection requests using the matrix decomposition algorithm, and showed the RNB conditions when $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers. The case with any number of m_1, m_2, and n is considered in Section 3.

The WSW1 switching fabric could be represented by the three-stage Clos network shown in Fig. 3. The Clos equivalent of the WSW1 switching fabric shown in Fig. 4 is presented in Fig. 5. The space switches in the first stage of the Clos network correspond to the first stage switches in the WSW1 switching fabric. Similarly, the space switches

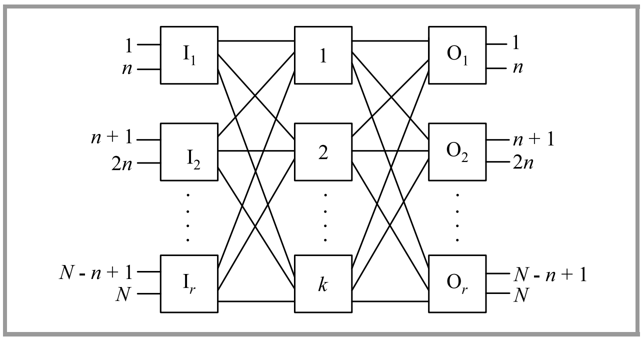

Fig. 3. Three-stage Clos network architecture.

in the third stage of the Clos network correspond to the third stage switches in the WSW1 switching fabric. Each FSU in the input fiber of the WSW1's switch I_i is represented by one input of switch I_i in the Clos network. Similarly, each FSU in the output fiber of the WSW1's switch O_j is represented by one output of switch O_j in the Clos network. In interstage links of the WSW1 fabric, each FSU corresponds to one center stage switch in the Clos network. Therefore, we have k switches in the center stage. The Clos network with these parameters can be as $C(k,n,r)$. It is known that if $k \geq n$ the Clos network is rearrangeable and if $k \geq 2n-1$ — it is strictly non-blocking [15]. The number of inputs and outputs to the Clos network is $N = nr$.

A matrix decomposition algorithm starts by deriving the $\mathbf{H}_n$ matrix of size $r \times r$, where each element $\mathbf{H}_n[i,j]$ denotes the number of connection requests at input switch I_i which are directed to output switch O_j. Because each first stage switch has n inputs, the sum of the entries in each row is n,

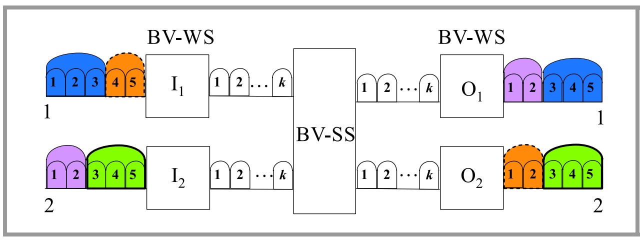

Fig. 4. 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_1[3], 3); (I_1[5], O_2[1], 2); (I_2[1], O_1[1], 2); (I_2[3], O_2[3], 3)\}$.

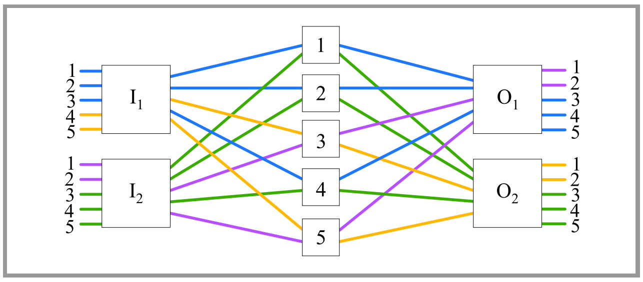

Fig. 5. Three-stage Clos network architecture with $\mathbb{C} = \{(I_1[1], O_1[3], 3); (I_1[5], O_2[1], 2); (I_2[1], O_1[1], 2); (I_2[3], O_2[3], 3)\}$.
(See color pictures online at www.nit.eu/publications/journal-jtit)

and since each last stage switch has n outputs, the sum of the entries in each column is also n.

