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DYNAMIC ANALYSIS OF CWR TRACK WORK ON ONE- AND TWO-PARAMETER ELASTIC FOUNDATION[1]

Włodzimierz Andrzej BEDNAREK*

*) Politechnika Poznańska, Instytut Inżynierii Lądowej

In the paper a dynamical analysis of the deflection of CWR track resting on one- and two-parameter is shown. This problem is considered in the subject matter of railroads literature [1, 7, 11, 17, 19-27]. A model of railway track on foundation without damping and calculation model with damping is analyzed. For one- and two-parameter foundation an appropriate calculation examples, reflecting an influence of velocity, damping, foundation parameters and track strength parameters changes are enclosed.

The paper has got also a review character, shows the formulas form on calculation of essential parameters for calculation of dynamic railway track deflections.

1. A DYNAMICAL FORCE TRANSFERRED FROM WHEEL ON THE RAIL

A transferred force from rail-vehicle wheel on the rail of track is mainly depended upon static load on the train axle, state of vehicle springing and imperfections arising in track (e.g. irregularity of track, unevenness on rolling surface of a rail or unevenness and in-homogeneity arising in ballasted roadbed) [3,4,18,20]. A quantity of dynamic load of wheel on the rail can be expressed in the following form [18]:

$$P_{dyn} = P_{st} + 0{,}75 \cdot P_p + 2{,}56 \cdot S_{nt} \tag{1.1}$$

where:

P_{st} – static load of wheel on a rail,

P_p – average load of wheel on a rail produced by vibrations of rail-vehicle springing mass,

S_{nt} – average of mean square deviation of wheel load on a rail produced by track irregularity.

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In this formula, a P_p quantity describes a form: $P_p = k_p \cdot z_{max}$, where: k_p – stiffness coefficient of springing mass of rail-vehicle falling on one wheel, z_{max} – additional, empirical deflection. A S_{nt} quantity describes a following dependence: $S_{nt} = 0{,}707 \cdot P_{nt}$, where: P_{nt} – maximal load produced by rolling wheel on track irregularity, according to formula:

$$P_{nt} = 0{,}8 \cdot 10^{-8} \cdot \beta \cdot \gamma \cdot l \cdot \left(P_{st} + 0{,}75 \cdot P_p\right) \cdot \sqrt{U \cdot L} \cdot \sqrt{q \cdot v} \qquad (1.2)$$

where:

β – coefficient regard to an influence of rail type on character of track irregularity,

γ – coefficient regard to an influence of ballast on character of track irregularity,

l – sleeper spacing, U – elasticity modulus of rail support,

$\frac{1}{L}$ – coefficient of relative stiffness of rail and rail support,

q – non-springing mass of rail-vehicle falling on one wheel,

v – motion velocity.

An equation for dynamical force of wheel on rail is also given in the following form:

$$P_{dyn} = \left(P_{st} + A \cdot k_p \cdot v^{\alpha}\right) \cdot \left(1 + B \cdot \sqrt{k_t} \cdot \sqrt{q} \cdot v\right) \qquad (1.3)$$

where:

P_{st} – static load of wheel on a rail,

A, α – parameters depend upon a rail-vehicle construction, characterizing its sensitivity on track irregularities,

k_p – stiffness coefficient of springing mass of rail-vehicle counted on one wheel,

B –empirical coefficient describing an influence of track stucture construction and its state on character track irregularities,

k_t – stiffness coefficient of rail springing.

Taking into consideration a vertical position of track, a dynamic force is counted from formula [16]:

$$P_{dyn} = 1{,}201 + 0{,}060 \cdot A + 0{,}051 \cdot (V - 50) \cdot (S - 0.5)\,[ton] \qquad (1.4)$$

where:

P_{dyn} – dynamic force of load on axle [ton],
A – static load on axle 5-22,5 [ton]
V – train speed 50-120 [km/h],

S – vertical position of track, with σ_{BMS} (standard deviation) for 0.5–3 [mm].

On the Fig. 1.1 a dynamic force growth due to increase of velocity and vertical track position for static load on axle (20 [ton], i.e. about 197 [kN]) is shown.

