Jodejko Anna
Wrocław University of Technology, Wroclaw, Poland

Maintenance problems of technical systems composed of heterogeneous elements

Keywords

multi-unit system, heterogeneous system, block maintenance policy, economic dependency

Abstract

The paper gives a short insight into the complexity of maintenance problems of a system composed of heterogeneous elements. The article proposes a simple algorithm to define parameters of block maintenance policy of multi-unit systems composed of elements with economic dependency. The procedure may be used especially in the case when elements of a system are not identical in the sense of their probability characteristics and analytical solution is inaccessible.

1. Introduction

An appropriate maintenance policy may have great benefits for every technical system. Many of them consist of a great number of elements. That is why multi-unit maintenance policy is concerned with the optimal strategy for the system as a whole not for every element or for a group of elements. Such an approach is necessary when machines or pieces of equipment depend on each other and single element maintenance models seem useless and should not be used [5]. As a result, correct maintenance policy determination is not an easy task. The complexity of the problem grows when elements of the multi-unit system are not identical in any sense, e.g. their probability characteristics, criticality for the system availability or importance for its economy. In order to choose the best maintenance policy for the whole system, it is necessary to define:

- kind of dependency that exists between the system, its elements and subsystems,
- reliability characteristics of system components,
- service/repair requirements of the system.

(i) Dependency that exists between the system, its elements and subsystems

A correct maintenance policy should be optimal or near optimal from the point of view of the entire system. The knowledge of relations between its components seems necessary. It is hard to precisely define all dependencies in any multi-unit system. The difficulty increases when system elements are not the same. The number of possible dependencies between components and a system grow very quickly. Every element usually has various criticalities for system reliability and any kind of dependency may have an affect on one another: structural dependency may have an influence on economic and stochastic dependency, the relation between an elements failures may affect cost results of maintenance activities, etc. The problem is to define at least the most important dependencies and take them into account when determining an optimal maintenance policy.

(ii) Components reliability characteristics

Probability characteristics of the system elements cause serious difficulties in mathematical maintenance modelling. This fact often prevents researchers from using optimal analytical solutions. The majority of accessible analytical models are solved for exponential lifetime cases, e.g. inventory of spare parts modelling [24] or even group replacement policy [16]. Heterogeneous elements have various probability distributions for their failure times. In order to estimate all probability characteristics it is necessary to gather much more data than when the elements are identical. Components failure rate may depend on different variables (time, work done, stress, etc.), moreover, probability characteristics may diversify the best maintenance policy among elements and the challenge is to join them effectively.

(iii) Service/repair requirements of the system
There are plenty of models that deal with service/repair needs in multi-unit systems. Heterogeneous components complicate the problem because every element may generate a demand for services of a different kind (e.g. not every repairman is able to fix every failure in a complex system). Service/repair requirements also determine the demand for spare parts. When the system consists of various elements, right inventory policy (taking into account budgetary or safety limitations) is difficult to determine because of the diversified demand for components.

The above characteristics are the basis of defining groups of "similar" components and to choose the best maintenance strategy for each of them. The next chapter contains some propositions on how to deal with the problem.

2. Maintenance policy of multi-component heterogeneous systems

The survey of maintenance models for multi-unit systems may be found in well known literature, e.g.: [5], [7], [16], [17], [19], [25]. However the number of models for heterogeneous multi-unit systems with some kind of dependency is relatively low. Some general methods of multi-unit system maintenance determination may be found, among others, in: [10], [18], [20], [22], [23], [26]. Because of the complexity of the problem, there are few propositions on how to deal with maintenance policy defining in a multi-unit, heterogeneous system.
Bevilacqua and Braglia [2] suggest that criticality analysis based on FMECA technique may be the first stage of maintenance strategy determining. Authors present the list of possible criteria of elements assessment: safety, machine importance for the process, spare machine/parts availability, maintenance cost, access difficulty, failure frequency, downtime length, machine type, operating conditions, propagation effect and production loss cost. They perform the analysis in the electrical power plant, composed of thousands of various elements. Six chosen criteria are evaluated with their importance indicators:
- safety machine x 1,5,
- machine importance for the process x 2,5,
- maintenance costs x 2,
- failure frequency x 1,
- downtime length x 1,5,
- operating conditions x 1.

