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CARRYING CAPACITY AND FRICTION FORCES IN A TRANSVERSE JOURNAL BEARING, LUBRICATED WITH NON-NEWTONIAN OIL

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Abstract

In this article, the authors present the results of numerical calculations. Calculations concern dimensionless carrying capacity and friction forces in a transverse journal bearing, lubricated by the oil of non-Newtonian properties. For analytical-numerical considerations a model of apparent viscosity changes based on exploitation time, pressure, temperature, shear rate was assumed The non-Newtonian properties of lubricating oil were characterized by increasing viscosity with increasing shear rate and described as an additional part in the constitutive equation$\beta_3 \cdot tr(\mathbf{A}_1^2)\mathbf{A}_1$.

Analytical-numerical calculations were performed for smooth, non-porous plain bearing with full angle of wrap. Non-isothermal, laminar and fixed flow of lubricant in the lubrication gap of the journal bearing was assumed. Numerical calculations of hydrodynamic pressure distribution were made for Reynolds boundary conditions. The finite difference method was used to determine the Reynolds equation and the successive approximation method by taking into account the influence of pressure, temperature and non-Newtonian properties on the change of apparent viscosity. The results of the calculations are presented in the form of graphs and tables illustrating the influence of relative eccentricity and pressure, temperature and non-Newtonian properties on changes in the dimensionless load and friction force. Analysis of the obtained results illustrates the high-pressure effect on the increase of the carrying capacity and friction force for high relative eccentricities. A similar situation is by considering the non-Newtonian properties.

Keywords: *slide journal bearing, hydrodynamic pressure, load carrying capacity, friction force, non-Newtonian oil*

1. Introduction

The issue of hydrodynamic lubrication of the journal bearings with the oils of Newtonian properties is already known and well researched. In the numerous papers following, influences at the lubrication oil viscosity: flow type, kind of sliding surfaces, temperature, pressure, exploitation time are taken into account.

Much more less research concerns the issue of hydrodynamic lubrication of the journal bearings with the oils of non-Newtonian properties. Especially in terms of numerical calculations of hydrodynamic pressure, carrying capacities and friction forces. Lubricants for slider bearings lubrication, during exploitation time are subject of deterioration (ageing process) and those properties change into non-Newtonian [4, 10-12]. Depending on lubricant's properties, different constitutive models are adopted, starting from the first order model [7, 13], through second order models [5, 8, 11], and ending with the third order models [1-3, 6, 9]. In this article, the authors attempted to determine the distribution of hydrodynamic pressure, carrying capacity, friction force taking into account apparent viscosity changes from pressure and non-Newtonian properties. For the analytical-numerical research, the first order constitutive mode has been adopted. This model takes viscosity increase by the shear rate increase into account.

2. Basic equations

Solving of the journal bearing lubrication problem, with neglecting of the mass forces, includes solution of the basic equations, which are: conservation of momentum, flow continuity equation and conservation of energy in the following form [10-14]:

$$\rho \frac{d\mathbf{v}}{dt} = \text{Div}\,\mathbf{S}\,, \tag{1}$$

$$\text{div}(\rho\mathbf{v}) = 0\,, \tag{2}$$

$$\text{div}(\kappa\,\text{grad}\,T) + \text{div}(\mathbf{vS}) - \mathbf{v}\text{Div}\mathbf{S} = \rho\frac{d(c_v T)}{d\,t}\,, \tag{3}$$

where:
c_v – specific heat at constant volume [J/(kg·K)],
t – time [s],
v – oil velocity vector [$m\cdot s^{-1}$],
T – oil temperature distribution in the lubrication gap [K],
ρ – oil density [$kg\cdot m^{-3}$],
κ – lubrication oil conductivity [W/(m·K)].