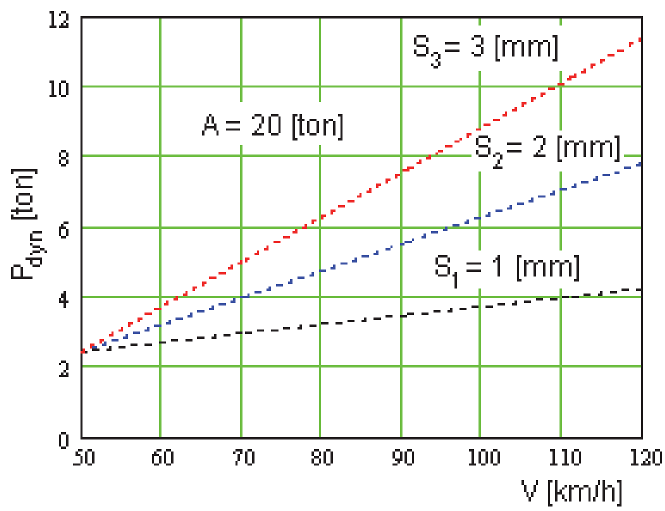

Fig. 1.1. A dynamic force with speed increase (according to 1.4 formula)

2. ANALYSIS OF INFLUENCE OF VELOCITY, TRACK MASS AND DAMPING ON RAILWAY TRACK DYNAMIC DEFLECTION ON 1-PARAMETER FOUNDATION

2.1. Differential equation without damping for track on Winkler's 1-parameter foundation

An analyzed railway track works under concentrated force load $P(t)$, moving with constant velocity (v parameter). Taking into consideration an influence inertia force of railway track with velocity growth, a differential equation for such system (fig. 2.1) by roadbed coefficient U, got the form [2,7,12]:

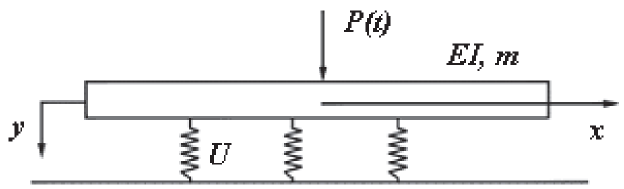

Fig. 2.1. Railway track on Winkler's one-parameter foundation without damping [12]

$$EI \cdot \frac{\partial^4 y}{\partial x^4} + m_t \cdot \frac{\partial^2 y}{\partial t^2} + U \cdot y = P(t) \cdot \delta(x) \tag{2.1}$$

where:
EI – beam stiffness [MNm^2],
m_t – beam mass per unit of track length [kg/m],
U – roadbed foundation [MPa],
$m_t \cdot \frac{\partial^2 y}{\partial t^2}$ – inertia force of track as external load.

Making the assumptions [2]:

– load keeps its quantity with constant v velocity of moving, then track deflection moves along the track axis with the same velocity as the load moves,
– with such assumption it's easily to consider a motion formula in co-ordinate system (ς, y) connecting with moving force,
– between ς and x exists a dependence: $\varsigma = x - v\,t$.

A motion equation in new co-ordinate system has got a form (concentrated force is considered next in boundary conditions):

$$EI \cdot \frac{d^4 y}{d\varsigma^4} + m \cdot v^2 \cdot \frac{d^2 y}{d\varsigma^2} + U \cdot y = 0 \tag{2.2}$$

Assuming denotations:

$$L^4 = \frac{4 \cdot EI}{U} \text{ and } \lambda^2 = \frac{4 \cdot EI}{m}, \tag{2.3}$$

we obtain:

$$\frac{d^4 y}{d\varsigma^4} + 4 \cdot \frac{v^2}{\lambda^2} \cdot \frac{d^2 y}{d\varsigma^2} + \frac{4}{L^4} \cdot y = 0 \tag{2.4}$$

Considering a particular case, when a concentrated force P is moving, while $\varsigma=0$, i.e. $x = v\,t$, then a deflection is counted from the formula:

$$y = \frac{P \cdot L^2}{8 \cdot EI \cdot \alpha} \cdot e^{-\alpha \cdot \varsigma} \cdot \left(cos(\beta \cdot \varsigma) + \frac{\alpha}{\beta} \cdot sin(\beta \cdot \varsigma) \right) \tag{2.5}$$

whereas:

$$\alpha = \sqrt{\left(\frac{1}{L}\right)^2 - \left(\frac{v}{\lambda}\right)^2} \;;\; \beta = \sqrt{\left(\frac{1}{L}\right)^2 + \left(\frac{v}{\lambda}\right)^2} \tag{2.6}$$

Deflection under force (i.e. $\varsigma=0$) amounts to:

$$y_0 = \frac{P \cdot L^2}{8 \cdot EI \cdot \alpha} = \frac{P}{2 \cdot U \cdot L} \cdot \frac{1}{L \cdot \sqrt{\left(\frac{1}{L}\right)^2 - \frac{m \cdot v^2}{4 \cdot EI}}} = y_{v=0} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{v_{kr}^2}}} \tag{2.7}$$

and critical velocity:

$$v_{kr} = \sqrt[4]{\frac{4 \cdot EI \cdot U}{m^2}} . \tag{2.8}$$

2.2. Calculation example

Data for calculations (60E1 rail; wooden sleeper, ballasted roadbed):

– $EI = 6{,}4155$ [MNm^2]; $m_t = 146$ [kg/m]; $P = 100$ [kN] (concentrated force),
– $U = 40$ [MPa] $\rightarrow L = 0{,}895$ [m],
– $v/v_{kr} = 0{,}32$ [-] with $v = 150$ [m/s].

