PRACE INSTYTUTU LOTNICTWA ISSN 0509-6669
233, s. 9–23, Warszawa 2014

ENTROPY-ENERGY MODEL OF FATIGUE DEFECTS

Nikolaj Delas, Vladimir Kasyanov
National Aviation University Kyiv

Abstract

The dependence is deduced on the basis of the maximum entropy method, for the distribution of the number of fatigue microcracks according to the classes of absorbed energy. The obtained ratio is non-Gaussian and is called "The extreme hyperbolic distribution law", which becomes asymptotically hyperbolic with the growth of argument. That curve allows the microcracks to "distribute" the absorbed energy in the most reasonable way. The energy balance for the microcrack's absorbed energy is made out, proceeding from the supposition that the increase of its area is in proportion to the gap energy of intercrystalline links, and the dissipation energy in the plasticity area is in proportion to the size of its borders. This gave an opportunity to obtain the ratios for the growth indicators of microcrack's area. These results and taking account of microcrack's distribution according to the categories of absorbed energy, help to obtain the evaluation formula for the fatigue curve.

Keywords: hyperbolic distribution, microcrack's distribution, microcrack's growth rate, fatigue curve formula

INTRODUCTION

A great number of operational destructions have a fatigue character; therefore the problems of the accumulation of damages are the most important and at the same time the most difficult in the theory of Strength. One can see the upward trend of material loading; therefore the life time of some constructional elements became to be limited by small-size defects. Prediction of the development of small-size cracks that are close to the dimensions of the material structure becomes more urgent.

Nowadays there are a lot of models that describe growth process of microcracks. They can be nominally divided into three groups. The models of the first group are based on the construction of calculation schemes of loading with the attempt taking into account the

microstructure of the material. The second group is based on the input of formal parameter of damageability and postulation for it some of the evolutionary equation which connects stress and growth rate of damage. In the models of the third group it is assumed that the dynamics of damageability has a certain "thermodynamical" properties, or rather the universal properties found in most complex systems. Approaches used in the present article, tend more to the third group. Here as reference points, variational principle of entropy maximum and the law of energy conservation are used.

THE AIM OF THE ARTICLE

In order to solve the problems of fatigue durability, there is a practical importance of the distribution of defects quantity from their size $n_i = f(l_i)$. So far an experiment was the only way to obtain fatigue curve.

This article raises two problems. The first one is to show that the relation $n_i = f(l_i)$ can be quite easily derived from the principle of entropy maximum. The second – to describe the dynamics of the microcracks growth on the basis of energy balance of the dissipation processes and bond rupture in the material. The solution to these two problems allows achieving the main goal - to obtain integral dependences for the characteristics of fatigue strength.

THE IDEA OF APPROACH

The paper [3] suggests an idea – to consider many of self-organized distributed systems, including the microcracks, from a unified point of view, namely, from the perspective that on the set of some of "consumers" is distributed a limited number of "resources" (in particular, on the set of microcracks is distributed the energy of destruction. Moreover, the distribution is carried out not arbitrarily, but according to the so-called "maximum-hyperbolic distribution law", derived in [3] on the basis of the entropy principle. The authors first published their materials of this subject the in May 2012 at the International Conference «ISDMCI '2012» in Eupatorium [4].

This unified approach allows getting closer to the explanation of the phenomenon known as non-Gaussian (power) distribution pattern in many of, very different by its nature, systems or communities. A review article [5] presents a dozen of various approaches for explanation the mechanism of non-Gaussian distribution patterns.

The entropy principle used herein can be effectively applied to the research of complex hard-formalized systems. It is based on the understanding that some distributed quantity that characterizes the system's conditions (in our case – it is the energy of formation of defects) distributes within the system in the most probable way. Therefore, the entropy of this distribution reaches its maximum value. Maximum entropy appears as the integral criterion, when from a set of options the system is implemented in this particular configuration.