Let us consider the WSW1 switching fabric in Fig. 4. This switching fabric serves 4 connections which occupy 2 or 3 FSUs. In Fig. 5, this WSW1 is modeled as a three-stage Clos network. In Fig. 4, we used different colors to recognize these connections. The connection marked with a solid line (blue connection) occupies 3 adjacent FSUs from I_1 to O_1, and it is represented in Fig. 5 by solid lines. Similarly, the connection marked with a dashed line (orange connection), which occupies 2 adjacent FSUs from I_1 to O_2, is marked orange in Fig. 5. Other connections from I_2 are represented in the same way. These connections can be represented by the connection matrix $\mathbf{H}_5 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

According to Neman's algorithm [13], this matrix can be decomposed into 5 permutation matrices: $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, and $\mathbf{P}_5$. Each permutation matrix represents one switch from the middle stage of the Clos network (see Fig. 5). FSUs belonging to connections represented by $\mathbf{P}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are set up through the first switch from the middle stage (or first FSUs in interstage links in the WSW1 switching fabric), FSUs belonging to $\mathbf{P}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are set up through the second switch, and so on for $\mathbf{P}_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{P}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\mathbf{P}_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. As a result, the blue connection which occupies 3 adjacent FSUs in the input link, is set up through 1st, 2nd, and 4th FSUs in the WSW1's interstage link. However, this is not correct, since these FSUs are not adjacent to each other. In general, the problem of routing connections in the WSW1 structure looks similar to routing connections in the three-stage Clos network. However, some important differences include the following:

- instead of finding connections which can be set up though one center stage switch, we have to find connections which can be set up using the same set of FSUs in the interstage link,

- connections which occupy several FSUs must use adjacent FSUs.

3. Rearrangeability Conditions

We consider 2×2 WSW1 switching fabric with the number of connection rates limited to 2, i.e. there are only m_x-slot connections, where $x = 1,2$. A set of compatible connections in $\mathbb{C}$ is represented by $\mathbf{H}^{m_x}$ matrices:

$$\mathbf{H}^{m_x} = \begin{bmatrix} h_{11}^{m_x} & h_{12}^{m_x} \\ h_{21}^{m_x} & h_{22}^{m_x} \end{bmatrix}, \qquad (1)$$

where $h_{ij}^{m_x}$ is equal to the number of m_x-slot connection requests from switch I_i to switch O_j. According to Algorithm 1 given in [12], the $\mathbf{H}^{m_x}$ matrix can be decomposed into $c_{\max}^{m_x}$ permutation matrices $\mathbf{P}_i^{m_x}$, where $c_{\max}^{m_x}$ represents the maximum number of m_x-slot connections in one input or output, while $c_{\min}^{m_x}$ represents the minimum number of such connections. We can use this algorithm to set up the set of connection requests given in Example 1 (see Fig. 2). In Table 1, steps of decomposition of the given set of connection requests are presented one by one. In the first row, connection matrices $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$ are given. In the next rows, matrices that result from decomposition of $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$ matrices are presented. For each decomposed matrix, the number of assigned FSUs are provided in second and fourth columns. As can be noticed, the number of occupied FSUs in interstage links is 18. After applying the decomposition algorithm, it is time to commence the merging operation which can reduce the required number of FSUs in the intestage links to 14. The final arrangement for the 8 connections mentioned is shown in Fig. 6.

Table 1
Assignment of FSUs to connections used in Example 1

Matrices representing m_1-slot connections	FSUs in interstage links	Matrices representing m_2-slot connections	FSUs interstage links
$\mathbf{H}^{m_1} = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}$	—	$\mathbf{H}^{m_2} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	—
$\mathbf{P}_1^{m_1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	1–2	$\mathbf{P}_1^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9–13
$\mathbf{P}_2^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	3–4	$\mathbf{P}_2^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	14–18
$\mathbf{P}_3^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	5–6	—	—
$\mathbf{P}_4^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	7–8	—	—

In [12], we proved that the WSW1 switching fabric presented in Fig. 1 with $r = 2$ is rearrangeably non-blocking when $m \in \{m_1; m_2\}$ if and only if:

$$k \geq n, \qquad (2)$$

where $m_1 < m_2$, and $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers. In this article we propose a new theorem to find value k that makes the WSW1 switching fabric RNB for a scenario of a more general nature.