From a formula (2.6) we obtain:

$$\alpha = \sqrt{\left(\frac{1}{L}\right)^2 - \left(\frac{v}{\lambda}\right)^2} = 1{,}0585 \left[\frac{1}{m}\right].$$

Static deflection therefore amounts:

$$y_{st} = \frac{P \cdot L^3}{8 \cdot EI} = \frac{P}{2 \cdot U \cdot L} = 1{,}397 \, [mm],$$

however a dynamical deflection (2.7) for $\varsigma = 0$:

$$y_0 = \frac{P \cdot L^2}{8 \cdot EI \cdot \alpha} = \frac{P}{2 \cdot U \cdot L} \cdot \frac{1}{L \cdot \sqrt{\left(\frac{1}{L}\right)^2 - \frac{m \cdot v^2}{4 \cdot EI}}} = y_{v=0} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{v_{kr}^2}}} = 1{,}474 \, [mm]$$

$$\text{where } \frac{1}{\sqrt{1 - \frac{v^2}{v_{kr}^2}}} = 1{,}0556 \, [-], \text{ i.e.}$$

$$y_0 = y_{st} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{v_{kr}^2}}} = 1{,}397 \, [mm] \cdot 1{,}0556 = 1{,}474 \, [mm].$$

2.3. Differential equation with damping on Winkler's one-parameter foundation

Model, from point 2.1 of the paper, was considered without damping (c_t). Taking into account a damping (model shown on fig. 2.2), a differential equation (2.2) has got a form [7,8,12]:

$$EI \cdot \frac{\partial^4 y(x,t)}{\partial x^4} + m_t \cdot \frac{\partial^2 y(x,t)}{\partial t^2} + c_t \cdot \frac{\partial y(x,t)}{\partial t} + k_t \cdot y(x,t) = P(t) \cdot \delta(x) \qquad (2.9)$$

where:
EI – beam stiffness [Nm2],
m_t – beam mass [kg/m],
c_t – damping coefficient [Ns/m2],
k_t = U – roadbed coefficient [N/m2],
q – continuous load [N/m],
P – concentrated force [N],
v – velocity [m/s],
x [m], t [s] – time and y [m] – beam deflection.

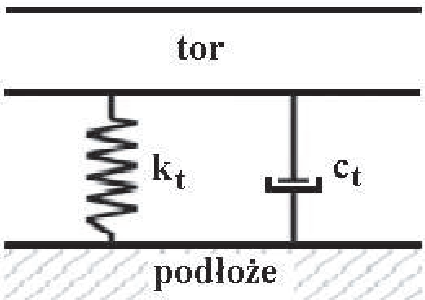

Fig. 2.2. Railway track on Winkler's foundation with damping [12]

Moving load $P\delta$(x-vt) will be considered in boundary conditions, hence we can write [8]:

$$EI \cdot \frac{\partial^4 y(x,t)}{\partial x^4} + m_t \cdot \frac{\partial^2 y(x,t)}{\partial t^2} + c_t \cdot \frac{\partial y(x,t)}{\partial t} + k_t \cdot y(x,t) = 0 \qquad (2.10)$$

Parameter c_t assumes the values in the range from 10,1 to 120 [kNs/m^2] [8]. In the paper [15] for railway track with rail 60E1 on wooden sleeper and ballasted roadbed, track, track stiffness EI=12,59 [MNm^2] and track mass 120,7 [kg/m] it's obtained a value c_t=92.2 [kNs/m^2] with velocity V=81,65 [km/h] of two EP09 locomotives. We improve the additional supportive parameters [8]:

$$s = \lambda(x - v \cdot t); \; \lambda = \left(\frac{k}{4 \cdot EI}\right)^{\frac{1}{4}} = \frac{1}{L} \qquad (2.11)$$

Derivatives of deflection in relation to *x* and *t* have got a form:

$$\frac{\partial y}{\partial x} = \lambda \cdot \frac{dy}{ds}; \frac{\partial y}{\partial t} = -\lambda \cdot v \cdot \frac{dy}{ds}, etc. \tag{2.12}$$

