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NEW SUFFICIENT CONDITIONS OF GLOBAL STABILITY OF NONLINEAR POSITIVE ELECTRICAL CIRCUITS

The global stability of electrical circuits composed of positive linear parts and nonlinear static element with given characteristic and positive gain feedbacks is investigated. New sufficient conditions for the global stability of this class of nonlinear positive electrical circuits are established. These new stability conditions are demonstrated on simples examples of positive nonlinear electrical circuits.

Keywords: global stability, sufficient conditions, positive, nonlinear ,electrical, circuit.

1. INTRODUCTION

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [1, 4, 9, 16]. Variety of models having positive behavior can be found in engineering, especially in electrical circuits [16], economics, social sciences, biology and medicine, etc. [4, 9].

The positive electrical circuits have been analyzed in [6, 9-13] and positive descriptor systems and electrical circuits in [2, 21] The controllabilityy and observability of standard and positive electrical circuits has been addressed in [16], A new class of normal positive linear electrical circuits has been introduced in [9]. Positive fractional linear electrical circuits have been investigated in [3, 5-7, 10, 11, 12, 15]. The superstabilization of positive linear electrical circuits by state-feedbacks has been analyzed in [14].

In this paper the global stability of nonlinear positive linear electrical circuits with positive state-feedbacks will be investigated.

The paper is organized as follows. In section 2 the basic definitions and properties of positive electrical circuits are recalled. The main result of the paper the new global stability conditions of the nonlinear positive electrical circuits with state feedbacks are established in section 3. Concluding remarks are given and some open problems are formulated in section 4.

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The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ real matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix.

2. POSITIVE ELECTRICAL CIRCUITS

Consider the linear continuous-time electrical circuit described by the state equations

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{2.1a}$$

$$y(t) = Cx(t) + Du(t), \tag{2.1b}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $y(t)\in\Re^p$ are the state, input and output vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $D\in\Re^{p\times m}$.

It is well-known [16] that any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources can be described by the state equations (2.1). Usually as the state variables $x_1(t),\ldots, x_n(t)$ (the components of the state vector $x(t)$) the currents in the coils and voltages on the capacitors are chosen.

Definition 2.1. [16] The electrical circuit (2.1) is called (internally) positive if $x(t)\in\Re_+^n$ and $y = y(t)\in\Re_+^p$, $t\in[0,+\infty]$ for any $x_0 = x(0)\in\Re_+^n$ and every $u(t)\in\Re_+^m$, $t\in[0,+\infty]$.

Theorem 2.1. [16] The electrical circuit (2.1) is positive if and only if

$$A\in M_n,\ B\in\Re_+^{n\times m},\ C\in\Re_+^{p\times n},\ D\in\Re_+^{p\times m}. \tag{2.2}$$

Theorem 2.2. [16] The linear electrical circuit composed of resistors, coils and voltage sources is positive for any values of the resistances, inductances and source voltages if the number of coils is less or equal to the number of its linearly independent meshes and the direction of the mesh currents are consistent with the directions of the mesh source voltages.

Theorem 2.3. [16] The linear electrical circuit composed of resistors, capacitors and voltage sources is not positive for all values of its resistances, capacitances and source voltages if each its branch contains resistor, capacitor and voltage source.

Theorem 2.4. [16] The *R*, *L*, *C*, *e* electrical circuits are not positive for any values of its resistances, inductances, capacitances and source voltages if at least one its branch contains coil and capacitor.

Definition 2.2. [16] The positive electrical circuit is called asymptotically stable if

$$\lim_{t\to\infty} x(t) = 0 \text{ for any } x_0\in\Re_+^n. \tag{2.3}$$

Theorem 2.5. [16,20] The positive electrical circuit (2.1) is asymptotically stable if and only if one of the equivalent conditions is satisfied:

1) All coefficient of the characteristic polynomial

$$\det[I_n s - A] = s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 \tag{2.4}$$

are positive, i.e. $a_k > 0$ for $k = 0,1,\dots,n-1$.