The empirical dependences of fatigue defects quantity from their sizes are given in [1]. Their non-Gaussian characters, as well as the existence of property of self-similarity, are noted there. One of the examples is shown in the figure (Fig. 2).

In the present article it is shown that based on the above-mentioned approach the power (hyperbolic) nature of the experimental curves can be derived.

DISTRIBUTION OF THE SIZES OF MICROCRACKS

Under the action of an alternating loading, part of the energy W is spent on the formation of fatigue defects. Their total number N can be divided into M classes; each of them consists of n_i representatives that consume the equal amount of energy ε_i. With such designations, the equalities are obvious:

$$\sum_{i=1}^{M} n_i = N \tag{1}$$

$$\sum_{i=1}^{M} n_i \varepsilon_i = W \tag{2}$$

If it is assumed that the energy of microcracks formation ε_i is proportional to some positive power γ of size l_i,

$$\varepsilon_i = \eta \cdot l_i^{\gamma} \tag{3}$$

Then the expression (2) will get the look:

$$\eta \cdot \sum_{i=1}^{M} n_i \cdot l_i^{\gamma} = W \tag{4}$$

Solution of the problem of finding $n_i = f(l_i)$ is reduced to the search of the distribution of defects quantity over the amount of energy of their formation $n_i = \varphi(\varepsilon_i)$. According to the principle of entropy, this dependence will be formed such as that for all selected M classes of defects, the distribution of energy $W_i = n_i \cdot \varepsilon_i$ will be realized with maximum expansion, and therefore will be reached the conditional maximum of entropy. For this purpose it is convenient to use the Shannon entropy:

$$H = -\sum_{i=1}^{M} p_i \cdot \ln(p_i) \tag{5}$$

Here p_i – frequency (probability) of i – event. In the case of energy distribution $W_i = n_i \cdot \varepsilon_i$ over the classes it is equal $p_i = \dfrac{n_i \cdot \varepsilon_i}{E}$.

Thus, the required distribution $n_i = f(\varepsilon_i)$ will be obtained as a result of solving the task on conditional maximum of entropy, written in the following form:

$$H_E(n_i) = -\sum_{i=1}^{M} \frac{n_i \cdot \varepsilon_i}{W} \cdot \ln \frac{n_i \cdot \varepsilon_i}{W} \tag{6}$$

Here (1) and (2) act as requirements.

We use Lagrange multipliers method to determine the conditional extremum. It is known, that if the entropy appears to be the studied function, this approach is sometimes called the Jaynes-Gibbs formalism. Its essence lies in the fact that to achieve the conditional maximum $H_{\mathcal{E}}(n_i)$, is sufficient to solve the problem of finding unconditional extremum of a new function $\Phi(n_i)$ that includes additively H, and constraint equations (1) and (2), weighted by Lagrange's multipliers α and β:

$$\Phi(n_i) = -\sum_{i=1}^{M} \frac{n_i \cdot \varepsilon_i}{W} \cdot \ln \frac{n_i \cdot \varepsilon_i}{W} + \alpha \cdot \left(\sum_{i=1}^{M} \frac{n_i}{W} - \frac{N}{W} \right) + \beta \cdot \left(\sum_{i=1}^{M} \frac{n_i \cdot \varepsilon_i}{W} - 1 \right) \tag{7}$$

Having equated to zero partial derivatives:

$$\frac{\partial \Phi(n_i)}{\partial n_i} = -\frac{\varepsilon_i}{W} \cdot \ln \frac{n_i \cdot \varepsilon_i}{W} - \frac{\varepsilon_i}{W} + \alpha \cdot \frac{1}{W} + \beta \cdot \frac{\varepsilon_i}{W} = 0$$

We'll get an expression which brings us to extremum (7):

$$n_i = \frac{C_1}{\varepsilon_i} \cdot \exp\left(\frac{\alpha}{\varepsilon_i}\right) \tag{8}$$

Where $C_1 = W \cdot \exp(\beta - 1)$.