Relationship, which describes correlation between coordinates of stress tensor **S** and shear rate coordinates $\mathbf{A}_1$ of the lubrication oil of non-Newtonian properties, was assumed in the following form:

$$\mathbf{S} = -p\mathbf{I} + [\eta + \beta_3 \text{tr}(\mathbf{A}_1^2)]\mathbf{A}_1 = -p\mathbf{I} + \eta_p\mathbf{A}_1, \quad \text{where} \quad \eta_p = \eta + \beta_3 \text{tr}(\mathbf{A}_1^2)\,, \tag{4}$$

where:
I – unity tensor,
p – hydrodynamic pressure [Pa],
β_3 – material coefficient [$Pa\cdot s^3$],
η – dynamic viscosity coefficient [Pa·s],
η_p – apparent viscosity coefficient [Pa·s].

In the equation (4) tensor $\mathbf{A}_1$, is described by the following relation [1-3], [5-8], [10-14]:

$$\mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^T,\ \mathbf{L} \equiv \text{grad}(\mathbf{v})\,, \tag{5}$$

where:
L – tensor of the velocity vector gradient [s^{-1}].

Apparent viscosity can be presented as a function dependent on the temperature, pressure, exploitation time and shear rate $\eta_p=\eta_p(p, T, t, \theta)$. Authors propose to present apparent viscosity η_p as a product of the dimensional value η_o and dimensionless dependencies of the several influences:

$$\eta_p = \eta_o \cdot \eta_{p1}\,,\ \ \eta_{p1} = \eta_1 + \eta_{1\theta}\,,\ \ \eta_1 = \eta_{1p}\cdot\eta_{1T}\cdot\eta_{1t}\,,\ \ \eta_{1p}(\varphi,z) = a\cdot e^{\delta_p\cdot p_o\cdot p_1} = a\cdot e^{\delta_{p1}p_1}\,,$$
$$\eta_{1T}(\varphi,z,r) \equiv b\cdot e^{-\delta_T(T-T_o)} = b\cdot e^{-Q_{Br}T_1}\,,\ \ \eta_{1t}(t) \equiv d\cdot e^{\delta_t\cdot t} = d\cdot e^{\delta_{t1}\cdot t_1}\,,\ \ \eta_{1\theta} = \left[\frac{\beta_3}{\eta_o}\text{tr}(\mathbf{A}_1^2)\right], \tag{6}$$

where:
η_1 – dimensionless function of the viscosity changes, dependent on the pressure, temperature and exploitation time,
η_{1p} – dimensionless function of the viscosity changes, dependent on the pressure,
η_{1t} – dimensionless function of the viscosity changes, dependent on the exploitation time,
η_{1T} – dimensionless function of the viscosity changes, dependent on the temperature,

$\eta_{1\theta}$ – dimensionless function of the viscosity changes, dependent on the shear rate,
η_{p1} – dimensionless function of the apparent viscosity changes,
δ_T, δ_t, δ_p – dimensional values of the material coefficients which include viscosity changes in temperature, time and pressure,
δ_{t1}, δ_{p1} – dimensionless values of the material coefficients which include viscosity changes in time and pressure,
Q_{Br} – dimensionless coefficient of the viscosity changes in temperature T,
T_1 – dimensionless function of the oil temperature,
T_o – dimensional characteristic value of the temperature [K],
a, b, d – dimensionless coefficients which include different values of the characteristic viscosity η_o designated in the research on the rheometer, depending on the different influences (temperature, pressure, exploitation time).

In order to make several quantities dimensionless and to estimate an order of magnitude of each part of the system of equations: conservation of momentum, flow continuity, conservation of energy and also track of the tensor $\mathbf{A}_1$, the following dimensional and dimensionless designations and characteristic numbers were assumed [11-14]:

$$t=t_o\cdot t_1,\ r=R(1+\psi r_1),\ z=bz_1,\ h_p=h_{p1}\cdot\varepsilon,\ p=p_o p_1,\ \kappa=\kappa_o\kappa_1,\ \rho=\rho_o\cdot\rho_1,\ v_\phi=Uv_1,\ v_r=U\psi v_2,\ \varepsilon=R'-R,$$