Theorem 1: The WSW1 switching fabric presented in Fig. 1 with $r = 2$ is rearrangeably non-blocking for m-slot connections, where $m \in \{m_1; m_2\}$, if:

$$k \geq \left\lfloor \frac{n}{m_2} \right\rfloor \cdot m_2 + \left(\left\lfloor \frac{n}{m_1} \right\rfloor - \left\lfloor \frac{n}{m_2} \right\rfloor \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1. \qquad (3)$$

Proof: Let $\mathbb{C}$ denote a set of compatible connections. We have two connection rates, m_1 and m_2, and all connections can be represented by $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$ matrices. According to the decomposition algorithm given in [12], $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$ can be decomposed into $c_{\max}^{m_1}$ and $c_{\max}^{m_2}$ permutation matrices $\mathbf{P}^{m_x}$, respectively. Each $\mathbf{P}^{m_x}$ matrix represents a set of m_x-slot connections which can be set up using the same m_x FSUs in interstage links. From these $\mathbf{P}^{m_x}$ matrices, only $c_{\min}^{m_1}$ and $c_{\min}^{m_2}$ matrices contain exactly one value 1 per each row and each column. Other $\left(c_{\max}^{m_1} - c_{\min}^{m_1}\right)$ matrices $\mathbf{P}^{m_1}$ and $\left(c_{\max}^{m_2} - c_{\min}^{m_2}\right)$ matrices $\mathbf{P}^{m_2}$ contain some rows and/or columns with 0 values only. Permutation matrices $\mathbf{P}^{m_1}$ with 0s only in certain row(s) or column(s) can be merged with matrices $\mathbf{P}^{m_2}$ with 0s only, but in other row(s) or column(s). In this case, at most $\left(c_{\max}^{m_2} - c_{\min}^{m_2}\right)$ matrices $\mathbf{P}^{m_2}$ can be merged with at most $\left(c_{\max}^{m_1} - c_{\min}^{m_1}\right)$ matrices $\mathbf{P}^{m_1}$. The required number of FSUs in interstage links k, which allows to set up all connections simultaneously, is given by the flowing formula:

$$\begin{aligned} k \geq\, & c_{\min}^{m_1} \cdot m_1 + c_{\min}^{m_2} \cdot m_2 + \left(c_{\max}^{m_2} - c_{\min}^{m_2}\right) \cdot m_2 \\ & + \left(\left(c_{\max}^{m_1} - c_{\min}^{m_1}\right) - \left(c_{\max}^{m_2} - c_{\min}^{m_2}\right) \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 \end{aligned} \qquad (4)$$

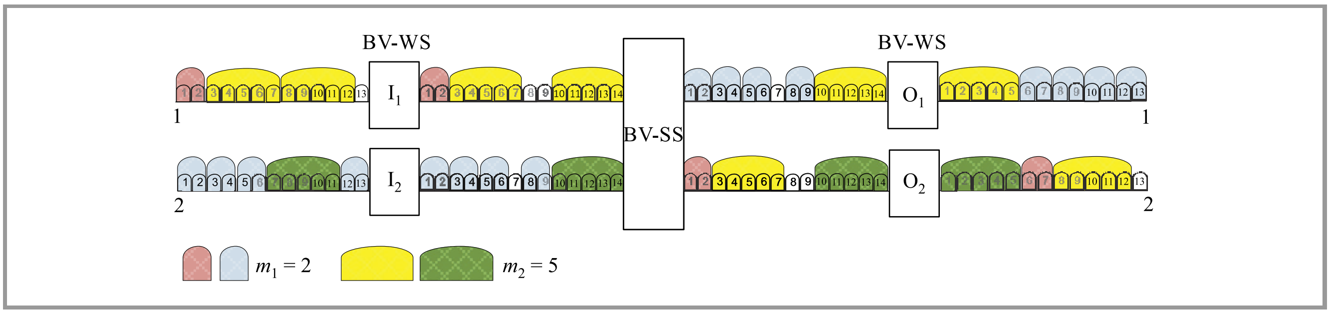

Fig. 6. 2×2 WSW1 switching fabric with set $\mathbb{C}$ of requested connections (see Fig. 1) set up with the number of FSUs decreased due to the merging operation.