The factor coming from this assessment gives the base for a defining maintenance programme (*Table 1*).

Table 1. Maintenance policy selection based on FMECA analysis [2]

Criticality index	Maintenance policy
> 395	Predictive
290 - 395	Preventive
< 290	Corrective

Such analysis may be a foundation for more precise maintenance policy and its parameters determination.
Basic strategies, often applied in multi-unit systems, are: group/block/opportunistic maintenance policies with their modifications. They are applied to components that should be replaced / repaired before their expected failures (predictive and preventive policy). They assume that some maintenance activities may be joined together in order to achieve some benefits coming from system internal dependencies [16]:
- economic (economies of scale, system down-time shortening),
- stochastic (failure dependency),
- structural (all elements of a subsystem, reliability structure of a system).

Some examples of these models may be found in: [3], [4], [8], [9], [13], [21] (block/group replacements optimisations) or [1], [6], [11], [12], [28] (opportunistic maintenance policy).
Group/block/opportunistic maintenance strategies are well known in maintenance theory. Nevertheless, the majority of models are complicated, especially when system components have various probability characteristics. Its solutions are usually limited to very few special cases. There is still a need to develop easily applied methods, which allow us to find parameters of a "good" solution within a large number of possibilities. Such an attempt is made in the next chapter. The proposed algorithm may be used when one needs to define parameters of block maintenance policy in a system composed of non-identical components.

3. Block maintenance policy of a system with economic dependency

This chapter contains a contribution to a block maintenance policy of a heterogeneous system with economic dependency. The block maintenance policy assumes that groups of units in the system are replaced at periodic intervals but each unit is also replaced upon failure [5]. The policy is used, when some kind of dependency between system elements exists. A simple example of common dependency is economy of scale, when the cost of joint replacements is inversely proportional to the number of maintained elements. If the system is composed of elements, which have various probability characteristics, the problem is to find a way for elements to be grouped in blocks (preventively replaced together). The number of possible solutions is usually huge and analytical optimisation models are inaccessible. The algorithm presented below proposes a simple procedure for element grouping. The basis to start a search is a single element age replacement policy (ARP). The procedure takes into consideration a single optimisation criteria – maintenance cost minimisation. It may be the first stage in creating a simple and effective multi-criteria tool for block maintenance policy determining.

3.1. Age replacement policy

The analysed system/subsystem is composed of M elements. All elements may have various probability characteristics. Time to failure of every element is random and may be described by c.d.f. $F_i(t)$, where $i = 1, 2,\ldots, M$. Element replacement/repair times are negligible. Block replacement policy in the system is applied. All elements are replaced or perfectly repaired at failure and at moments $k\cdot T_i$, ($k = 1, 2,\ldots,\infty$; T_i - time interval between two consecutive preventive replacements of ith element). The system incurs failure cost k_u, when any element of the system fails, k_w/n_i cost of preventive replacement (k_w - single preventive replacement cost, n_i – number of elements replaced together with ith element) and the cost of the element (spare part) purchase k_{zi}. The cost k_w/n_i of preventive replacement represents an economy of scale (economic dependency) in the system.
Every ith element is characterised by variable: $F_i(t)$, T_i^*, n_i , k_{zi}.
If elements are independent, the best time interval between two preventive replacements may be obtained according to the well known formula of ARP [14], [27]. A modified expression taking into consideration the cost of the new element purchase is:

$$K_i = \frac{k_u \cdot F_i\left(T_i^*\right) + k_w \cdot \left(1 - F_i\left(T_i^*\right)\right) + k_z}{\int_0^{T_i^*}\left(1 - F_i(t)\right)dt}, \qquad (1)$$

where T_i^* is optimal time interval between two consecutive preventive replacements of ith element.
The optimal time interval has to satisfy:

$$\lambda_i\left(T_i^*\right)\cdot \int_0^{T_i^*}\left(1 - F_i(t)\right)dt - F_i\left(T_i^*\right) = \frac{k_w + k_{zi}}{k_u - k_w}, \qquad (2)$$

where

$f_i(t)$ is $\dfrac{dF_i(t)}{dt}$ and $\lambda_i(t) = \dfrac{f_i(t)}{\left(1 - F_i(t)\right)}$.

