

Opportunities for Making Production-Related Decisions on the basis of Shadow Prices

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Abstract

This article describes how shadow prices can be used as active constraints (in this case constraints of mine production capacity) to address and support production-related decision-making. This is an algorithm from a post-optimal analysis developed by the author as part of a method for rationalising production decisions for a formal group (PGG, a company) of hard coal mines. Opportunities for using shadow prices are presented using examples of actual mines. The developed algorithm provides a quick way of obtaining information, with no need to solve the problem again, about possible gains or losses resulting from an increase or a decrease in a selected production limit, to determine how changes to such constraints will affect the profits and production and sales structures for specific coal sizes.

Keywords: shadow prices, post-optimal analysis, simplex algorithm

Introduction

The method presented here relies on what is known as the simplex tableau, generated on the basis of an optimisation model developed by the author. It constitutes an optimum solution for the production and sales of coal by mines [2, 3, 4, 5].

The tableau contains a complete set of balance-sheet equations and coefficients representing objective function sensitivity to changes in decision variables. In addition, it is a starting point for the post-optimal analysis, where shadow prices play a very important role, since they help assess the extent and direction of changes to the optimum value of the criterion function as constraints change. Shadow prices make it possible to predict, with no need to solve the problem again, the possible gains or losses resulting from an increase or a decrease in a selected (production or sales) limit [7]. This is particularly important for hard-coal mining, where mines frequently have spare production capacities. The practical example of using shadow prices to make rational production decisions, as presented here, shows how specific decisions could affect companies' or mines' optimum coal production and sales plans.

This method can be successfully used in the decision making process for planning mining works in hard coal mines [1].

The nature of shadow prices

The coal production and sales optimisation model, as developed by the author, has the following canonical form [2, 3, 4, 5]:

$$A \cdot X = B, \quad (1)$$

$$J = c^T \cdot X, \quad (2)$$

$$X \geq 0, \quad (3)$$

where:

A – constraint matrix (simplex tableau),

X – vector of decision variable values together with inequality

constraint slack variables (1),

B – vector of the right-hand sides of the equation;

c – vector of objective function coefficients and zero coefficients for slack variables,

J – objective function (quality coefficient).

The solution to the problem (1-3) is vector X divided into sub-vectors of basic variables with positive values, and vector of the other variables, referred to as non-basic, whose optimum values are zeroes. This solution is a simplex tableau.

The optimal solution, which – in relation to the basic and nonbasic variables and the quality coefficient – is represented by the following equations [4, 5, 6]:

$$x^B = [A^B]^{-1} \cdot B - [A^B]^{-1} \cdot A^N \cdot x^N \quad (4)$$

$$J = c^{BT} \cdot [A^B]^{-1} \cdot B - [c^{BT} \cdot [A^B]^{-1} \cdot A^N]^T \cdot x^N \quad (5)$$

where:

A^B, A^N – submatrices of the A matrix (A – matrix of the constraint coefficients);

c^B, c^N – subvectors of objective-function coefficients;

x^B, x^N – vectors of basic and nonbasic decision variables, respectively.

In the index maximisation task (2) optimum coefficient values in formula (5) with non-basic variables are non-positive.

With x^{NO} and x^{BO} representing, respectively, optimum values of basic and non-basic decision variables, and the following symbols:

$$A^O = [A^B]^{-1} \cdot A^N \quad (6)$$

$$c^O = [c^{BT} \cdot [A^B]^{-1} \cdot A^N]^T - c^N \quad (7)$$

and disregarding all non-basic variables x^{NO} (as in the opti-

Tab. 2. Comparison of selected values characterising calculation variants [Source: own study]
 Tab. 2. Porównanie wybranych wielkości charakteryzujących warianty obliczeniowe [Źródło: opracowanie własne]