Equation (4) can be simplified into the following:

$$k \geq c_{\max}^{m_2} \cdot m_2 + \left(c_{\max}^{m_1} - \left(c_{\max}^{m_2} - c_{\min}^{m_2} \right) \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 \,. \quad (5)$$

Equation (5) must be maximized through all possible sets $\mathbb{C}$. Since $c_{\max}^{m_x}$ represents the maximum number of m_x-slot connections in one of the inputs or outputs, the number of such connections in one input/output will never be greater than $\left\lfloor \frac{n}{m_x} \right\rfloor$. When $c_{\max}^{m_x}$ value is maximized, the value of $c_{\min}^{m_x}$ is minimized. When we put $c_{\max}^{m_x} = \left\lfloor \frac{n}{m_x} \right\rfloor$ and $c_{\min}^{m_x} = 0$ to Eq. (5) we get:

$$k \geq \left\lfloor \frac{n}{m_2} \right\rfloor \cdot m_2 + \left(\left\lfloor \frac{n}{m_1} \right\rfloor - \left\lfloor \frac{n}{m_2} \right\rfloor \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 \quad (6)$$

which gives number of FSUs in each interstage links. ■

4. Examples of Algorithm's Operation

Let us present a few examples rendering the idea behind the proof presented clearer. The first example for the 2×2 WSW1 switching fabric was already presented in Section 2.

Example 2. In this example the WSW1 switching fabric has the following parameters: $r = 2$, $n = 12$, $z = 2$, $m_1 = 3$, $m_2 = 5$, and the set of connection requests $\mathbb{C}$ is shown in Fig. 7. All connections from the set of requested connections $\mathbb{C}$ can be represented by matrices $\mathbf{H}^{m_1} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{H}^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. There are $c_{\max}^{m_1} = 4$ and $c_{\max}^{m_2} = 2$ permutation matrices for $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$, respectively. The matrices that do not contain one element 1 in each row and each column can be merged together because it means that they represent connections which are at different inputs and are directed to different outputs. After decomposition, the number of permutation matrices received from $\mathbf{H}^{m_1}$ and $\mathbf{H}^{m_2}$ that cannot be merged with other matrices, is equal to $c_{\min}^{m_1} = 0$ and $c_{\min}^{m_2} = 0$, respectively. But this does not mean that all of the permutation matrices can be merged together, and this is because values of $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ (or at least the third value) are not integers. Generally, we can merge only $\left\lfloor \frac{m_2}{m_1} \right\rfloor$ $\mathbf{P}^{m_1}$ matrices with one matrix $\mathbf{P}^{m_2}$, since connections in $\mathbf{P}^{m_2}$ occupy m_2 FSUs, while connections in $\mathbf{P}^{m_1}$ – only m_1 FSUs. In the presented example, we have $m_2 = 5$ and $m_1 = 3$, so $\left\lfloor \frac{m_2}{m_1} \right\rfloor = 1$ and only one $\mathbf{P}^{m_1}$ can be merged with one $\mathbf{P}^{m_2}$.

In the first step of $\mathbf{H}^{m_1}$ decomposition, we get $\mathbf{P}_1^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\mathbf{H}_1^{m_1} = \mathbf{H}^{m_1} - \mathbf{P}_1^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$. The next permutation matrix is $\mathbf{P}_2^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\mathbf{H}_2^{m_1} = \mathbf{H}^{m_1} - \mathbf{P}_2^{m_1} = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$. Finally, $\mathbf{H}_2^{m_1}$ can be decomposed into two equal permutation matrices $\mathbf{P}_3^{m_1} = \mathbf{P}_4^{m_1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

For $\mathbf{H}^{m_2}$, the first permutation matrix is $\mathbf{P}_1^{m_2} = \mathbf{P}_2^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and the second permutation matrix is obtained from $\mathbf{H}_1^{m_2} = \mathbf{H}^{m_2} - \mathbf{P}_1^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \mathbf{P}_2^{m_2}$. When no merging operation is performed, we need 22 FSUs in interstage links, i.e. four $\mathbf{P}^{m_1}$ matrices, each uses three FSUs, and two $\mathbf{P}^{m_2}$ matrices occupying five FSUs each. When two $\mathbf{P}^{m_2}$ matrices are merged with two $\mathbf{P}^{m_1}$ matrices, the number of required FSUs is reduced to 16, as shown in Fig. 7.