$$v_z=\frac{U}{L_1}v_3,\ \psi\equiv\frac{\varepsilon}{R}\cong 10^{-3},\ L_1\equiv\frac{b}{R},\ \mathrm{Re}\equiv\frac{U\varepsilon\rho_o}{\eta_o},\ p_o\equiv\frac{RU\eta_o}{\varepsilon^2},\ \mathrm{Str}\equiv\frac{R}{Ut_o},\ T=T_o+T_o\mathrm{Br}T_1,\tag{7}$$

$$\mathrm{Gz}=\frac{\varepsilon^2\rho_o\omega\cdot c_v}{\kappa_o},\ \mathrm{Br}\equiv\frac{U^2\eta_o}{\kappa_o T_o},\ 0<Q_{Br}\equiv\mathrm{Br}T_o\delta_T<1,$$

where:
Br – dimensionless Brinkman number,
Gz – Graetz number, which describes forced heat convection,
L_1 – dimensionless bearing length,
R – journal radius [m],
R' – bushing radius [m],
Re – Reynolds number, which describes type of the flow,
Str – Strouhal number, which describes unsteady flow,
U – dimensional value of the perimeter velocity [$m\cdot s^{-1}$],
2b – bearing length [m],
h_p – dimensional height of the lubrication gap, which depends on relative eccentricity and axes skew [m],
h_{p1} – dimensionless height of the lubrication gap, which depends on relative eccentricity and axes skew,
p_o – dimensional value of the characteristic pressure [Pa],
p_1 – dimensionless value of the hydrodynamic pressure,
r – radial coordinate in the lubrication gap [m],
r_1 – dimensionless radial coordinate,
t_o – dimensional time [s],
t_1 – dimensionless time,
z – lengthwise coordinate [m],
z_1 – dimensionless lengthwise coordinate,
ε – radial clearance [m],
κ_o – dimensional value of the lubricants heat conduction coefficient [$W\cdot m^{-1}\cdot K^{-1}$],
κ_1 – dimensionless value of the lubricants heat conduction coefficient,
λ – relative eccentricity,

υ_o – dimensional value of the lubricants convective heat transfer coefficient [$W\cdot m^{-2}\cdot K^{-1}$],
υ_1 – dimensionless value of the lubricants convective heat transfer coefficient,
ρ_o – dimensional value of the lubricants density [$kg\cdot m^{-3}$],
ρ_1 – dimensionless value of the lubricants density,
φ – perimeter coordinate,
ψ – dimensionless value of the relative radial clearance,
ω – angular velocity of the bearings journal [s^{-1}].

By neglecting parts in order of relative radial clearance $\psi\approx0.001$ we get:

$$\mathrm{tr}(\mathbf{A}_1^2)=\frac{2U^2}{\varepsilon^2}\left[\left(\frac{\partial v_1}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial r_1}\right)^2\right]. \tag{8}$$

By the substitution of the estimated equation (16) into equation (10_6) we get dimensionless function of the viscosity changes, depending on non-Newtonian properties in a form:

$$\eta_{1\Theta}=\left[2\frac{\beta_3}{\eta_o}\frac{U^2}{\varepsilon^2}\left[\left(\frac{\partial v_1}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial r_1}\right)^2\right]\right]=\left[2\cdot D_\beta\left[\left(\frac{\partial v_1}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial r_1}\right)^2\right]\right]. \tag{9}$$

where:

$D_\beta=\dfrac{\beta_3}{\eta_o}\dfrac{U^2}{\varepsilon^2}$ – dimensionless number.