Therefore a differential equation has got a following form:

$$\frac{d^4 y}{ds^4}(s) + 4 \cdot s^2 \cdot \frac{d^2 y}{ds^2}(s) - 8 \cdot \alpha \cdot \beta \cdot \frac{dy}{ds}(s) + 4 \cdot y(s) = 0 \tag{2.13}$$

where:

$$\alpha = \frac{v}{2 \cdot \lambda} \cdot \left(\frac{m}{EI}\right)^{1/2} [-];\ \beta = \frac{c}{2 \cdot m} \cdot \left(\frac{m}{k}\right)^{1/2} [-] \tag{2.14}$$

and:

λ – converse of beam replacing length,

α – coefficient between actual velocity and critical velocity,

β – coefficient between actual damping and critical damping.

Making substituting: $y = e^{\gamma s}$, we obtain:

$$\gamma^4 + 4 \cdot \alpha^2 \cdot \gamma^2 - 8 \cdot \alpha \cdot \beta \cdot \gamma + 4 = 0 \tag{2.15}$$

with solutions in following form [8]:

$$s \geq 0\ w = A_1 \cdot e^{\gamma_1 \cdot s} + A_2 \cdot e^{\gamma_2 \cdot s}\ \ s < 0\ w = A_3 \cdot e^{\gamma_3 \cdot s} + A_4 \cdot e^{\gamma_4 \cdot s} \tag{2.16}$$

The constants A_i we calculate from boundary conditions, e.g. in matrix notation in form:

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ \gamma_1 & \gamma_2 & -\gamma_3 & -\gamma_4 \\ \gamma_1^2 & \gamma_2^2 & -\gamma_3^2 & -\gamma_4^2 \\ \gamma_1^3 & \gamma_2^3 & -\gamma_3^3 & -\gamma_4^3 \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \frac{P}{8 \cdot EI \cdot \lambda^3} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 8 \end{bmatrix} \tag{2.17}$$

Analytical form of differential equation solution (2.18) (where ρ – unit mass of beam, c_t – damping coefficient):

$$E_S \cdot I_\varsigma \cdot \frac{\partial^4 y}{\partial x^4} + \rho \cdot \frac{\partial^2 y}{\partial t^2} + c_t \cdot \frac{\partial y}{\partial t} + k \cdot y = P(x,t), \tag{2.18}$$

is given in publication [10]. For differential equation of infinity beam on Winkler's foundation with moving concentrated force with constant velocity, is assumed a solution in form [10]:

$$y = sin\alpha(x - v \cdot t) \tag{2.19}$$

with solution in form $y_1(x)$ for x<0:

$$y_1(x)=\frac{P\cdot\lambda}{2\cdot k}\cdot\left[\frac{\eta\cdot e^{\eta\cdot\lambda\cdot x}}{\eta^4+(\eta\cdot\theta)^2+\frac{1}{2}\cdot\left(\frac{\theta\cdot\beta}{\eta}\right)^2}\right]\cdot\left[\begin{array}{l}\dfrac{-\left(\frac{\theta\cdot\beta}{\eta}+\eta^2\right)\cdot sin\left[\left(2\cdot\theta^2+\eta^2-2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{2}}\cdot(\lambda\cdot x)\right]}{\eta\cdot\left(2\cdot\theta^2+\eta^2-2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{4}}}+\\ +cos\left[\left(2\cdot\theta^2+\eta^2-2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{2}}\cdot(\lambda\cdot x)\right]\end{array}\right] \quad (2.20)$$

and $y_2(x)$ for x>0:

$$y_2(x)=\frac{P\cdot\lambda}{2\cdot k}\cdot\left[\frac{\eta\cdot e^{-\eta\cdot\lambda\cdot x}}{\eta^4+(\eta\cdot\theta)^2+\frac{1}{2}\cdot\left(\frac{\theta\cdot\beta}{\eta}\right)^2}\right]\cdot\left[\begin{array}{l}\dfrac{-\left(\frac{\theta\cdot\beta}{\eta}-\eta^2\right)\cdot sin\left[\left(2\cdot\theta^2+\eta^2+2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{2}}\cdot(\lambda\cdot x)\right]}{\eta\cdot\left(2\cdot\theta^2+\eta^2+2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{4}}}+\\ +cos\left[\left(2\cdot\theta^2+\eta^2+2\cdot\frac{\theta\cdot\beta}{\eta}\right)^{\frac{1}{2}}\cdot(\lambda\cdot x)\right]\end{array}\right] \quad (2.21)$$