Example 3. In this example, most connections in I_1 are m_2-slot connections, and the rest of FSUs are used by one m_1-slot connection. In I_2, all FSUs are occupied by m_1-slot connections This switching fabric with $n = 12$, $z = 2$, $m_1 = 2$, and $m_2 = 5$, as well as the set of connection requests $\mathbb{C}$ are shown in Fig. 8. Set $\mathbb{C}$ is represented by matrices $\mathbf{H}^{m_1} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ and $\mathbf{H}^{m_2} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$. The number of permutation matrices for $\mathbf{H}^{m_1}$ is $c_{\max}^{m_1} = 6$, and for $\mathbf{H}^{m_2}$ is $c_{\max}^{m_2} = 2$. After decomposition, for $\mathbf{H}^{m_1}$ we get the following set of permutation matrices:$\mathbf{P}_1^{m_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\mathbf{P}_2^{m_1} = \mathbf{P}_3^{m_1} = \mathbf{P}_4^{m_1} = \mathbf{P}_5^{m_1} = \mathbf{P}_6^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, while for $\mathbf{H}^{m_2}$ we get two permutation matrices: $\mathbf{P}_1^{m_2} = \mathbf{P}_2^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. We can merge $\mathbf{P}_1^{m_2}$ with $\mathbf{P}_2^{m_1}$ and $\mathbf{P}_3^{m_1}$, and $\mathbf{P}_2^{m_2}$ with $\mathbf{P}_4^{m_1}$ and $\mathbf{P}_5^{m_1}$, to get number of FSUs in the interstage links $k = 14$ instead of $k = 22$.

Example 4. In the fourth example, all connections in I_1 are m_2-slot connections, and in I_2 we have one m_2-slot connection and the rest of FSUs are occupied by m_1-slot connections. This switching fabric with $n = 11$, $z = 2$, $m_1 = 2$, $m_2 = 5$, and the set $\mathbb{C}$ are shown in Fig. 9. It can be represented by matrices $\mathbf{H}^{m_1} = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}$ and $\mathbf{H}^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. The

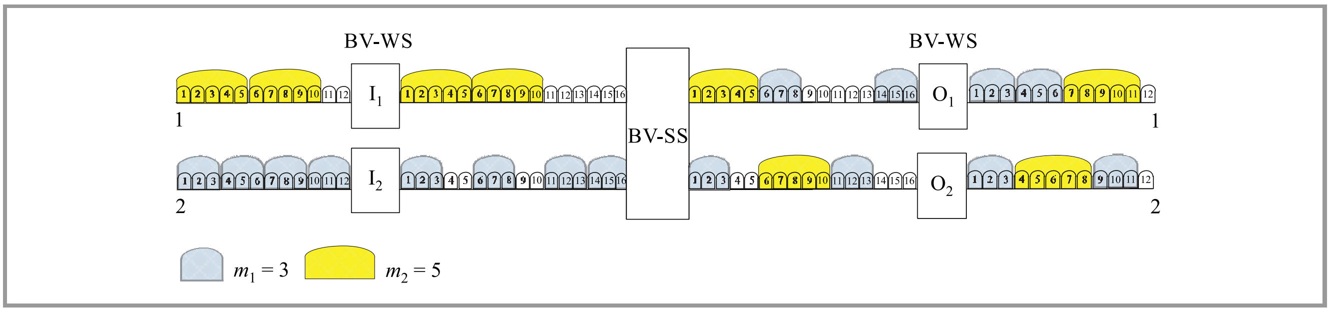

Fig. 7. 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_1[7], 5);\ (I_1[6], O_2[4], 5);\ (I_2[1], O_1[1], 3);\ (I_2[4], O_1[4], 3);\ (I_2[7], O_2[1], 3);\ (I_2[10], O_2[9], 3)\}$, where all connections from input 1 are of size m_2 and from input 2 are of size m_1.