The physical meaning of the multiplier α becomes clear after the determination the function extremum $n_i = f(\varepsilon_i)$ given by (8). It is easier to find the extremum of proper continuous distribution $n = \varphi(\varepsilon)$ that is obtained when $M \to \infty$. By assumption $\frac{dn}{d\varepsilon} = 0$ follows, that $\alpha = \varepsilon_*$, and constant $C_1 = n_* \ \varepsilon_* \cdot e$. Here $\varepsilon \approx 2.718$, and ε_*, n_* – the coordinates of the point at which the distribution (8) reaches its maximum.

As a result of the distribution $n_i = \varphi(\varepsilon_i)$, can be written following:

$$\frac{n_i}{n_*} = \frac{\varepsilon_*}{\varepsilon_i} \cdot \exp\left(1 - \frac{\varepsilon_*}{\varepsilon_i}\right) \tag{9}$$

Below (fig. 1) are given graphs.

Since with the growth of the argument ε_i influence of the exponential multiplier in (9) practically has no effect (leveled), therefore it approaches asymptotically to a power (hyperbolic) dependence:

$$n_i = \frac{n_* \cdot \varepsilon_* \cdot e}{\varepsilon_i} = \frac{C_1}{\varepsilon_i} \tag{10}$$

Therefore, in [3] the relation (9) is called as *extreme hyperbolic distribution law*. In the present work there are also represented formulas for calculation of parameters n_* and ε_* included in (9).

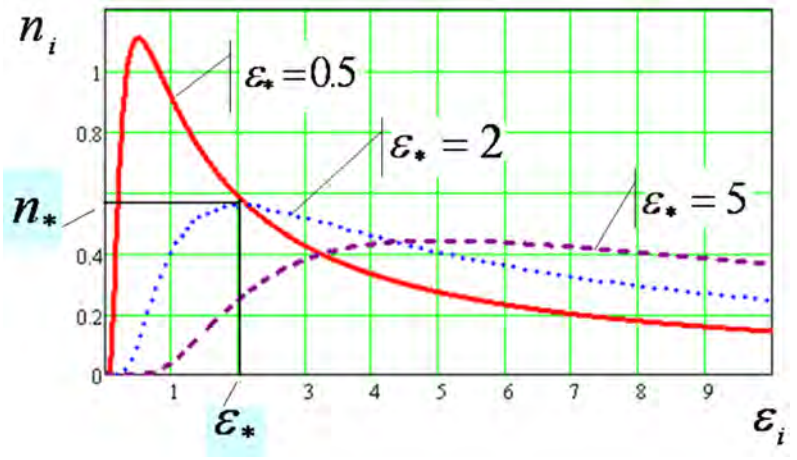

Fig. 1. Extreme hyperbolic distribution law

In [3] authors insist on their point of view, that it is the extreme hyperbolic distribution law that has the versatility of manifestation, but not a power (hyperbolic) law. Deviation from the pure hyperbola can be almost negligible for the experiment in those cases, when the value of the parameter ε_* is too small. It takes place at sufficiently big quantity of "carriers" and not too large volume of distributed "resource".

It is remarkable that the same result can be obtained by writing the expression for the entropy in the form proposed by Hartley:

$$S = g \cdot \ln G \tag{11}$$

Here g – multiplier, that takes into account the dimension, and G – statistical weight, that specifies the number of possible permutations of the various portions of energy $\varepsilon_I = \Delta\varepsilon_I \cdot i$, except for the number of trivial combinations within each class:

$$G = \frac{\left(\sum_{i=1}^{M} n_i \cdot i\right)!}{\prod_{i=1}^{M} (n_i \cdot i)!} = \frac{I!}{\prod_{i=1}^{M} (n_i \cdot i)!} \tag{12}$$