Solutions to the above equations may easily be obtained thanks to numerical methods for various probability distributions [14]. The vector of optimal times, where $T_1^* \le T_2^* \le T_i^* \le T_M^*$ is the starting point for element grouping. The total cost per unit time that the system incurs, is:

$$K = \sum_{i=1}^{M} K_i , \qquad (3)$$

3.2. The search algorithm

The proposed algorithm assumes economic dependency between elements (k_w/n_i). It means that the replacement of a few components together may be profitable for the system. The cost of preventive replacement may result for various reasons, e.g. a production line stoppage. If one is able to estimate the cost of the system unavailability, economic criteria may also express availability measure.
The possible saving should consider the lower cost of preventive replacement but also the costs of new component purchases and failure cost, which may increase. The procedure (*Figure 2*) proposes to combine replacement activities that may give the highest total savings for the system:

(i) Estimation of profit if ith element time interval T_i^* is reduced or extended to the nearest available value: T_{i-1}^* and T_{i+1}^* (*Figure 1*).

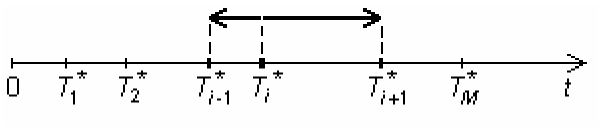

Figure 1. Reduction and extension of time interval T_i^* for ith element

Profit estimation may be calculated according to the following equations:
- shortening of time T_i^* to time T_{i-1}^*:

$$\Delta K_{i-} = \Delta K_{ui-} + \Delta K_{wi-} + \Delta K_{zi-}, \qquad (4)$$

$$\Delta K_{ui-} = k_u\left(\frac{F_i\left(T_i^*\right)}{\overline{T}_i} - \frac{F_i\left(T_{i-1}^*\right)}{\overline{T}_{i-1}}\right), \qquad (5)$$

