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Matlab, non-linearity's, vibration, machine

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MODELLING SUPPRESSION OF NON-LINEAR MACHINE VIBRATION

In this paper, the coupled non-linear differential equations of the model of machine as non-linear dynamical two degree of freedom vibrating system including cubic non-linearity are solved. The system consists of the main system and the absorber. Consider the non-linearity dynamical vibrating system as a model machine, which is compiled of a driven motor, gearbox and mechanisms with elastic and damping parts and driven parts. The absorber is used to control the main system vibrations when subjected to external excitation force. This system represents many applications in machine tools, ultrasonic cutting process, etc. Optimum working conditions for the absorber are by the same frequencies $\omega = \omega_2$ [1] of the action force $F = F_0 \sin\omega t$ and main mass m_1. The effects of different parameters of the system are studied numerically with Matlab.

1. INTRODUCTION

Different mechanical, electrical and other systems can be represented by one degree of freedom non-linear systems. The vibration of such systems at resonance cases can be reduced using absorbers. Mechanical systems are inherently non-linear with many sources of non-linearities present in any machine under consideration. Material and constitutive non-linearities are frequently encountered in vibration isolators in which polymeric materials are extensively reduce the deleterious effects of unwanted vibration. The simplified model of machine with a reduced main mass m_1 with displacement y_1 located on cushion with coefficient of elasticity k_1, non-linear coefficient of elasticity k_{1n}, linear and non-linear damping coefficient b_1, b_{1n} and the affiliate mass m_2 of the absorber with displacement y_2, located on a spring with coefficient of elasticity k_2, non-linear coefficient of elasticity k_{2n}, linear and non-linear damping coefficient b_2, b_{2n} is possible to illustrate as a two mass system (Fig. 1). The non-linear stiffness k_{1n} comprises a hardening spring defined: $+ k_{1n}y_1^3$ and k_{2n} constitutes a softening spring defined: $- k_{2n}y_2^3$. The main mass m_1 is exited by external forces $F=F_0 \sin\omega t$ where F_0 and ω are the forcing amplitude and frequency. The model of this system takes the convenient simplified form a two degree of freedom Duffing

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oscillator system, where the stiffness general $k_1>0$, respective $k_2>0$. For $k_1>0$, respective $k_2>0$ the Duffing oscillator can be interpreted as a forced oscillator with a spring whose restoring force is written as $F_1=- k_1 y_1- k_{1n} y_1^3$, respective $F_2=- k_2 y_2- k_{2n} y_2^3$. When, $k_{1n}>0$, respective $k_{2n}>0$ this spring is called a hardening $F_2=- k_2 y_2- k_{2n} y_2^3$, and when $k_{1n}<0$, respective $k_{2n}<0$, it is called a softening spring although this interpretation is valid only for small y_1, respective y_2.

The amplitude of the induced vibration is a function of the applied force and its frequency. An exciting force has the greatest effect when applied at the fundamental frequency of the system. The system is then excited at resonance, and in the case of a lightly damped system, the induced movement can be many times greater than the deflection caused by the equivalent static force.

Vibrations in a structure have this effect. The very high peak accelerations can mean that the effective weight of the vibrating system increases several-fold, and this may cause its destruction. The vibration absorber is advantageous primarily in that it reduces the amplitude of the vibrations in the system by an oscillating force F acting, for example from an unbalanced wheel on the system $F=me\omega^2 \sin\omega t=F_0 \sin\omega t$, where m is the mass of the unbalanced wheel e is the eccentricity of the unbalanced rotor and ω is the angular velocity of the unbalanced rotor.

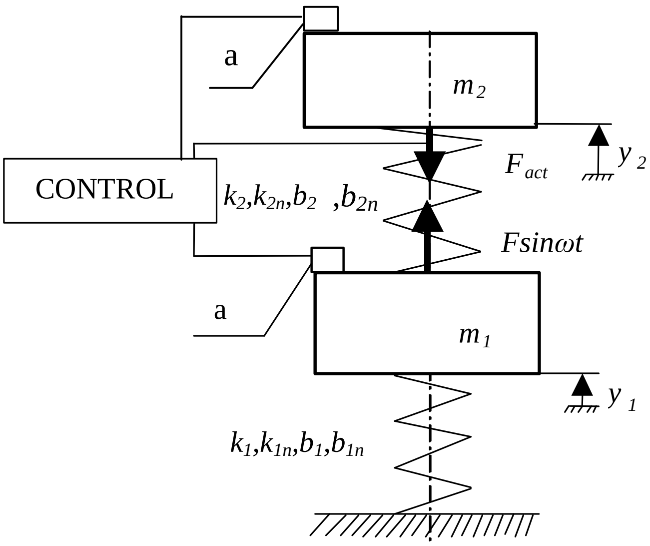

Fig. 1. Simplified model of the machine (mass m_1) with absorber (mass m_2) as two degree freedom system