Adequate parameters denote [10]:
k – coefficient of Winkler's foundation [kPa], ρ – unit mass of beam [kg/m],
c_t – damping coefficient [kNs/m^2] and:

$$v_{kr}=\left(\frac{4\cdot k\cdot EI}{\rho^2}\right)^{\frac{1}{4}};\ \boxed{\theta=\frac{v}{v_{kr}}};\ \lambda=\left(\frac{k}{4\cdot EI}\right)^{\frac{1}{4}};\ c_{kr}=2\cdot(k\cdot\rho)^{\frac{1}{2}};\ \boxed{\beta=\frac{c_t}{c_{kr}}} \quad (2.22)$$

Dimensionless coefficient η satisfies an equation:

$$\eta^6+2\cdot\theta^2\cdot\eta^4+\left(\theta^4-1\right)\cdot\eta^2-\theta^2\cdot\beta^2=0 \quad (2.23)$$

Due to the lack of damping $\beta = 0$, we obtain: $\eta = (1-\theta^2)^{1/2}$.

2.4. Calculation example

Data for calculation (60E1 rail; wooden sleeper, ballasted roadbed):
– EI = 6,4155 [MNm^2]; k=U = 40 [MPa] → L = 0,895 [m],
– P = 100 [kN] (concentrated force).

On fig. 2.3 the diagrams of track deflections for constant velocity (θ = 1,0 case) and damping changes (parameter β) are shown, therefore on fig 2.4 an influence of velocity growth (parameter θ) and damping (parametr β) on the shape of rail deflections are presented:

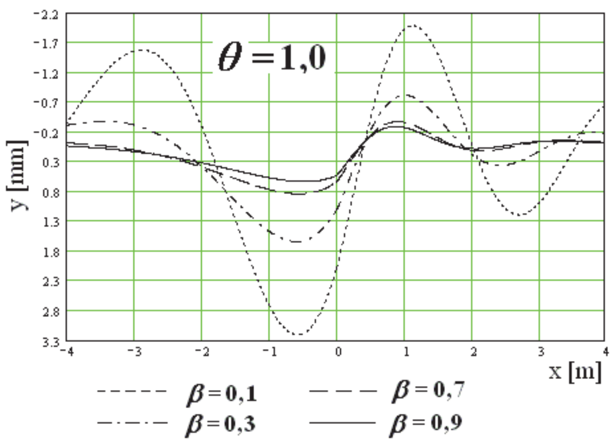

Fig. 2.3. Changes of rail deflections for θ = 1,0 (v = 1,0 v_{kr})
and damping increase (β = 0,1; β = 0,3; β = 0,7; β = 0,9)

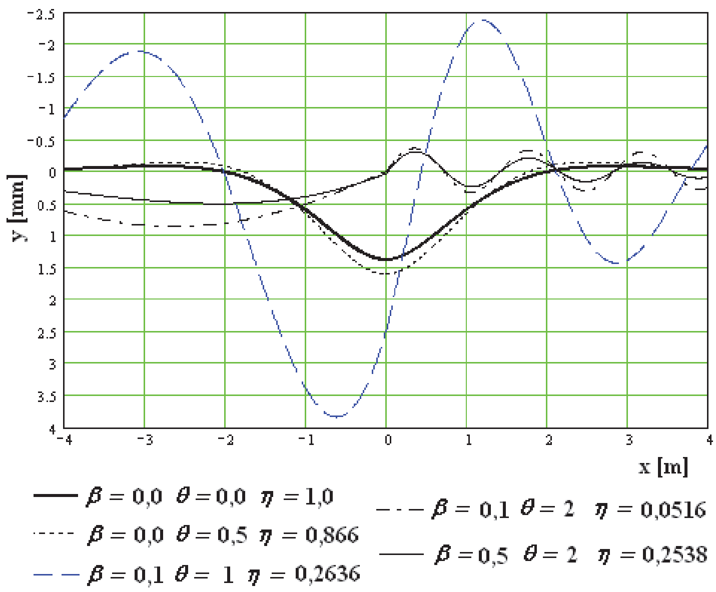

Fig. 2.4. Dynamical deflection of rail under moving concentrated force
for various coefficients of damping (β) and velocity (θ)