By applying well-known Stirling's formula

$$\ln I! \approx I \cdot (\ln I - 1) \tag{13}$$

The expression (11) can be written as:

$$S = g \cdot \left[I \cdot (\ln I - 1) - \sum_{i=1}^{M} (n_i \cdot i \cdot (\ln(n_i \cdot i) - 1)) \right]$$

or

$$S = g \cdot [I \cdot \ln I - \sum_{i=1}^{M} n_i \cdot i \cdot \ln(n_i \cdot i)]$$

Conditional maximum of entropy S will be defined by finding distribution n_i, by analogy with (7), leading to the extremum of the function:

$$\Psi(n_i) = -g \sum_{i=1}^{M} \frac{n_i \cdot i}{I} \cdot \ln\left(\frac{n_i \cdot i}{I}\right) + \vartheta \cdot \frac{N}{I}\left(\sum_{i=1}^{M} \frac{n_i}{N} - 1\right) + \eta\left(\sum_{i=1}^{M} \frac{n_i \cdot i}{I} - 1\right)$$

Its solution is $n_i = \frac{C_2}{i} \cdot \exp\left(\frac{\vartheta / g}{i}\right) = \frac{\Delta\varepsilon \cdot C_2}{\Delta\varepsilon \cdot i} \cdot \exp\left(\frac{\Delta\varepsilon \cdot \vartheta / g}{\Delta\varepsilon \cdot i}\right) = \frac{C_1}{\varepsilon_i} \cdot \exp\left(\frac{\alpha}{\varepsilon_i}\right)$, that coincides with the expression (8).

Such independence of result from the form of entropy is quite remarkable and demonstrates the validity of the very idea of using the entropy principle for many hard formalized problems.

The distribution of the number of defects n_i on their size l_i can be obtained by substituting the expression (3), which binds the size of the defect with the energy of its formation $\varepsilon_i = \eta \cdot l_i^{\gamma}$ in (14). We will get sought distribution:

$$n_i = n_* \cdot \left(\frac{l_*}{l_i} \right)^{\gamma} \exp\left(1 - \left(\frac{l_*}{l_i} \right)^{\gamma} \right) \tag{14}$$

here $l*$ and $n*$ – the extremum point coordinates for the curve.

Figure 2 shows the empirical distribution data of fatigue defects, taken from [1]. A calculation by formula (14) corresponds quite well with these curves (Fig. 3).

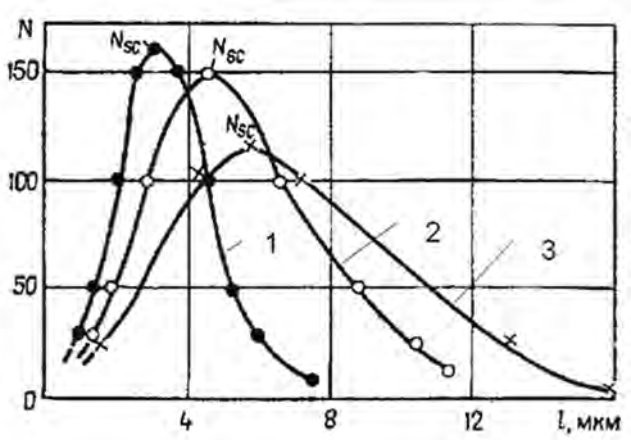

Fig. 2. An example of empiric curves of defects distribution [1]

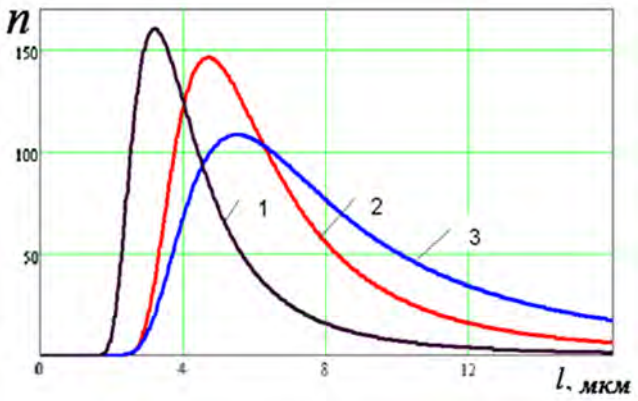

Fig. 3. Calculations by formula (14)