Problem:	Kopalnia	XM0301	X040107	X040113	X040116	X050116	QS01	QS05
OBJ	3,3E+08	54,332	2,206	17,496	14,946	0,0	3,42	20,791
XM0101	160 022,5	0,0	0,0	0,0	0,0	0,0	0,11	0,0
XM0201	21 821,3	0,0	0,0	0,0	0,0	0,0	0,015	0,0
X030113	264 764,5	1,0	0,0	0,0	0,0	0,0	0,182	0,0
X040109	0,0	0,0	1,0	1,0	1,0	0,0	0,0	0,0
XM0401	675 004,0	0,0	0,0	0,0	0,0	0,0	0,464	0,0
X050113	317 135,5	0,0	0,0	0,0	0,0	1,0	0,218	0,0
X060115	16 002,3	0,0	0,0	0,0	0,0	0,0	0,011	0,0
X010214	112 672,7	0,012	0,0	-0,001	-0,001	0,0	0,005	0,011
XM0202	250 787,5	0,027	0,0	-0,002	-0,002	0,0	0,011	0,024

imum solution these take zero values) the result is:

- optimum values of basic variables in the form:

$$x^{BO} = [A^B]^{-1} \cdot B \quad (8)$$

- optimum value of quality indicator:

$$J^O = c^{BT} \cdot [A^B]^{-1} \cdot B = W^T \cdot B \quad (9)$$

where:

W - weight vector, which represents the impact of individual constraint values B on the quality indicator.

When correlations (4) and (5) are substituted to equations (6-9), we obtain formulas directly used in post-optimal analysis [5, 6]:

$$x^B = x^{BO} = x^{BO} - A^O \cdot x^N \quad (10)$$

and

$$J = J^O - c^{OT} \cdot x^N \quad (11)$$

where:

c^o - non-basic variable shadow prices, ≥ 0 for the maximisation of the quality indicator,

A^O - optimum solution coefficient matrix,

J^O - optimum value of the quality indicator.

Let us determine, using $J(B)$, the optimum value of the objective function for the primal problem, as produced by the constraint $B = [B_i]$ adopted for the purposes of these calculations. The optimum value of shadow price is a partial derivative of function $J(B)$ in relation to limit B_i [7]:

$$c_i^o = \frac{\partial J(B)}{\partial B_i} \quad (12)$$

and expresses a reaction of the optimum value of the primal problem objective function to slight (marginal) changes in limit B_i .

In practice, marginal interpretations are replaced with incremental interpretations. What the optimum variable value refers to here is a change in the optimum value of the objective function of the primal problem caused by a change in the absolute term B_i of the i-th primal problem constraint by a unit, namely:

$$c_i^o = \frac{\Delta J(x^o)}{\Delta B_i} \quad \text{przy } \Delta B_i = 1 \quad (13)$$

If in the i-th constraint of the problem the absolute term increases by a unit (while not causing the constraints to be not satisfied), then the optimum value of the objective function of the primal problem $f(x^o)$ increases by c_i^o units, i.e., to $J(x^o) +$ and vice versa [7].

Generally, shadow price is treated as the measure of how efficiently a limited resource B_i of the i-th production factor is used. When shadow price is interpreted on the basis of the partial derivative in formula 12, shadow price c_i^o can be considered to be the marginal productivity of the i-th factor. The above interpretations are correct if they refer to certain production factors with predefined limits („ \geq ” model constraints) [7].

Shadow prices for active inequality constraints (constraints on the productive capacity of the mine) directly show how changes to these constraints affect the quality indicator [6].

Let k denote constraint number in system (10) corresponding to the production capacity of the k-th mine, and j the number of the corresponding slack variable. Matrix A^N (Formula 4) has in its j-th column zero elements, except a single one in the k-th row (this results directly from the definition of the slack variable). Based on this and on correlation (5), it is possible to demonstrate that weight W in formula (9) is equal to the shadow price for variable j, i.e. $W = c_j^o$. Assuming a negative value of the j-th non-basic variable, one can calculate the increase in the quality indicator obtained by increasing the k-th constraint by the module of this value. The acceptable value of this increase and its impact on basic variables are determined in the same way as for positive increases in non-basic variables.

Analysis of shadow prices

Having an optimum solution in the form of a simplex tableau, one can quickly see the measurable effect produced by tapping into the unused production capacities in mines. Moreover, it is possible to analyse the consequences (to the optimum production and sales plans) to which such an increase by a specific value could lead. This was presented on the basis of the coal company Alfa [5].