It was assumed for the further analysis of the basic equations (1)-(3) that dimensionless heat transfer coefficient $\kappa_1=1$, dimensionless convective heat transfer coefficient $\upsilon_1=1$ and dimensionless density $\rho_1=1$ are constant and independent on the temperature and pressure [4, 5, 9, 21]. Neglected are the inertia forces in the momentum equations – elements multiplicated by $Re\cdot\psi$. The elements, which are multiplicated by the Graetz number Gz, concern forced convection and are also neglected. This neglection is reasonable in the low- and medium speed bearings [4, 5, 9, 21]. Another assumption is steady and stationary flow, so the elements, which include derivatives relative to time, were neglected. Neglected were also parts in the same order as relative radial clearance $\psi\approx0.001$. After these simplifications, momentum equations get a form [11-14]:

$$0=-\frac{\partial p_1}{\partial\varphi}+\frac{\partial}{\partial r_1}\left(\eta_{p1}\frac{\partial v_1}{\partial r_1}\right), \tag{10}$$

$$\frac{\partial p_1}{\partial r_1}=0, \tag{11}$$

$$0=-\frac{\partial p_1}{\partial z_1}+\frac{\partial}{\partial r_1}\left(\eta_{p1}\frac{\partial v_3}{\partial r_1}\right), \tag{12}$$

$$\frac{\partial v_1}{\partial\varphi}+\frac{\partial v_2}{\partial r_1}+\frac{1}{L_1^2}\frac{\partial v_3}{\partial z_1}=0, \tag{13}$$

$$\frac{\partial}{\partial r_1}\left(\kappa_1\frac{\partial T_1}{\partial r_1}\right)+\eta_{p1}\left[\left(\frac{\partial v_1}{\partial r_1}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial r_1}\right)^2\right]=0, \tag{14}$$

where: $0<r_1<h_{p1}$, $0<\varphi<2\pi$, $-1<z_1<+1$.

Characteristic dimensionless height of the lubrication gap $h_{p1}(\phi,z)$ in the cylindrical bearing, assumed as a function of the perimeter- and longitudinal variable, so nonparallelism of the bushing axis in relation to journal axis was taken into account. Dimensional height h_p of the lubrication gap depends on relative eccentricity λ and nonparallelism of the journal axis in relation to bushing axis γ [11]:

$$h_p(\varphi,z)=\varepsilon\cdot\left[1+\lambda\cdot\cos(\varphi)+a_\gamma z_1\cos(\varphi)\right]\ ,\ a_\gamma=\frac{L_1}{\psi}\tan\left(\gamma\right),\tag{15}$$

where:

a_γ – dimensionless skew coefficient.

With the assumption in the first calculation step, that the apparent viscosity does not depend on pressure, temperature and shear rate, we designate from the system of equations (10)-(14), by the proper integration and applying of the classical boundary conditions, dimensionless components of the velocity vector, Reynolds-type equation and dimensionless temperature. From the Reynolds-type equation, with the proper numerical method, we designate hydrodynamic pressure. Designated in the first step values of the pressure will be used to designate the components of the velocity vector, temperature and apparent viscosity in the second calculation step. From the second step, we will get components of the velocity vector and temperature. These values we will use to designate adjusted apparent viscosity and once again, we designate hydrodynamic pressure from the Reynolds-type equation. We proceed these activities until we get convergent results.

For the oil velocity vector components, by the stationary lubrication, the following boundary conditions are assumed:

$$\begin{aligned}&v_1=0,\quad v_2=0,\quad v_3=0\qquad \text{na panewce}\qquad r_1=h_{p1},\\&v_1=1,\quad v_2=0,\quad v_3=0\qquad \text{na czopie}\qquad r_1=0.\end{aligned}\ .\tag{16}$$

For the hydrodynamic pressure distribution in oil, by the stationary lubrication, the following Reynolds boundary conditions are assumed in a form [10-14]:

$$p_1=0 \text{ for } \varphi=\varphi_p,\ \ p_1=0 \text{ for } \varphi\ge\varphi_k,\ \frac{\partial p_1}{\partial\varphi}=0 \text{ for } \varphi=\varphi_k,\ p_1=0 \text{ for } z_1=+1 \text{ and } z_1=-1.\tag{17}$$