2) All principal minors $\overline{M}_i$, $i = 1,\dots,n$ of the matrix $-A$ are positive, i.e.

$$\overline{M}_1 = |-a_{11}| > 0,\ \overline{M}_2 = \begin{vmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{vmatrix} > 0,\ \dots,\ \overline{M}_n = \det[-A] > 0. \tag{2.5}$$

3) There exists strictly positive vector $\lambda^T = [\lambda_1 \ \cdots \ \lambda_n]^T$, $\lambda_k > 0$, $k = 1,\dots,n$ such that

$$A\lambda < 0 \text{ or } A^T\lambda < 0. \tag{2.6}$$

3. GLOBAL STABILITY OF STANDARD NONLINEAR FEEDBACK SYSTEMS

Consider the nonlinear feedback electrical circuit shown in Fig. 3.1 which consists of the positive linear part, the nonlinear element with characteristic $u = f(e)$ and the feedback with positive gain h. The linear part is described by the equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}, \tag{3.1}$$

where $x = x(t) \in \Re_+^n$, $u = u(t) \in \Re_+$, $y = y(t) \in \Re_+$ is the state vector, input and output and $A \in M_n$, , $B \in \Re_+^{n\times 1}$, $C \in \Re_+^{1\times n}$.

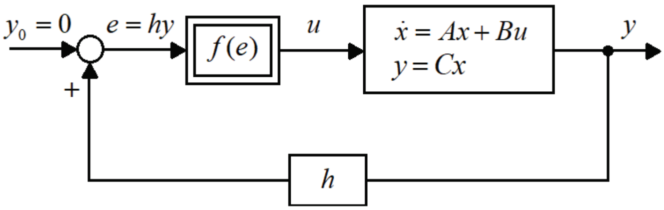

Fig. 3.1. The nonlinear feedback system

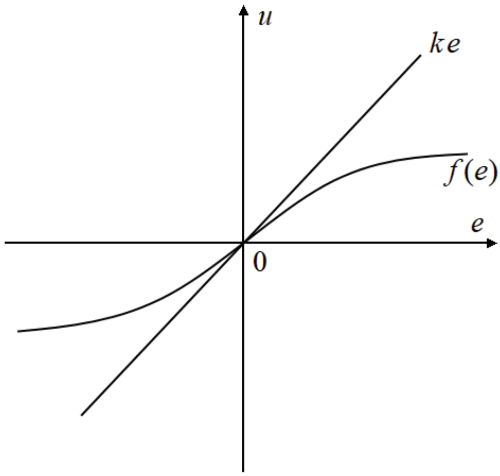

Fig. 3.2. Characteristic of the nonlinear element

The characteristic of the nonlinear element is shown in Fig. 3.2 and it satisfies the condition

$$0 \le \frac{f(e)}{e} \le k < \infty. \tag{3.2}$$

It is assumed that the positive linear part is asymptotically stable (the matrix $A \in M_n$ is Hurwitz).

Definition 3.1. The nonlinear positive electrical circuit is called globally stable if it is asymptotically stable for all nonnegative initial conditions $x(0) \in \Re_+$.

The following theorem gives sufficient conditions for the global stability of the positive nonlinear electrical circuit.

Theorem 3.1. The nonlinear electrical circuit consisting of the positive linear part, the nonlinear element satisfying the condition (3.2) and the feedback with positive gain h is globally stable if

$$A + khBC \in M_n. \tag{3.3}$$

Proof. The proof will be accomplished by the use of the Lyapunov method [18,19]. As the Lyapunov function $V(x)$ we choose

$$V(x) = \lambda^T x \ge 0 \text{ for } x \in \Re_+^n, \tag{3.4}$$

where λ is strictly positive vector, i.e. $\lambda_k > 0$, $k = 1,..., n$.

Using (3.4) and (3.1) we obtain

$$\dot{V}(x) = \lambda^T \dot{x} = \lambda^T (Ax + Bu) = \lambda^T (Ax + Bf(e)) \le \lambda^T (A + khBC)x \tag{3.5}$$

since $u = f(e) \le ke = khCx$.

From (3.5) it follows that $\dot{V}(t) < 0$ if the condition (3.3) is satisfied and the nonlinear positive electrical circuit is globally stable.

To find the maximal value of k satisfying the condition (3.2) for the nonlinear positive electrical circuit the following procedure can be used.

Procedure 3.1.

Step 1.Using the matrices A, B, C of the positive electrical circuit and h compute the matrix

$$A_1 = A + khBC. \tag{3.6}$$

Step 2.Compute the characteristic polynomial of the matrix (3.6) with coefficient depending on k

$$\det[I_n s - A_1]. \tag{3.7}$$

Step 3.Using the condition 1) of Theorem 2.5 find the maximal k for which all coefficients of (3.7) are positive.

Example 3.1. Consider the nonlinear positive electrical circuit shown in Fig. 3.3 with given resistances $R_1 = 3$, $R_2 = 2$, inductances $L_1 = L_2 = 1$, the characteristic $u = f(i)$ satisfying the condition

$$0 \le \frac{f(i)}{i} \le k \tag{3.8}$$

and with the feedback gain $h = 0.5$.