In [1] are given distributions of quantity of defects on their size for different ways of loading, different materials and different kinds of deformation. Almost all results are similar to the curves on fig.2. They have strongly marked extremum and their descending branches have hyperbolic shape. The physical sense of this nature becomes clear only within proposed in [3] the extreme hyperbolic law. It turns out that exactly this kind of the curve enables microcracks "to distribute among themselves" the absorbed energy in the most rational way.

PROPAGATION OF MICROCRACKS

As a result of many experimental studies (eg, [6, 7, 8]) came the realization that the fatigue microcracks propagation is closely related to the process of plastic material deformation at the edges of these defects. On one hand, such zone of localized plasticity "sets the stage" for the growth of microcracks sizes, and on the other – it is also a main consumer of that part of external energy that is known as the energy of dissipation. As shown by measurements [7, 8], performed by the method of infrared thermographs, the most intense heat generation takes place exactly on the border of cracks.

Given fact leads to a suggestion that dissipation energy is proportional to the size of the plastic zone. Based on this assumption, and taking into account the results obtained in the previous article section, we build an approximate "energy" model of fatigue microcracks propagation.

Consider the simplest case of the longitudinal cyclic loading (tension – compression) of the rod of constant section. We consider that microcracks are distributed over the entire cross section of the rod. They have a flat shape (the form of puddles of different sizes), and oriented perpendicular to the load. Conditionally we select small section of rod with length x, contained between two cross-sections, which are in the process of cyclic loading experiencing deformation with an amplitude equal $\Delta x_{\max}$. In the course of one cycle, where k its number, the volume of this part $\delta V = x \cdot S$ perceives energy:

$$W_k = \bar{W}_k \cdot \delta V,$$

where $\bar{W}_k$ – specific strain energy. For given simple outline it equals:

$$\bar{W}_k = E \cdot \left(\frac{\Delta X_{\max}}{X} \right)^2 = E \cdot \bar{\Delta}^2_{\max} \tag{15}$$

Here E – Young's modulus, $\bar{\Delta}_{\max}$ the amplitude of the relative strain (usually denoted as $\varepsilon_{\max}$, but in this article this symbol is already reserved for the energy portions).

Consider for all cycles load conditions are unchanged, thus $W_k = W = const$ Portion of this energy:

$$\tilde{W} = q \cdot W = q \cdot E \cdot \bar{\Delta}^2_{\max} \cdot \delta V \tag{16}$$

does not go back into the environment in the form of the work of elastic forces, but remains in the material. This is the absorbed energy (q – its share from the total strain energy). One of its components is spent on rupture of intercrystalline connections $\tilde{W}_{(break)}$, another - on the dissipation in the plastic zone $\tilde{W}_{(diss)}$:

$$\tilde{W} = \tilde{W}_{(break)} + \tilde{W}_{(diss)} \tag{17}$$

Using the scheme, accepted in the beginning of the article (2), can be written:

$$\tilde{W} = \sum_{i=1}^{M} n_i \varepsilon_i = \sum_{i=1}^{M} n_i \varepsilon_{i(break)} + \sum_{i=1}^{M} n_i \varepsilon_{i(diss)} \tag{18}$$

where n_i – number of defects in i – th class.