After designation of the velocity vector components from the equation (10)-(13), using boundary conditions (16) and integration of the equation (13) after that and applying of the proper boundary condition (16), the following Reynolds-type equation has been obtained:

$$\frac{\partial}{\partial\varphi}\left[\frac{h_{p1}^3}{\eta_{p1}}\left(\frac{\partial p_1}{\partial\varphi}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_{p1}^3}{\eta_{p1}}\left(\frac{\partial p_1}{\partial z_1}\right)\right]=6\frac{\partial h_{p1}}{\partial\varphi}.\tag{18}$$

Temperature distribution in the lubrication gap we get by the double integration of the equation (14) and applying of the proper boundary conditions [11]:

$$\begin{aligned}T_1(r_1,\varphi,z_1)&=1+\frac{1}{2}\eta_{p1}(1-2s)-q_{1c}h_{p1}s-\frac{1}{6}h_{p1}^2\left(\frac{\partial p_1}{\partial\varphi}\right)s\,(3-3s+s^2)+\\&-\frac{1}{2}\eta_{p1}\left[\left(v_1\right)^2+\frac{1}{L_1^2}\left(v_3\right)^2\right]+\frac{1}{24\eta_{p1}}h_{p1}^4\left[\left(\frac{\partial p_1}{\partial\varphi}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial p_1}{\partial z_1}\right)^2\right]s^3(s-2),\end{aligned}\tag{19}$$

where $s\equiv r_1/h_{p1}$ while $0\le s\le +1$, $0\le\varphi<2\pi$, $-1\le z_1\le +1$, q_{1c} – dimensionless heat flow in the journal.

With the assumption s=1, equation (19) returns unknown function of the temperature distribution on the bearing f_{1p}, dependent on the angle of wrap and bearing length.

Carrying capacity and friction force are to be designated from the following relation [11, 14]:

$$C = C_1 \cdot bR\eta_o\omega/\psi^2 \text{ where } C_1 = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1 \cos\gamma \sin\varphi \, d\varphi\right)dz_1\right)^2 + \left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1 \cos\gamma \cos\varphi \, d\varphi\right)dz_1\right)^2},$$

$$Fr = Fr_1 \cdot bR\eta_o\omega/\psi = Fr_1 = \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{p1}\frac{\partial v_1}{\partial r_1}\right)_{r_1=h_{p1}} d\varphi\right]dz_1 . \quad (20)$$

3. Numerical calculations

Numerical calculations of the hydrodynamic pressure and carrying capacity and friction force after that, have been proceed for the relative eccentricity from $\lambda = 0.1$; to $\lambda = 0.9$, dimensionless bearing length $L_1 = 1$ and the angle between journal and bushing axis $\gamma = 0$. Calculations have been performed in Mathcad 15 software, using own calculation procedures, with the assumption of the exploitation time of $\tau = 20{,}000$ km, coefficient $\delta_\tau = 8\cdot10^{-6}\ \text{km}^{-1}$ dimensionless number $De_\beta = = 0.0064$, and dimensionless values of the material coefficients, which are taking viscosity changes in pressure $\delta_{p1} = 0.00756$ into account, dimensionless coefficient of the viscosity changes in temperature $Q_{Br} = 0.1575$, dimensionless heat flow $q_{c1} = -0.5$.

Changes of the dimensionless carrying capacity and friction force in the function of relative eccentricity and kind of influences are presented on Fig. 1. Numerical values of these changes and theirs percentage changes are presented in Tab. 1.