However with assumption: $c_t = 0$ and $v = 0 \rightarrow \theta = 0$, $\beta = 0 \rightarrow \eta = 1{,}0$, we obtain a transition to static analysis of single concentrated force on beam at infinite length:

$$y_1(x) = \frac{P \cdot \lambda}{2 \cdot k} \cdot \left\{ e^{\lambda \cdot x} \cdot \left[-\sin(\lambda \cdot x) + \cos(\lambda \cdot x) \right] \right\} \text{ for } x < 0 \qquad (2.24)$$

$$y_2(x)=\frac{P\cdot\lambda}{2\cdot k}\cdot\left\{e^{-\lambda\cdot x}\cdot\left[sin(\lambda\cdot x)+cos(\lambda\cdot x)\right]\right\} \text{ for } x>0 \text{ with}$$
$$\lambda=\frac{1}{L}\left[m^{-1}\right]. \tag{2.25}$$

Making use from formulas (2.24) and (2.25) it is possible to present a relation of dynamic deflection to static deflection in the influence of velocity and damping (fig. 2.5):

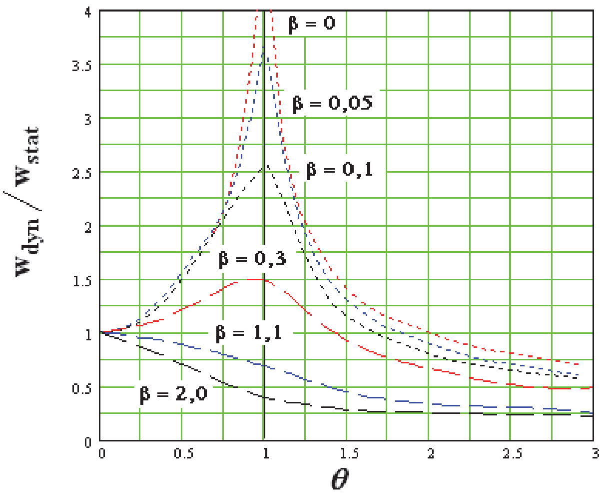

Fig. 2.5. Changes of dynamic deflections in dependence upon velocity for moving P force [8,10]

3. ANALYSIS OF INFLUENCE OF VELOCITY, TRACK MASS AND DAMPING ON DYNAMIC RAILWAY TRACK DEFLECTION ON 2-PARAMETER FOUNDATION

3.1. Differential equation with damping on 2-parameter foundation

Differential equation for track resting on 2-parameter foundation has got a form [9,13,14]:

$$EI\cdot\frac{\partial^4 w}{\partial x^4}-k_1\cdot\frac{\partial^2 w}{\partial x^2}+k\cdot w+\rho\cdot\frac{\partial^2 w}{\partial t^2}+c_t\cdot\frac{dw}{dt}=P(x,t) \tag{3.1}$$

where:

$w(x,t)$ – beam deflection [m],
EI – beam stiffness [Nm2],
k – first parameter of foundation [N/m^2],
k_1 – second parameter of foundation [N],
ρ – unit mass of beam [kg/m],
c_t – damping coefficient [Ns/m^2]
P – moving concentrated force [N],
v – speed [m/s], x [m], t [s] – time.

Introducing a Dirac delta we obtain [14]:

$$EI \cdot \frac{\partial^4 w}{\partial x^4} - k_1 \cdot \frac{\partial^2 w}{\partial x^2} + k \cdot w + \rho \cdot \frac{\partial^2 w}{\partial t^2} + c_t \cdot \frac{dw}{dt} = P\delta(x - vt) \tag{3.2}$$

Dividing by *EI* we got:

$$\frac{\partial^4 w}{\partial x^4} - \frac{k_1}{EI} \cdot \frac{\partial^2 w}{\partial x^2} + \frac{k}{EI} \cdot w + \frac{\rho}{EI} \cdot \frac{\partial^2 w}{\partial t^2} + \frac{c_t}{EI} \cdot \frac{dw}{dt} = \frac{P}{EI}\delta(x - vt) \tag{3.3}$$

while [14]:

$$a = \frac{\rho}{2 \cdot EI};\ b^2 = \frac{k}{EI};\ c_1 = \frac{k_1}{2 \cdot EI};\ d = \frac{c_t}{EI}. \tag{3.4}$$

Therefore we obtain a equation form of (3.3):

$$\frac{\partial^4 w}{\partial x^4} - 2 \cdot c_1 \cdot \frac{\partial^2 w}{\partial x^2} + b^2 \cdot w + 2 \cdot a \cdot \frac{\partial^2 w}{\partial t^2} + d \cdot \frac{dw}{dt} = \frac{P}{EI}\delta(x - vt). \tag{3.5}$$

Boundary conditions have got a following form:

$$\begin{gathered} w_1(0) = w_2(0);\ w_1'(0) = w_2'(0);\ w_1''(0) = w_2''(0); \\ w_1'''(0) - w_2'''(0) = \frac{P}{EI} \end{gathered} \tag{3.6}$$