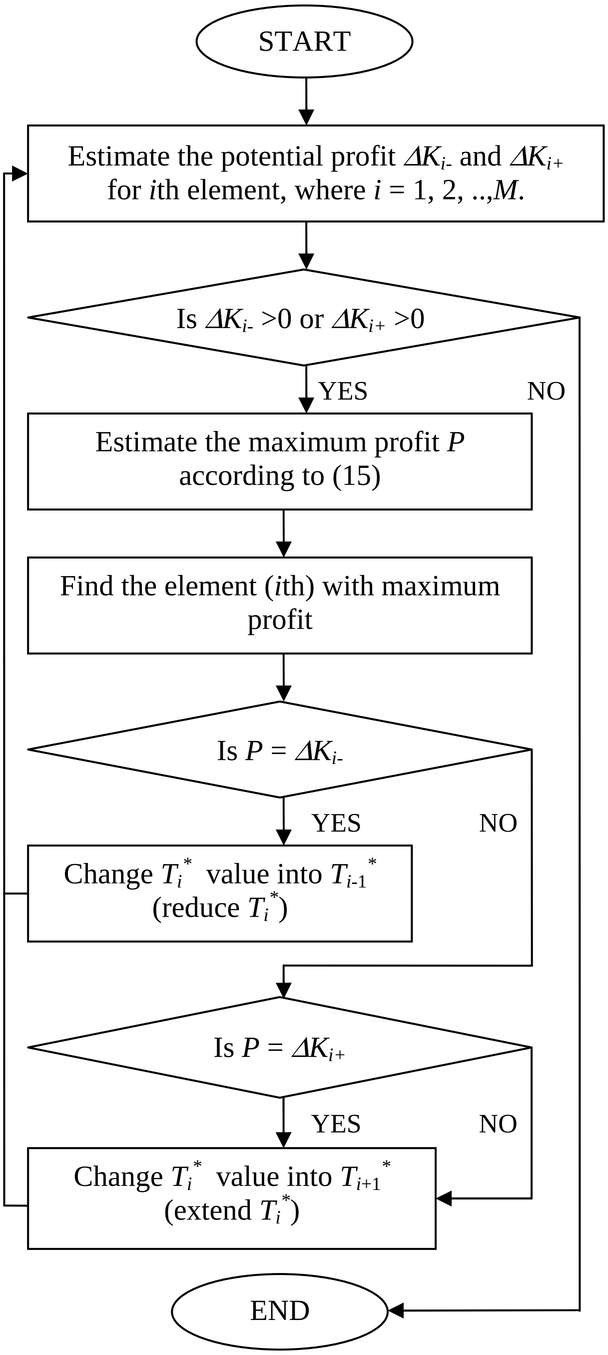

Figure 2. The scheme of element grouping process

$$\Delta K_{w-} = k_w \left(\frac{1 - F_i\left(T_i^*\right)}{n_i \cdot \overline{T}_i} - \frac{1 - F_i\left(T_{i-1}^*\right)}{n_{i-1} \cdot \overline{T}_{i-1}} \right), \tag{6}$$

$$\Delta K_{zi-} = k_{zi} \left(\frac{\overline{T}_{i-1} - \overline{T}_i}{\overline{T}_i \cdot \overline{T}_{i-1}} \right), \tag{7}$$

$$\overline{T}_i = \int_0^{T_i^*} \left(1 - F_i(t)\right) dt , \tag{8}$$

$$\overline{T}_{i-1} = \int_0^{T_{i-1}^*} \left(1 - F_i(t)\right) dt , \tag{9}$$

where: ΔK_{i-} is the total profit of the system resulting from shortening of time T_i^* to time T_{i-1}^*, ΔK_{ui-} is the profit resulting from lower failure probability, ΔK_{wi-} is the loss of profit resulting from higher probability of preventive replacement, ΔK_{zi-} is the loss of profit resulting from an increase in the number of element purchases, $F_i(T_i^*)$ is the c.d.f. of time to failure value of ith element in T_i^* moment, $F_i(T_{i-1}^*)$ is the c.d.f. of time to failure value of ith element in T_{i-1}^* moment, n_i is the number of elements replaced preventively together with ith element, if the element is replaced at T_i^* moment (present group) and n_{i-1} is the number of elements replaced preventively together with ith element, if the element is replaced at T_{i-1}^* moment (group after T_i^* reduction).

- lengthening of time T_i^* to time T_{i+1}^* should be calculated in the same way:

$$\Delta K_{i+} = \Delta K_{ui+} + \Delta K_{wi+} + \Delta K_{zi+} , \tag{10}$$

$$\Delta K_{ui+} = k_u \left(\frac{F_i\left(T_i^*\right)}{\overline{T}_i} - \frac{F_i\left(T_{i+1}^*\right)}{\overline{T}_{i+1}} \right), \tag{11}$$

$$\Delta K_{w+} = k_w \left(\frac{1 - F_i\left(T_i^*\right)}{n_i \cdot \overline{T}_i} - \frac{1 - F_i\left(T_{i+1}^*\right)}{n_{i+1} \cdot \overline{T}_{i+1}} \right), \tag{12}$$

$$\Delta K_{zi+} = k_{zi} \left(\frac{\overline{T}_{i+1} - \overline{T}_i}{\overline{T}_i \cdot \overline{T}_{i+1}} \right), \tag{13}$$