For separate *i – th* class microcrack:

$$\varepsilon_{ik} = \varepsilon_{ik(break)} + \varepsilon_{ik(diss)} \tag{19}$$

Here ε_{ik} – the energy, absorbed at *k – th* cycle by one microcrack of *i – th* class. One part of it – $\varepsilon_{ik(break)}$ is spent on destruction of intercrystalline connections, second – $\varepsilon_{ik(diss)}$ is converted into the heat in the zone of plasticity. It's important to understand that constancy ε_{ik} for each cycle (*by definition* its value depends only on $\tilde{W}$ and on the chosen quantity of classes *M*) does not mean, that its separate summands in the formula (19) will remain the same from cycle to cycle as well. Actually, as far as the area of microcrack increases, there is a process of gradual redistribution of energy ε_{ik} from the first summand $\varepsilon_{ik(break)}$ to the second due to increase of the length of the zone of plasticity. This mechanism is modeled below.

In order to continue using the energy balance equation (19), we take two assumptions that define further scheme:

a) the increase of the area of microcrack of i – th class ΔS_{ik} is proportional to the energy it receives, that is spent for rupture of intercrystalline connections:

$$\Delta S_{ik} = \lambda \cdot \varepsilon_{ik(break)} \tag{20}$$

(this is justified by the fact that there is an equal amount of these connections per unit area).

b) the energy of dissipation $\varepsilon_{ik(diss)}$ (the second summand in (19)), is proportional to the size (length) of plastic zone of crack l_{ik}, accumulated after k loading cycles:

$$\varepsilon_{ik(diss)} = \mu' \cdot l_{ik} \tag{21}$$

Arguments in favor of this hypothesis are given in the beginning of partition.

The physical meaning of multipliers: λ – the incrementation of rupture area of microcrack, by giving it an unit of energy; μ' – energy of dissipation which is absorbed in the plastic zone by the area of unit length.

Perimeter l_{ik} of the flat microcrack is connected with an area $\sum_{j=1}^{k} \Delta S_{ij}$ by a quadratic dependence, accumulated during *k* loading cycles:

$$l_{ik} = a \cdot \sqrt{\sum_{j=1}^{k} \Delta S_{ij}} \tag{22}$$

For the flat cracks, with a circular shape in plan, $a = 2 \cdot \sqrt{\pi}$

Taking into account these assumptions, the expression (19) takes the form:

$$\varepsilon_{ik} = \frac{1}{\lambda} \cdot \Delta S_{ik} + \mu \cdot \sqrt{\sum_{j=1}^{k} \Delta S_{ij}} \tag{23}$$

New designation is introduced here: $\mu' = \mu' \cdot a$.

Expression (23) is the original dependence for the further calculation of parameter, we are interested into. It can be considered as a discrete record of a nonlinear integral equation for the increment of microcrack area of the *i-th* class according to the number *k*of preceding loading cycles. Let's carry out some transformations here.

Let us denote: $\sum_{j=1}^{k} \Delta S_{ij} = S_i(k)$

Considering that the smallest possible increment value: $\Delta k = 1$, we can write down:

$$\Delta S_{ik} = \frac{\Delta S_{ik}}{\Delta k} \cdot \Delta k = \frac{\Delta S_i(k)}{\Delta k} \cdot 1$$

And also, due to the previously conditioned constancy ε_{ik} at change of k:

$$\varepsilon_{ik} = \varepsilon_i$$

After these designations equation (23) can be rewritten in the form:

$$\lambda \cdot \varepsilon_i = V_i(k) + \lambda \cdot \mu \cdot \sqrt{S_i(k)} \tag{24}$$

Here $V(k)_i = \frac{\Delta S_i(k)}{\Delta k}$ – the rate of growth of microcrack by the number of cycles k.