2. NUMERICAL SOLUTION

The equations of motion for the masses m_1, m_2 are:

$$m_1\ddot{y}_1=-k_1y_1-k_{1n}y_1^3+k_2(y_2-y_1)+k_{2n}(y_2-y_1)^3-b_1\dot{y}_1-b_{1n}\dot{y}_1^3+b_2(\dot{y}_2-\dot{y}_1)+b_{2n}(\dot{y}_2-\dot{y}_1)^3+F_0\sin\omega t,$$
$$m_2\ddot{y}_2=-k_2(y_2-y_1)-k_{2n}(y_2-y_1)^3-b_2(\dot{y}_2-\dot{y}_1)-b_{2n}(\dot{y}_2-\dot{y}_1)^3. \quad (1)$$

Numerical results are presented in graphical forms (using Matlab) as steady state amplitudes against detuning parameters and the time response for both main system and absorber. In the following sections, the effects of the different parameters on response and stability will be investigated. A good criterion of the stability and presence of dynamic chaos is the phase-plane trajectories, which are shown for some cases. Different primary resonance cases are studied and discussed.

For the given values of linear and non-linear coefficients of stiffness k_1, k_2, k_{1n}, k_{2n} and linear and non-linear dampening coefficients $b=b_1=b_2$, b_{1n} , b_{2n} of the individual components and the given value of main mass m_1 and absorber mass m_2 (Fig. 1), we look for the amplitude frequency response curves using numeric calculation method with the Matlab program (Fig. 2) by means of the transfer function.

The parameters of this mechanical system are:

m_1=100 kg, m_2=5 kg, $k_1=5\cdot10^6$ Nm^{-1}, $k_2=2{,}465\cdot10^6$ Nm^{-1}, $k_{1n}=1\cdot10^{18}$ Nm^{-1}, $k_{2n}=5\cdot10^{18}$ Nm^{-1}, b=100 Nsm^{-1}, $b_{1n}= b_{2n}=0$, F_0=0,1N.

The system compiled with machine and absorber is stable and free from dynamic chaos that the steady state amplitude of the machine is greatly reduced with some chaos when the absorber is effective , i.e. when $\omega= \omega_2$ [1]. The system designed for the machine and the absorber applies for the displacement machines approaching zero, the relationship [2]:

$$\omega_2 = \sqrt{\frac{k_2}{m_2}}. \qquad (2)$$

From equation (2) it is possible to calculate the size of angular velocityω. m file of the solution of equations (2) is:

```
global F0 w k b kn bn m_1;
N = 1;
W = 200;%logspace(1,3,N);
f = zeros(1,N);
for i=1:N
me = 10;
e = 1e-3;
w = W(i);
F0 = 0.1;%me*e;
k1 = 5e6;
k2 = 2.465e5;
b1 = 100;
b2 = 100;
k1n = 0;%1e18;
k2n = 0;%5e18;
m1 = 100;
m2 = 5;
k  = [ -(k1+k2) k2; k2 -k2 ];
b  = [ -(b1+b2) b2; b2 -b2 ];
kn = [ -k1n k2n; 0 -k2n ];
bn = [ -b1 b2; 0 0 ];
m  = [ m1 0; 0 m2 ];
m_1 = inv(m);
```

```
sim('karpacz10');
L = 10000;
Fs = 1000;
NFFT = L;

Y = fft(y.signals(1).values, NFFT)/L;
X = fft(y.signals(3).values(:,1), NFFT)/L;
G = Y./X;
f = Fs/2*linspace(0,1,NFFT/2)*2*pi;

figure(2);
semilogy(f,2*abs(G(1:NFFT/2)) ); hold on;
axis([150 300 5e-12 1e-8]);
end
```