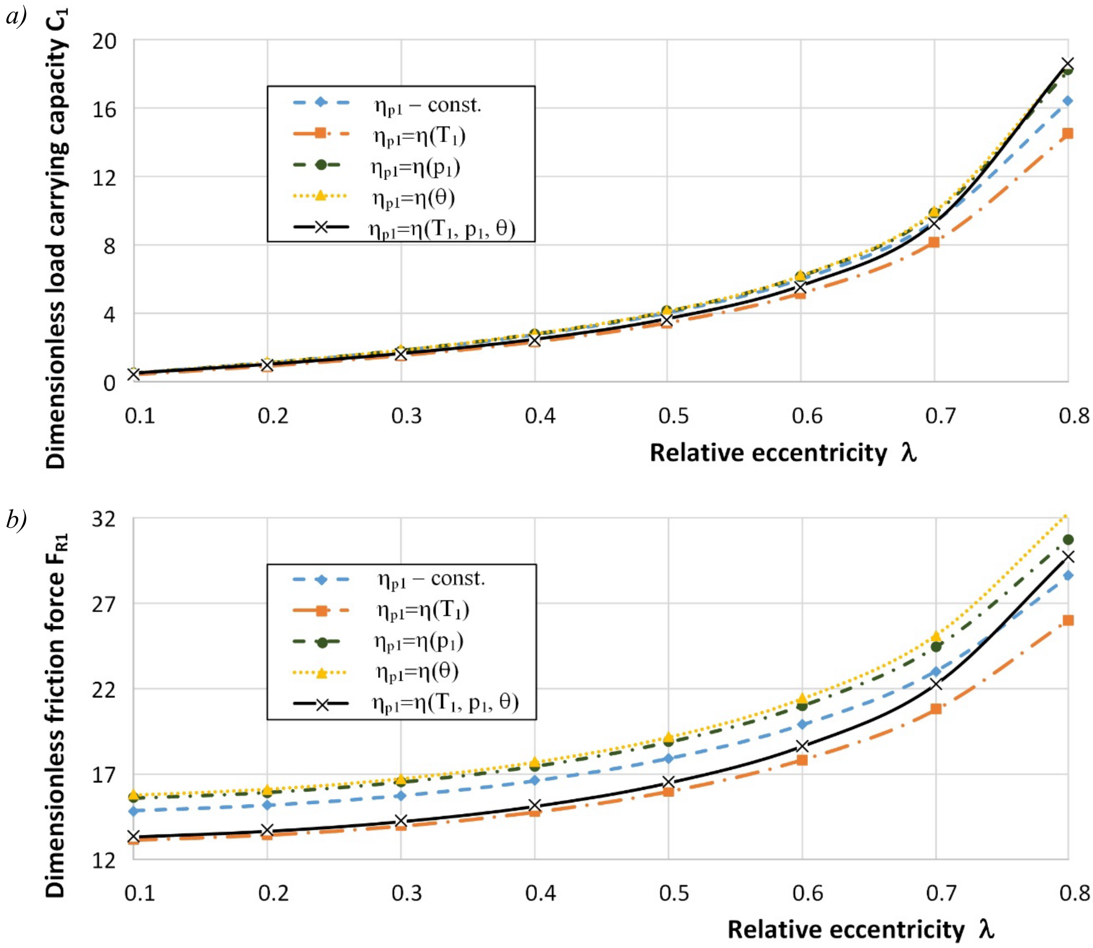

Fig. 1. Carrying capacity changes (a) and friction force changes (b) in a function of relative eccentricity and type of influence on the viscosity

Tab. 1. Numerical values and percentage changes of the dimensionless carrying capacity and friction force in a function of relative eccentricity and type of influence on the oil viscosity changes