To estimate the solution of this equation, we study the dynamics of the behavior of its some continuous analogue. For this purpose instead of a discrete variable k, we introduce a new dimensionless continuous value K. Then, instead of (24), we write its continuous approximation:

$$S_i(K) = \frac{1}{(\lambda \cdot \mu)^2} \cdot \left(\lambda \cdot \varepsilon_i - V_i(K)\right)^2.$$

After differentiation, we obtain:

$$\frac{dV_i(K)}{dk} \cdot \left(1 - \frac{\lambda \cdot \varepsilon_i}{V_i(K)}\right) = \frac{(\lambda \cdot \mu)^2}{2} \tag{25}$$

The exact solution of this equation, taking into account initial conditions:

$$\begin{aligned} S_i^{(0)} &= S_i(K)\Big|_{K=0} = 0 \\ V_i^{(0)} &= V_i(K)\Big|_{K=0} = \lambda \cdot \varepsilon_i \end{aligned} \tag{26}$$

can be expressed as:

$$\frac{e^{\overline{V}_i(K)}}{\overline{V}_i(K)} = e^{1 + \frac{b}{\varepsilon_i} K}, \tag{27}$$

where $\overline{V}_i(K) = \frac{V_i(K)}{\lambda \cdot \varepsilon_i}$, $b = \frac{\lambda \cdot \mu^2}{2}$.

However, the solution (27) has an implicit form. To express $\overline{V}_i(K)$ in the explicit form, we expand the left part (27) in the series. Small value of $\overline{V}_i(K)$ (changes within the limits

$0 < \bar{V}_i(K) \le 1$), provides a reasonable accuracy, leaving only the first two terms of the expansion. Approximate solution for the rate of growth of microcracks, satisfying the boundary conditions, can be written as:

$$V_i(K) \approx \frac{\lambda \cdot \varepsilon_i}{e^{1+(b/\varepsilon_i)\cdot K} - e + 1} \tag{28}$$

For comparison there are graphs (fig.4) of inversely dependence $K = f\left(\bar{V}_i\right)$, corresponding to the exact (27) and approximate (28) solutions.

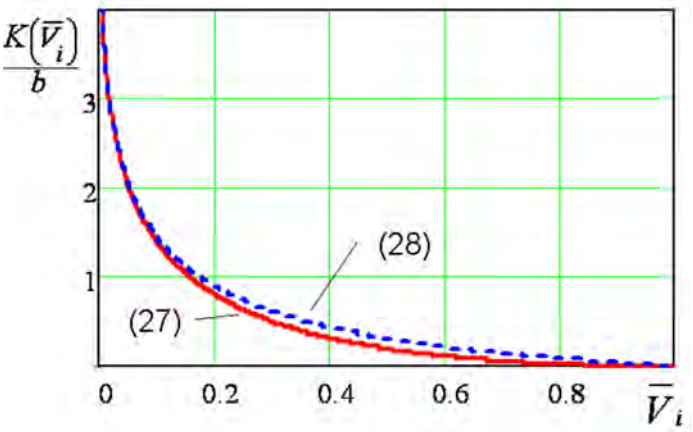

Fig. 4. Comparison of exact and approximate solutions, represented by the formulas (27) and (28)

The area enclosed under the curve is numerically equal to the accumulated area of microcrack $S_i(K)$. Its value can be obtained directly from equation (25) by substituting the value of $V_i(K)$from (28):

$$S_i(K) = \left(\frac{\varepsilon_i}{\mu}\right)^2 \cdot \left(1 - \frac{1}{e^{1+(b/\varepsilon_i)\cdot K} - e + 1}\right)^2 \tag{29}$$

Thus, we have the expression (28) – for the speed, and (29) – for the area of microcrack depending on the number of cycles $k = [K]$, portion of energy ε_i, value of λ, characterizing durability of intercrystalline connections, as well as the parameter μ, characterizing the magnitude of the energy dissipation in the area of plasticity.