Fig. 2. Matlab program of solution equation (1)

We solve the courses of deviations y_1/F_0 of main mass m_1 (Fig. 3) using the numeric calculation for three cases of models of the mechanical systems:

1) with linear stiffness of the springs, undamped: k_1=5·10^6 Nm^{-1}, k_2=2,465·10^6 Nm^{-1}, $k_{1n}= k_{2n}$=0, $b_1= b_2$=0, $b_{1n}= bn_2$=0 (solid line),

2) with linear stiffness of the springs, damped: k_1=5·10^6 Nm^{-1}, k_2=2,465·10^6 Nm^{-1}, $k_{1n}= k_{2n}$=0
and linear damping of the springs $b=b_1=b_2$=100 Nsm^{-1}, $b_{1n}= bn_2$=0 (dotted line),

3) with non-linear stiffness of the springs, undamped: k_{1n}=1·10^{18}Nm^{-1}, k_{2n}=5·10^{18} Nm^{-1} and k_1=5·10^6 Nm^{-1}, k_2=2,465·10^6 Nm^{-1}, $b_1= b_2$=0, $b_{1n}= bn_2$=0 (dash line).

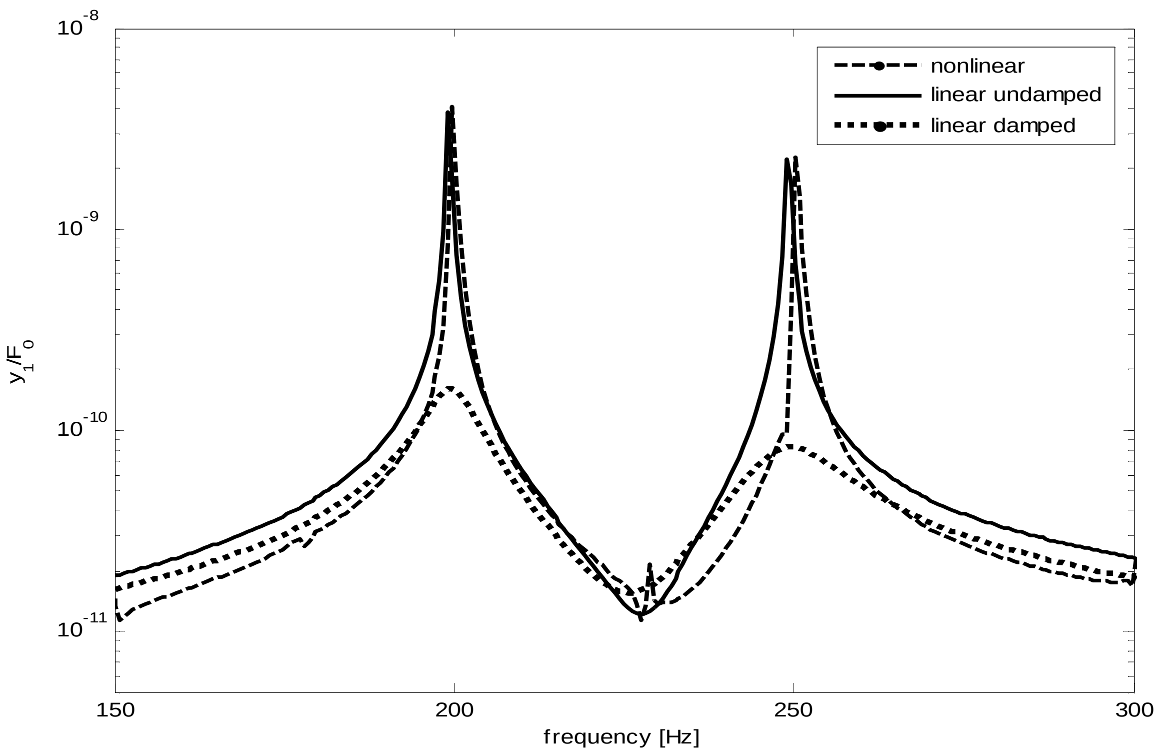

Fig. 3. Courses of deviations y_1/F_0 of main mass m_1

3. RESULTS OF THE SOLUTION

The aim of the paper is to acquaint the order with design of the incorporated absorber with linear and non-linear springs to the vibration of system, which makes possible the reduction of the vibration of the main mass of the system to a minimum. Numerical results are presented in graphical forms as steady state amplitudes y_1/F_0 against detuning parameters and the time response for main system (Fig. 3). The displacement y_1 of the main mass for all three cases of models is the smallest when the frequency $\omega = \omega_2 = 222\ \mathrm{sec}^{-1}$ (2) and Fig. 3.