Shadow prices for active inequality constraints (constraints on the productive capacity of the mine) directly show how changes to these constraints affect quality indicator levels. In Table 1, production capacity constraints correspond to non-basic variables QS01 and QS05, (for mines „A” and „E”

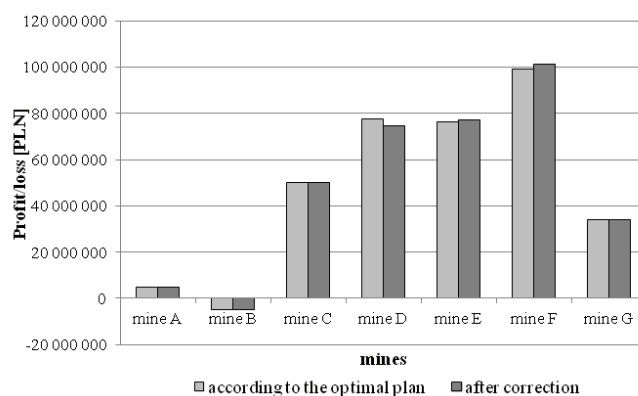
Tab. 2. Optymalny plan produkcji po korekcie zdolności produkcyjnych; Źródło: opracowanie własne
 Tab. 2. Optimal production plan after the correction of the production capacity; Source: Own elaboration

Company „Alpha”				
Max. Extraction: 15,949,350 ton		Profit: 336,778,099 PLN		
Sold: 11,423,865 ton		Company reserves: 1,896,203 ton		
Mine „A”				
Max. Extraction: 1,454,750 ton		Profit: 4,843,298 PLN		
Sold: 597,902 ton		Mine reserves: 0 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Dust kettles	fine coal I	264,764	264,764	0
Dust kettles	fine coal II	317,135	317,135	0
Grates 4	slurry	16,002	16,002	0
Dumping coal	cobble	160,023	160,023	0
Dumping coal	nut coal	21,821	21,821	0
Dumping coal	fine coal IIA	675,004	675,004	0
Mine „B”				
Max. Extraction: 793,500 ton		Loss: -4,873,866 PLN		
Sold: 112,783 ton		Mine reserves: 429,690 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Grates 3	fine coal II	112,783	112,673	110
Dumping coal	coaking coal	251,027	250,787	240
Mine „C”				
Max. Extraction: 1,110,900 ton		Profit: 49,931,963 PLN		
Sold: 996,546 ton		Mine reserves: 114,354 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 5	coaking coal	130,74	130,74	0
Coking plants 3	coaking coal	865,806	865,806	0
Mine „D”				
Max. Extraction: 3,174,000 ton		Profit: 74,459,137 PLN		
Sold: 1,801,825 ton		Mine reserves: 1,352,159 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 1	coaking coal	24,324	24,324	0
Export 2	coaking coal	287,249	287,359	-110
Export 3	coaking coal	233,299	233,299	0
Indv. consumers 2	cobble	40,042	40,512	-470
Indv. consumers 3	fine coal IIA	695,179	703,929	-8,750
Grates 3	fine coal II	9,097	9,207	-110
Coking plants 2	coaking coal	78,136	78,136	0
Coking plants 1	coaking coal	387,109	398,899	-11,790
Chamber grates 1	fine coal IIA	47,39	47,39	0
Dumping coal	fine coal I	20,016	20,256	-240

Mine „E”				
Max. Extraction: 2,988,850 ton		Profit: 76,944,673 PLN		
Sold: 2,962,438 ton		Mine reserves: 0 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 2	coaking coal	32,987	32,877	110
Export 8	nut coal	38,985	38,855	130
Indv. consumers 2	cobble	215,917	215,197	720
Dust kettles	fine coal I	206,921	206,231	690
Dust kettles	fine coal IIA	1,550,405	1,545,235	5,17
Dust kettles	fine coal II	866,668	863,778	2,89
Grates 4	slurry	47,178	47,178	0
Chamber grates 2	slurry	3,377	3,377	0
Dumping coal	slurry	36,412	36,122	290
Mine „F”				
Max. Extraction: 3,385,600 ton		Profit: 101,119,561 PLN		
Sold: 3,027,320 ton		Mine reserves: 0 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 7	cobble	175,801	175,551	250
Export 8	nut coal II	51,751	51,881	-130
Export 9	fine coal II	998,845	998,845	0
Indv. consumers 2	cobble	67,718	67,968	-250
coking plants 1	coaking coal	263,03	251,24	11,79
Dust kettles	fine coal I	243,52	243,52	0
Dust kettles	fine coal IIA	23,675	23,675	0
Dust kettles	fine coal II	1,202,980	1,202,980	0
Dumping coal	coaking coal	274,742	286,532	-11,79
Dumping coal	nut coal	9,129	8,999	130
Dumping coal	slurry	74,409	74,409	0
Mine „G”				
Max. Extraction: 3,041,750 ton		Profit: 33,978,285 PLN		
Sold: 1,925,051 ton		Mine reserves: 0 ton		
Name of consumer group	Coal size grade	adjusted amount of sales [ton]	The basic amount of sales [ton]	Difference + increase – decrease [ton]
Export 7	cobble	62,32	62,57	-250
Export 9	fine coal IIA	206,632	206,632	0
Indv. consumers 3	fine coal II	619,903	611,153	8,75
Dust kettles	fine coal IIA	12,155	12,155	0
Dust kettles	fine coal II	990,615	999,365	-8,75
Chamber grates 2	fine coal II	33,426	33,426	0
Dumping coal	cobble	44,035	43,785	250
Dumping coal	nut coal	15,194	15,194	0
Dumping coal	coaking coal	1,057,470	1,057,470	0