Writing ξ = x – vt, we obtain:

$$\frac{\partial^4 w}{\partial \xi^4} - 2 \cdot c_1 \cdot \frac{\partial^2 w}{\partial \xi^2} + b^2 \cdot w + 2 \cdot a \cdot v^2 \cdot \frac{\partial^2 w}{\partial \xi^2} - d \cdot v \cdot \frac{dw}{d\xi} = 0 \tag{3.7}$$

Substituting: $w = e^{m\xi}$, we obtain:

$$m^4 - 2 \cdot c_1 \cdot m^2 + 2 \cdot a \cdot v^2 \cdot m^2 + b^2 - d \cdot v \cdot m = 0. \tag{3.8}$$

Writing a critical damping as:

$$d_{cr} = 2\sqrt{2} \cdot b \cdot \sqrt{a}, \tag{3.9}$$

that for $d < d_{cr}$ a critical velocity of moving load: $v_{cr} = \sqrt{(b + c_1)/a}$, the roots of equation (3.8) are in following form [14]:

$$\begin{gathered} m_1 = -p + i \cdot q;\ m_2 = -p - i \cdot q;\ m_3 = p + i \cdot r; \\ m_4 = p - i \cdot r. \end{gathered} \tag{3.10}$$

Hence a solution of track deflections form is written in shape:

$$w_1(\xi)=e^{-p\cdot\xi}\cdot(A\cdot cos\,q\xi+B\,sin\,q\xi)\ \text{for}\ \ \xi<0, \tag{3.11}$$

$$w_2(\xi)=e^{p\cdot\xi}\cdot(C\cdot cos\,r\xi+D\,sin\,r\xi)\ \text{for}\ \ \xi<0. \tag{3.12}$$

Making use from boundary conditions (3.6), we obtain the equations system:

$$\begin{gathered} A-C=0, \\ -2\cdot p\cdot A+B\cdot q-D\cdot r=0, \\ r^2-q^2\cdot A-2\cdot p\cdot q\cdot B-2\cdot p\cdot r\cdot D=0, \\ p\cdot\left(3\cdot q^2-2\cdot p^2+3\cdot r^2\right)\cdot A+q\cdot 3\cdot p^2-q^2\cdot B+r\cdot\left(r^2-3\cdot p^2\right)\cdot D=\frac{P}{EI}. \end{gathered} \tag{3.13}$$

Solving the equations system (3.13), we obtain the analytical dependences on required parameters: *A*, *B*, *C* and *D* (3.14):

$$\begin{gathered} A=4\cdot P\cdot\frac{p}{EI\cdot\left(16\cdot p^4+8\cdot p^2\cdot q^2+q^4-2\cdot q^2\cdot r^2+8\cdot p^2\cdot r^2+r^4\right)}, \\ B=\frac{P}{EI\cdot\left(16\cdot p^4+8\cdot p^2\cdot q^2+q^4-2\cdot q^2\cdot r^2+8\cdot p^2\cdot r^2+r^4\right)}\cdot\frac{\left(-q^2+4\cdot p^2+r^2\right)}{q}, \\ C=4\cdot P\cdot\frac{p}{EI\cdot\left(16\cdot p^4+8\cdot p^2\cdot q^2+q^4-2\cdot q^2\cdot r^2+8\cdot p^2\cdot r^2+r^4\right)}, \\ D=P\cdot\frac{\left(-4\cdot p^2-q^2+r^2\right)}{EI\cdot\left(16\cdot p^4+8\cdot p^2\cdot q^2+q^4-2\cdot q^2\cdot r^2+8\cdot p^2\cdot r^2+r^4\right)\cdot r}. \end{gathered} \tag{3.14}$$

3.2. Calculation example

Data for calculations (rail 60E1; wooden sleepers, ballasted roadbed):

I. 1-parameter foundation:
– EI = 6,4155 [MNm2]; ρ = 133 [kg/m]; P = 100 [kN] (concentrated force),
– k = 50,78 [MPa] → L = 0,843 [m] and λ = 1,186 [1/m],
– ckr = 164,362 [kNs/m2].

II. 2-parameter foundation [5,6]:
– EI = 6,4155 [MNm2]; ρ = 133 [kg/m]; P = 100 [kN] (concentrated force),
– k = 50,78 [MPa]; k1 = 1,669 [MN]; ckr = 164,362 [MNs2/m/m],
For v = 0,25*vkr and ct = 0,1*ckr for 2-parameter foundation we obtain:
– p = 1,1751; q = 1,2278; r = 1,1671
– A = 1,1784*10-3; B = 1,0981*10-3; C = 1,1784*10-3; D = – 1,2177*10-3,

For v = 1,0*vkr and ct = 0,3*ckr for 2-parameter foundation we obtain:

– p = 0,5392; q = 2,2373; r = 1,0977

– A = 1,11262*10-3; B = – 6,0817*10-4; C = 1,11262*10-3; D = – 2,3326*10-3.