In the cases 1 and 2, when the frequency ω of the acting force F driving the system is the same as frequency ω_2 and $\omega_2 = 222\ \mathrm{sec}^{-1}$ (Fig. 3) of the vibration of the affiliate mass m_2 absorber see you the time courses y_1 of the main mass m_1 and y_2 of the absorber on the Fig. 4.

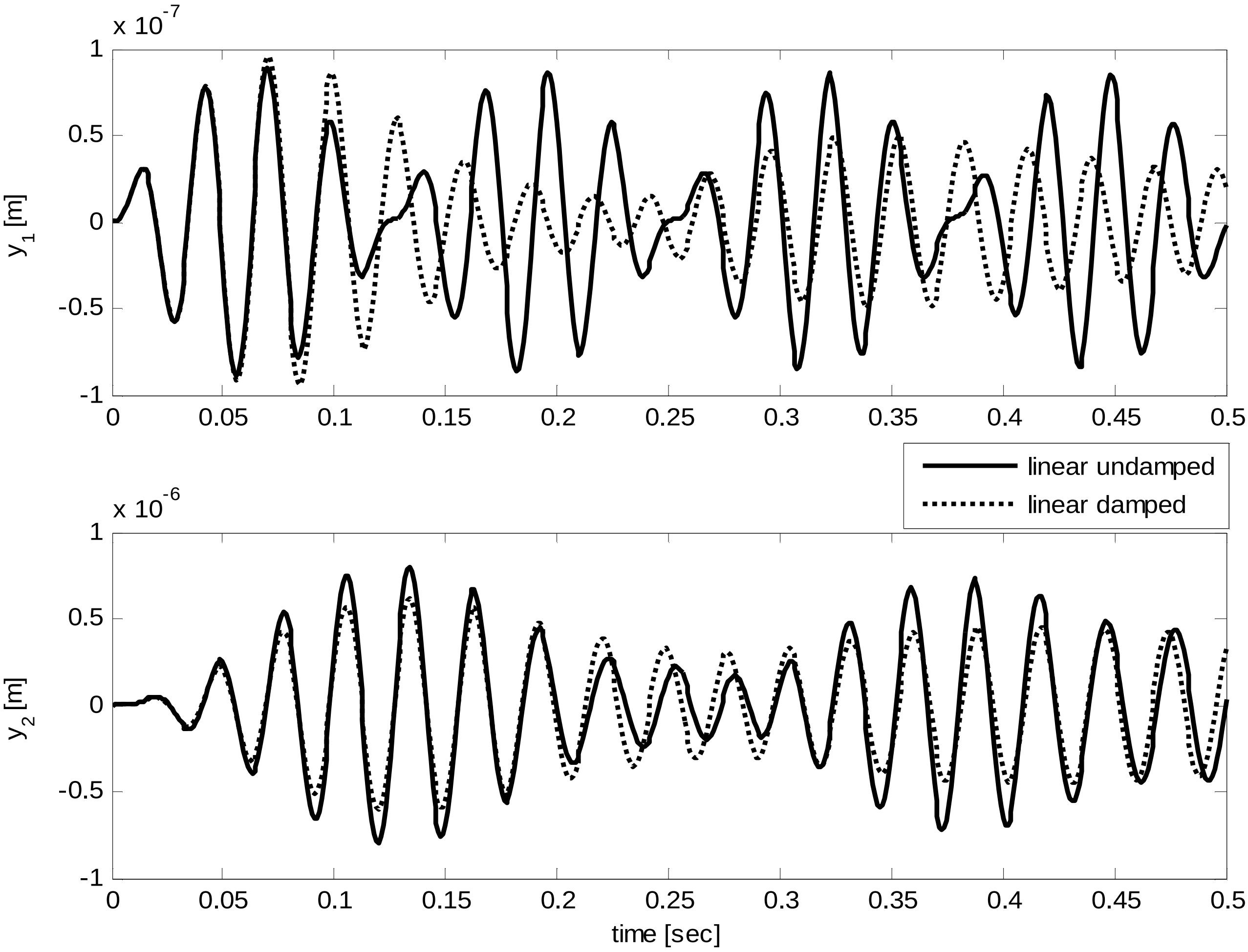

Fig. 4 Time courses y_1 of the main mass m_1 and y_2 of the linear absorber with mass m_2 (case1 and 2)