$$\overline{T}_{i+1} = \int_0^{T_{i+1}^*} \left(1 - F_i(t)\right) dt , \tag{14}$$

where: ΔK_{i+} is the total profit of the system resulting from the shortening of time T_i^* to time T_{i-1}^*, ΔK_{ui+} is the loss of profit resulting from higher failure probability, ΔK_{w+} is the profit/loss of profit resulting from higher probability of preventive replacement and combining/separating group replacement, ΔK_{z+-} is the profit resulting from rarer element purchase, $F_i(T_{i+1}^*)$ is the c.d.f. of time to failure value of ith element in T_{i+1}^* moment, n_{i+1} is the number of elements replaced preventively together with ith element, if the element is replaced at T_{i+1}^* moment (group after T_i^* extension).

(ii) If the reduction or extension to any element T_i^* is profitable ($\Delta K_{i-} > 0$ or $\Delta K_{i+} > 0$), maximum savings should be found:

$$P = (\max(\Delta K_{i-}, \Delta K_{i+})), \tag{15}$$

The time interval T_i^* of the element with maximum profit should be:
- shortened into T_{i-1}^* , if $P = \Delta K_{i-}$,
- lengthened into T_{i+1}^* , if $P = \Delta K_{i+}$,
and the previous step (i) should be repeated.

(iii) If $P \le 0$, there is no further possibility to obtain a cheaper solution.

3.3. Numerical example

The considered system is composed of $M = 5$ elements. The time to failure of ith element is described by Weibull c.d.f.:

$$F_i(t) = 1 - e^{-(t/Bi)^{Ai}} \tag{16}$$

Assumed parameters of element probability distributions $F_i(t)$, new element purchase cost, failure and preventive replacement costs, and optimal interval T_i^*, calculated according to equation (2), are:
$k_u = 30$,
$k_w = 1$,
$A_i = \{3{,}3;\ 3{,}3;\ 3{,}3;\ 3{,}3;\ 3{,}3\}$,
$B_i = \{100;\ 200;\ 300;\ 400;\ 500\}$,
$k_{zi} = \{0, 0, 0, 0, 0\}$,
$T_i^* = \{28, 56, 84, 112, 140\}$.
The minimization process of the total system maintenance cost is NP-hard task. If start vector T_i^* is based on the ARP policy, there are 55 possibilities of grouping for the presented 5-elements system. If one wants to start element grouping, taking into account other initial values of T_i^*, the number of possible solutions seems infinite. An optimal solution seems to be inaccessible even in this easy case. Therefore this simple procedure allows us to limit the number of analysed solutions, shifting it in the most profitable direction. *Table 2* presents the procedure functioning for the above assumptions.
In order to assess the algorithm results, all 55 possibilities were examined. Total cost per unit time that the system incurs for various variants of grouping is presented in *Figure 3*. The solution obtained according to the procedure is marked by ■.

Table 2. Steps of element grouping process:

No of element	T_i^*	ΔK_{i-}	ΔK_{i+}	P
STEP 1, K = 0,117				
1	28	0	$-3{,}3\cdot10^{-2}$	0
2	56	$0{,}62\cdot10^{-2}$	$-0{,}14\cdot10^{-2}$	$0{,}62\cdot10^{-2}$
3	84	$0{,}61\cdot10^{-2}$	$0{,}25\cdot10^{-2}$	0
4	112	$0{,}48\cdot10^{-2}$	$0{,}27\cdot10^{-2}$	0
5	140	$0{,}39\cdot10^{-2}$	0	0
STEP 2, K = 0,093				
1	28	0	0	0
2	28	0	$-0{,}63\cdot10^{-2}$	0
3	84	$0{,}47\cdot10^{-2}$	$0{,}25\cdot10^{-2}$	0
4	112	$0{,}48\cdot10^{-2}$	$0{,}27\cdot10^{-2}$	$0{,}48\cdot10^{-2}$
5	140	$0{,}39\cdot10^{-2}$	0	0
STEP 3, K = 0,083				
1	28	0	0	0
2	28	0	$-0{,}44\cdot10^{-2}$	0
3	84	$-0{,}11\cdot10^{-2}$	0	0
4	84	0	$-0{,}21\cdot10^{-2}$	0
5	140	$0{,}53\cdot10^{-2}$	0	$0{,}53\cdot10^{-2}$
STEP 4, K = 0,073				
1	28	0	0	0
2	28	0	$-0{,}35\cdot10^{-2}$	0
3	84	$-0{,}31\cdot10^{-2}$	0	0
4	84	0	$-0{,}21\cdot10^{-2}$	0
5	84	0	0	0