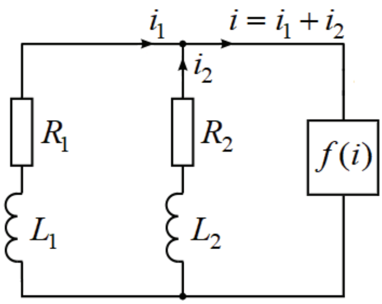

Fig. 3.3. Nonlinear electrical circuit

Find the maximal value of k for which the nonlinear electrical circuit is globally stable.

Using Procedure 3.1 we obtain the following.

Step 1. The positive linear part is described by the equations

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + Bu, \quad y = C\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}, \tag{3.9a}$$

where

$$A=\begin{bmatrix}-\frac{R_1}{L_1} & 0\\ 0 & -\frac{R_2}{L_2}\end{bmatrix}=\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix},\quad B=\begin{bmatrix}\frac{1}{L_1}\\ \frac{1}{L_2}\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix},\quad C=[1\quad 1].\tag{3.9b}$$

The matrix (3.6) has the form

$$A_1=A+khBC=\begin{bmatrix}-3 & 0\\ 0 & -2\end{bmatrix}+k\begin{bmatrix}0.5 & 0.5\\ 0.5 & 0.5\end{bmatrix}.\tag{3.10}$$

Step 2. The characteristic polynomial of the matrix (3.10) has the form

$$\det[I_2 s-A_1]=\begin{vmatrix}s+3-0.5k & -0.5k\\ -0.5k & s+2-0.5k\end{vmatrix}=s^2+(5-k)s+6-2.5k.\tag{3.11}$$

Step 3. Therefore, the maximal value of k for which all the coefficient of (3.11) are positive is $k<2.4$.

Example 3.2. Consider the nonlinear positive electrical circuit shown in Fig. 3.4 with given resistances $R_1=R_2=2$, capacitances $C_1=0.2$, $C_2=0.1$, the characteristic $i=f(u)$ satisfying the condition

$$0\le\frac{f(i)}{i}\le k\tag{3.12}$$

and the feedback gain $h=0.2$.

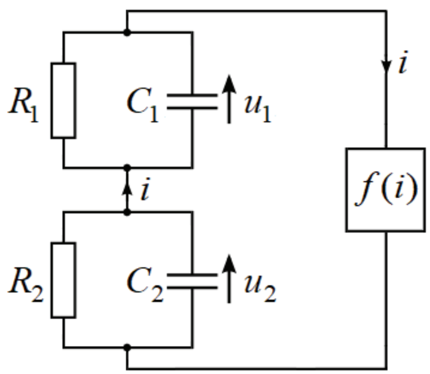

Fig. 3.4. Nonlinear electrical circuit

Check if the positive nonlinear electrical circuit is globally stable for $k=0.5$. The positive linear part of the nonlinear circuit is described by the equations

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + Bu, \quad y = C\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{3.13a}$$

where

$$A = \begin{bmatrix} -\frac{1}{R_1C_1} & 0 \\ 0 & -\frac{1}{R_2C_2} \end{bmatrix} = \begin{bmatrix} -2.5 & 0 \\ 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}, \quad C = [1 \quad 1]. \tag{3.13b}$$

In this case the matrix (3.6) has the form

$$A_1 = A + khBC = \begin{bmatrix} -2.5 & 0 \\ 0 & -5 \end{bmatrix} + 0.1\begin{bmatrix} 5 \\ 10 \end{bmatrix}[1 \quad 1] = \begin{bmatrix} -2 & 0.5 \\ 1 & -4 \end{bmatrix}. \tag{3.14}$$

and its characteristic polynomial

$$\det[I_2 s - A_1] = \begin{vmatrix} s+2 & -0.5 \\ -1 & s+4 \end{vmatrix} = s^2 + 6s + 7.5 \tag{3.15}$$

has positive coefficients.
Therefore, by condition 1 of Theorem 2.5 and Theorem 3.1 the nonlinear positive electrical circuit is globally stable.

4. CONCLUDING REMARKS

The global stability of electrical circuits composed of positive linear parts and nonlinear static element with given characteristic and positive feedbacks has been investigated. New sufficient conditions for the global stability of this class of nonlinear positive electrical circuits have been established. These new stability conditions are demonstrated on simples examples of positive nonlinear electrical circuits. The considerations can be extended to fractional positive nonlinear electrical circuits and to discrete-time nonlinear positive systems. An open problem is an extension of these considerations by the use of the Kudrewicz approach [17].

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