	Dimensionless load carrying capacity C_1								
Relative eccentricity	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
η_{p1} – const	0.540	1.137	1.852	2.776	4.074	6.037	9.416	16.430	38.450
$\eta_{p1} = \eta(T_1)$	0.455	0.961	1.570	2.365	3.493	5.219	8.225	14.540	34.560
$\eta_{p1} = \eta(p_1)$	0.541	1.142	1.866	2.812	4.144	6.221	9.888	18.250	60.580
$\eta_{p1} = \eta(\theta)$	0.546	1.151	1.879	2.830	4.171	6.247	10.01	18.620	60.080
$\eta_{p1} = \eta(T_1, p_1, \theta)$	0.463	0.979	1.608	2.441	3.653	5.593	9.295	18.660	100.600
	Dimensionless friction forces F_{R1}								
η_{p1} – const	14.84	15.15	15.71	16.58	17.89	19.86	22.99	28.57	41.77
$\eta_{p1} = \eta(T_1)$	13.15	13.44	13.96	14.77	15.98	17.82	20.75	26.02	38.60
$\eta_{p1} = \eta(p_1)$	15.60	15.93	16.53	17.46	18.86	20.98	24.40	30.72	49.13
$\eta_{p1} = \eta(\theta)$	15.77	16.11	16.72	17.69	19.15	21.40	25.11	32.32	55.20
$\eta_{p1} = \eta(T_1, p_1, \theta)$	13.34	13.67	14.23	15.12	16.48	18.61	22.21	29.71	64.27
	Percent change in the dimensionless load carrying capacity C_1								
$\Delta C_1(T_1)$ [%]	15.7	15.5	15.2	14.8	14.3	13.5	12.6	11.5	10.1
$\Delta C_1(p_1)$ [%]	–0.2	–0.4	–0.6	–1.3	–1.7	–3.0	–5.0	–11.1	–57.6
$\Delta C_1(\theta)$ [%]	–1.1	–1.2	–1.5	–1.9	–2.4	–3.5	–6.3	–13.3	–56.3
$\Delta C_1(T_1, p_1, \theta)$ [%]	14.3	13.9	13.2	12.1	10.3	7.4	1.3	–13.6	–161.6
	Percent change in the dimensionless friction forces F_{R1}								
$\Delta F_{R1}(T_1)$ [%]	11.4	11.3	11.1	10.9	10.7	10.3	9.7	8.9	7.6
$\Delta F_{R1}(p_1)$ [%]	–5.1	–5.1	–5.2	–5.3	–5.4	–5.6	–6.1	–7.5	–17.6
$\Delta F_{R1}(\theta)$ [%]	–6.3	–6.3	–6.4	–6.7	–7.0	–7.8	–9.2	–13.1	–32.2
$\Delta F_{R1}(T_1, p_1, \theta)$ [%]	10.1	9.8	9.4	8.8	7.9	6.3	3.4	–4.0	–53.9

Calculation example of the percentage changes of the dimensionless carrying capacity or friction force:

$$\Delta C_1(T_1) = \frac{C_1(\eta - \text{const.}) - C_1(\eta(T_1))}{C_1(\eta - \text{const.})} \cdot 100\%; \quad \Delta F_{R1}(p_1) = \frac{F_{R1}(\eta - \text{const.}) - F_{R1}(\eta(p_1))}{F_{R1}(\eta - \text{const.})} \cdot 100\%.$$

4. Conclusions

Analysing the obtained results can be stated:

- Carrying capacity (friction force) decreases in case of taking viscosity changes in temperature into account, in comparison to the carrying capacity (friction force) without taking viscosity changes and non-Newtonian properties into account. The biggest changes can be observed for the low relative eccentricities and the smaller by the highest relative eccentricities.
- Carrying capacity (friction force) increases in case of taking viscosity changes in pressure into account in comparison to the carrying capacity (friction force) without taking viscosity changes and non-Newtonian properties into account. The biggest changes can be observed for the high relative eccentricities and the smaller by the lower relative eccentricities. These changes may result from the assumed exponential model of the viscosity changes in pressure.

– Carrying capacity (friction force) increases in case of taking non-Newtonian properties into account in comparison to the value of carrying capacity (friction force) without taking viscosity changes and non-Newtonian properties into account. The biggest changes can be observed for the high relative eccentricities and the smaller by the lower relative eccentricities. The non-Newtonian properties were taken into account only by the part with the pseudo-viscosity coefficient β_3.

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