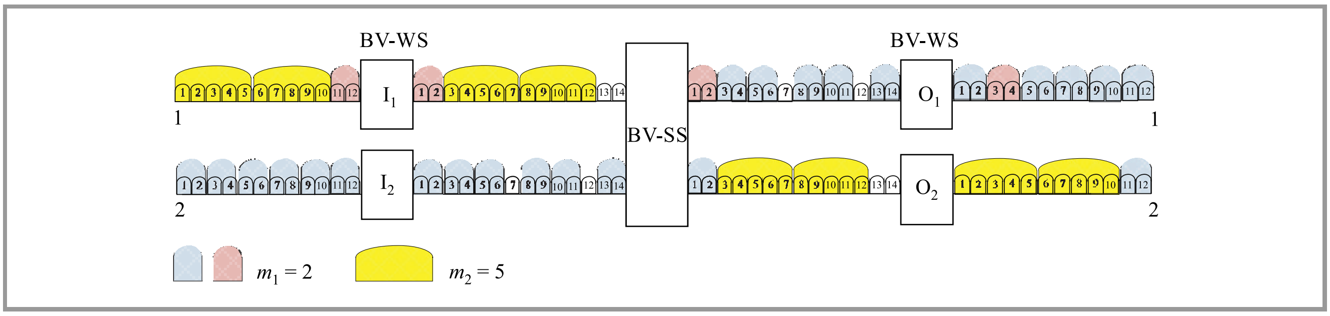

Fig. 8. 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_2[6], 5);\ (I_1[6], O_2[1], 5);\ (I_1[1], O_1[3], 2);\ (I_2[1], O_1[1], 2);\ (I_2[3], O_2[11], 2);\ (I_2[5], O_1[1], 2);\ (I_2[7], O_1[5], 2);\ (I_2[9], O_1[7], 2);\ (I_2[11], O_1[11], 2)\}$, where all connections from input 1 are of size m_2 or m_1, and from input 2 are of size m_1.

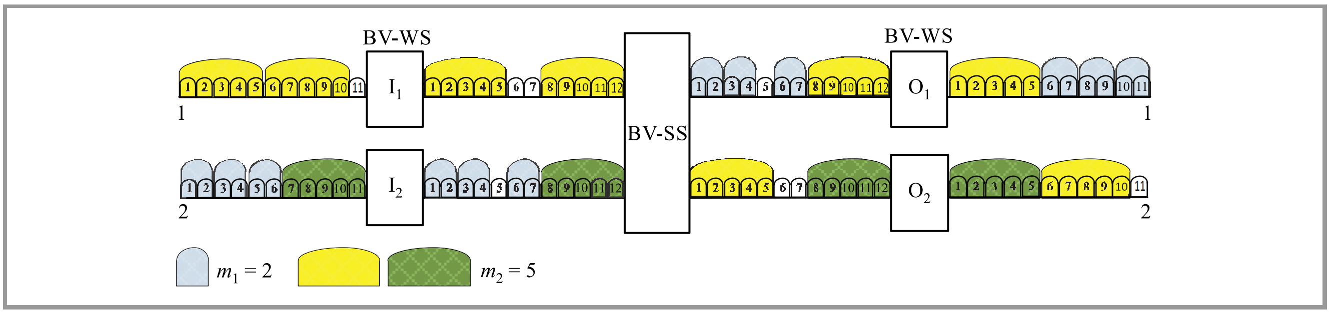

Fig. 9. 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_1[1], 5);\ (I_1[6], O_2[6], 5);\ (I_2[1], O_1[6], 2);\ (I_2[3], O_1[8], 2);\ (I_2[5], O_1[10], 2);\ (I_2[7], O_2[1], 5)\}$, where all connections from input 1 are of size m_2 and from input 2 are of size m_1 or m_2.

number of permutation matrices for $\mathbf{H}^{m_1}$ is $c^{m_1}_{\max} = 3$, and for $\mathbf{H}^{m_2}$ is $c^{m_2}_{\max} = 2$. After decomposition, the set of permutation matrices obtained from $\mathbf{H}^{m_1}$ is $\mathbf{P}^{m_1}_1 = \mathbf{P}^{m_1}_2 = \mathbf{P}^{m_1}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, while form $\mathbf{H}^{m_2}$: $\mathbf{P}^{m_2}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{P}^{m_2}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. In this example, we can merge matrix $\mathbf{P}^{m_2}_2$ with matrices $\mathbf{P}^{m_1}_1$ and $\mathbf{P}^{m_1}_2$. The number of FSUs in the interstage links is $k = 12$ instead of $k = 16$ without merging.

5. Conclusions

In this article, we considered the rearrangeability of WSW1 switching fabrics for elastic optical network nodes. Up till now, rearrangeable conditions for switching fabrics with two inputs, two outputs, two connection rates, and when $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers, have been given. We extended these conditions to the case with any relations between values of n, m_1, and m_2. We described also a few examples which show the operation of the proposed algorithms and how merging operation results in reducing the required number of FSUs in interstage links.

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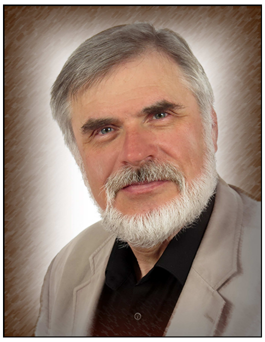

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