The results of calculations are shown on Fig. 3.1.

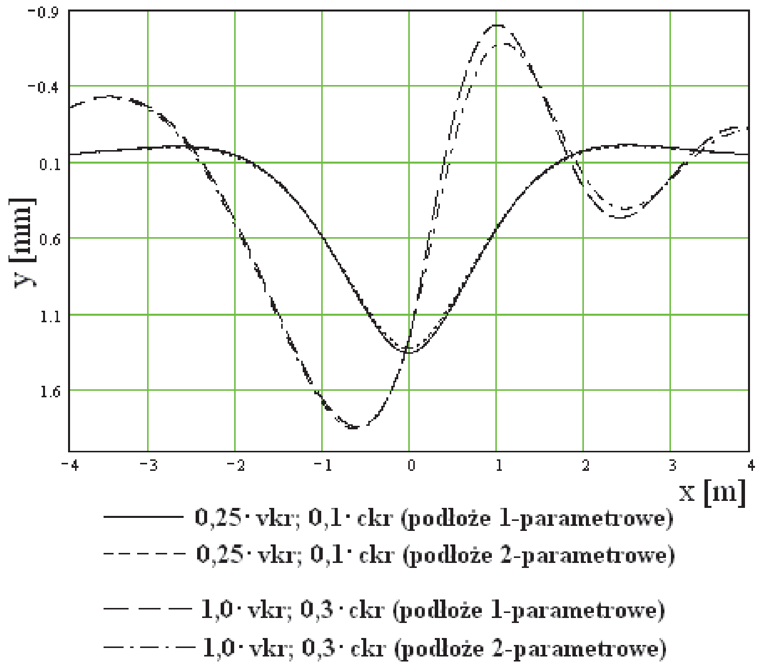

Fig. 3.1.Track deflections for moving concentrated force *P* for 1 – and 2 – parameter foundation

4. CONCLUSIONS

In the paper an analysis of influence of moving concentrated force on dynamic deflection of railway track resting on one- and two-parameter is presented. The specific stages of theoretical considerations in the paper are illustrated by adequate calculation examples (showing an influence of particular parameters on dynamic deflections of railway track).

Particularly an unfavorable for track deflection is the case of large velocity of moving concentrated force (v≈vkr) with lower value of damping coefficient (c≈(0.1÷0,3)ckr), what clearly can be seen on figs. 2.4 and 3.1.

As can be seen on fig. 2.5 a damping coefficient growth ($\beta>1$) induces a values decrease of dynamic railway track deflections. On other hand a damping coefficient value (while $\beta\to0$) induces a significant growth of railway track deflection.

Particular meaning for dynamic deflection of railway track has got first of all a value of velocity and damping (fig. 3.1). However an influence of assumed calculation model of elastic foundation (two-parameter foundation and one-parameter foundation) has got a less meaning on form and obtained values of railway track dynamic deflections.

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DYNAMICZNA ANALIZA PRACY BEZSTYKOWEGO TORU KOLEJOWEGO NA SPRĘŻYSTYM PODŁOŻU JEDNO- I DWUPARAMETROWYM

Streszczenie

W pracy przedstawiono analizę dynamicznych ugięć bezstykowego toru kolejowego spoczywającego na podłożu jedno- i dwuparametrowym, rozpatrywaną w literaturze z zakresu dróg kolejowych [1, 7, 11, 17, 19-27]. Przeanalizowano model toru na podłożu bez tłumienia oraz model obliczeniowy z uwzględnieniem tłumienia. Dla podłoża jedno- i dwuparametrowego zamieszczono stosowne przykłady obliczeniowe, obrazujące wpływ zmian prędkości, tłumienia i parametrów podłoża oraz parametrów wytrzymałościowych toru.

Praca posiada także charakter przeglądowy, prezentuje postać wzorów na obliczanie niezbędnych parametrów do obliczeń dynamicznych ugięć toru kolejowego.

Dane autora:
dr hab. inż. Włodzimierz Andrzej Bednarek
Politechnika Poznańska
Instytut Inżynierii Lądowej
e-mail: wlodzimierz.bednarek@put.poznan.pl
telefon: +48 61 665 24 07
fax: +48 61 665 26 38