respectively). An increase in the production capacity of any of these mines causes its profit to increase by an amount equal to the value of the increased production capacity multiplied by the shadow price for that variable. As can be seen, the most beneficial option is to increase the production capacity in mine „E”, as this generates an additional PLN 20.8 for every additional tonne [5].

For example, we calculated the increase in the quality indicator that can be achieved as a result of increasing the production capacity in mine „E” by 10,000 tonnes. The results of these calculations are presented as adjusted coal production and sales plans for company Alfa in Table 2 and Figure 1.



Rys. 1. Zysk/strata kopalni według planu optymalnego I po korekcie zwiększenia zdolności produkcyjnej kopalni „E”; Źródło: opracowanie własne
 Fig. 1. Mining profit/loss according to the optimal plan and after adjusting the production capacity of the „E”; mine; Source: Own elaboration

Impact assessment of the conducted dual price analysis

The applied adjustment has caused the following changes in comparison to the optimal plan [3]:

1. The company's profit earned owing to the applied adjustment increased by 0.06% (increase by PLN 208,000); sales remained unchanged; production reserves increased by 1.1%.
2. As for the „A” and „C” mines, the optimal plan remained unchanged.
3. In the “B” mine, the loss rate decreased slightly by 0.13%. The volume of coal sales by 0.1%, the mine's production reserves decreased by 0.08%.
4. The profit of „D” mine decreased by 4.8%. Sales decreased by 1.16% and production reserves decreased by 1.6%.
5. In the “E” mine, as a result of the increase in production capacity, the profit increased by 1.1% and the sales volume increased by 0.32%.

6. In the “F” mine there was an increase in profit by 2.08% as a result of an increase in the sales volume by 0.39%.

7. In the “G” mine, profit decreased by 1.6%, and the sales volume by almost 0.02%.

Summary

The presented adjustment and the solutions described in [5] make it possible to adjust production plans to reflect the actual internal and external conditions in which mines are expected to function. This can help significantly reduce computation time while not requiring the optimisation procedure to be run again with new constraints.

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Możliwości podejmowania decyzji produkcyjnych z wykorzystaniem cen dualnych

W artykule zaprezentowano sposób wykorzystania cen dualnych aktywnych ograniczeń (w tym przypadku ograniczeń zdolności wydobywczych kopalń) do rozwiązywania i wspomagania decyzji produkcyjnych. Jest to jeden z algorytmów opracowanej przez autora analizy postoptymalnej opracowanej w ramach metody racjonalizacji decyzji produkcyjnych dla sformalizowanej grupy (spółki, PGG) kopalń węgla kamiennego. Możliwości wykorzystania cen dualnych przedstawiono na rzeczywistym przykładzie kopalń. Dzięki opracowanemu algorytmowi można szybko uzyskać informację, bez konieczności ponownego rozwiązywania zadania, co uzyskalibyśmy lub stracili zwiększając lub zmniejszając wybrany limit produkcji, i określić wpływ zmian tych ograniczeń na osiągnięty zysk i strukturę wielkości produkcji i sprzedaży poszczególnych sortymentów węgla.

Słowa kluczowe: ceny dualne, analiza postoptymalna, algorytm Simpleks