In the case 3, when the frequency ω of the acting force F driving the system is the same as frequency ω_2 and $\omega_2 = 222\ \mathrm{sec}^{-1}$ (Fig. 3) of the vibration of the affiliate mass m_2

absorber see you the time courses y_1 of the main mass m_1 and y_2 of the absorber on the Fig. 5.

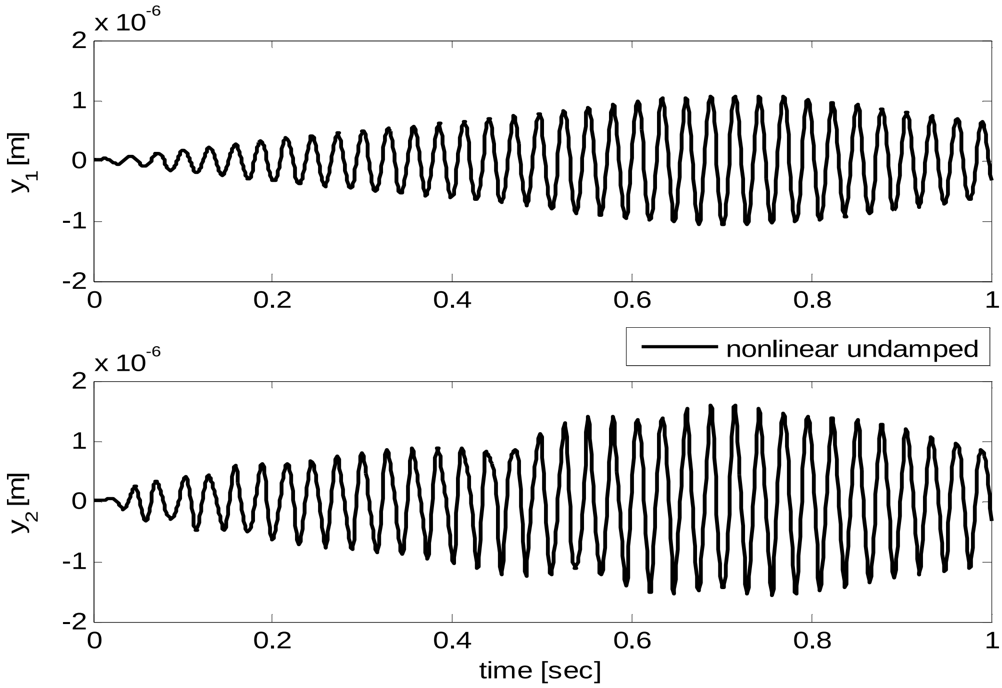

Fig. 5. Time courses y_1 of the main mass m_1 and y_2 of the non-linear absorber with mass m_2 (case 3)

4. CONCLUSION

In the domain of mechanical vibration research, dynamic absorbers have extensive application in reducing vibration of machine. The critical vibration of such machine at resonance cases can be effectively reduced using linear (case 1 and 2) absorbers (Fig. 4). In the domain of many vibration machine the coupled non-linear second order differential equations which are solved numerically. The model of the machine with absorber is two degree of freedom vibration system with non-linear spring and non-linear damping (Fig. 1). Non-linearities necessarily introduce a whole range of phenomena that are not found in linear systems. The vibration of the model including cubic non-linearities can be controlled applying non-linear absorber (case 3). Numerical solution with Matlab (Fig. 2) is applied to determine approximate solutions of coupled non-linear differential equations to second order approximations (Fig. 5). Generally speaking, the nonlinearity in this case 3 to control the vibration damage - hard to match and the result are hard to estimate.

REFERENCES

[1] VONDRICH J., THÖNDEL E., 2006, *Modelling and design of Absorber,* XI International Conference Computer Solution in Machine Design-COSIM2006, Warsaw University of Technology, Poland, 367-364.