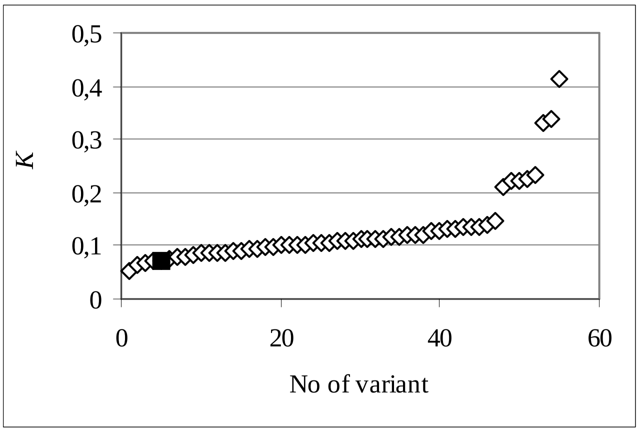

Figure 3. Total cost per unit time for various variants of grouping

The same analysis was executed for 15 various vectors of k_{zi}. The mean value of solutions obtained according to individual ARP optimization is 40% (the worst - 75%) higher than the cheapest group maintenance policy that was found. The mean solution achieved according to the presented algorithm is 6% higher and the worst is 26% higher than the cheapest one.

The investigation conducted for various relations of failure and preventive replacement costs is shown in *Figure 4*. The economic results of the system are ca 40% better than in the case of ARP policy applied separately for every single component.

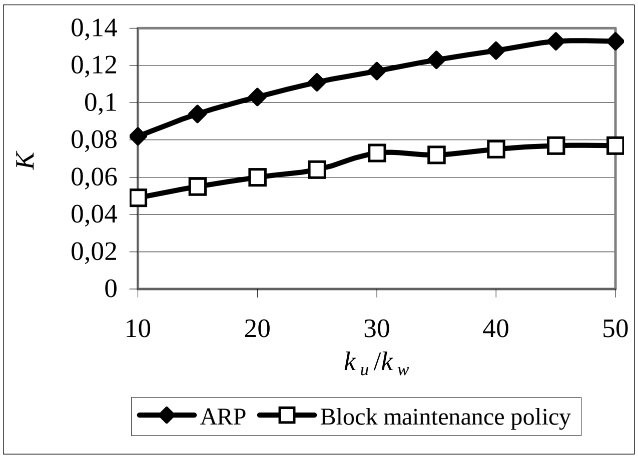

Figure 4. Total cost per unit time for various relations of failure and preventive replacement costs

4. Conclusion

Maintenance of systems composed of heterogeneous elements seems to be a challenge for scientists and practitioners. There is a great need to search understandable models and techniques that may be effectively applied in practice. This paper gives a short insight into the complexity of the problem. Moreover it proposes a simple method of heterogeneous element grouping, in systems with economic dependency.

The proposed algorithm is a tool, which may be used to define preventive replacement times in systems composed of heterogeneous units. On the basis of single element ARP policy, it allows us to easily find a much better (cheaper) solution of block replacement strategy. The algorithm should be developed in order to generate optimal (not near optimal) solutions for a given variant of T_i^* vector. It may also be the starting point for the next procedures, taking into account other factors that have an influence on system results, e.g. availability analysis, structural and probabilistic dependencies. It should also be verified in practise and this task will be done in future.

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