One can see, that *with the unlimited increase of numbers of cycles,* the area of microcrack tends to the value:

$$S_i^{(\infty)} = \lim_{K \to \infty} S_i(K) = \left(\frac{\varepsilon_i}{\mu}\right)^2 \tag{30}$$

Considering (30), expression (29) can be written as a relative area (Fig. 5)

$$\bar{S}_i(K) = \frac{S_i(K)}{S_i^{(\infty)}} = \left(1 - \frac{\frac{1}{e}}{e^{(b/\varepsilon_i)\cdot K} - 1 + \frac{1}{e}}\right)^2 \tag{31}$$

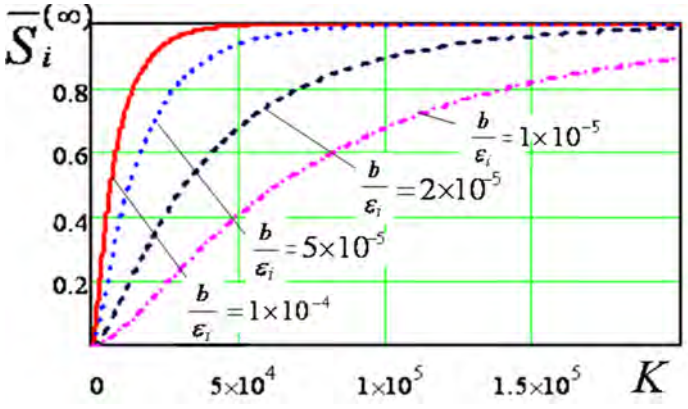

Fig. 5. Dependency of relative area of microcrack $\bar{S}_i(k) = \frac{S_i(k)}{S_i^{(\infty)}}$ ***on the number of cycles***

Analysis of the results allows receiving some conclusions. Thus, from (30) it follows that the maximum area a microcrack can develop $S_i^{(\infty)}$, is proportional to the square of the portion of energy, received during every cycle of its loading, and inversely proportional to the square of the dissipation parameter. From (Fig. 5) one can see, that with decrease of the ratio b/ε_i, a microcrack grows in continuation of the bigger number of cycles.

FATIGUE CURVE

According to the scheme accepted above, we will consider a very short section of the rod – such, that is filled with flat microcracks, which are located only in one layer perpendicular to the axes of action of loading. In our design scheme we will assume that they are all located in the same one section. From formula (29) for all of these *N* microcracks can be calculated their total area $S_\Sigma(K)$. For this purpose we sum $S_i(K)$ by the number of all n_i representatives of each of *M* classes (*see the beginning of article*):

$$S_\Sigma(K) = \sum_{i=1}^{M} n_i \cdot S_i(K). \tag{32}$$

Using solution (29), we will get:

$$S_\Sigma(K) = \frac{1}{\mu^2} \sum_{i=1}^{M} n_i \cdot \varepsilon_i^2 \cdot \left(1 - \frac{\frac{1}{e}}{e^{(b/\varepsilon_i)\cdot K} - 1 + \frac{1}{e}}\right)^2. \tag{33}$$

For the practical use of this formula the information about distribution of microcracks quantity n_i at M classes is required. Such distribution, shown in the beginning of this article, falls into line with the extreme hyperbolic law (9). Substituting (9) into (33), can be obtained the dependence that allows to define the numerical value of $S_\Sigma(K)$.

To obtain an analytical calculation formula $S_\Sigma(K)$, in first approximation can be assumed, that the sizes of all microcracks are approximately equal. At the same time $M = 1, \quad n_i = n = 1, \quad \varepsilon_i = \varepsilon = \frac{\tilde{W}}{N}$.

Using formula (16), we will define

$$\varepsilon = \frac{\tilde{W}}{N} = \left(\frac{q}{N / \Delta V} \right) \cdot E \cdot \bar{\Delta}^2_{\max}, \tag{34}$$

where N – number of microcracks in the selected volume ΔV. Then it follows from (31) that:

$$S_\Sigma(K) = \left(\frac{E \cdot \bar{\Delta}^2_{\max}}{\psi} \right)^2 \cdot \left(1 - \frac{1/e}{e^{\frac{\psi}{E \cdot \bar{\Delta}^2_{\max}} \cdot \frac{\lambda \cdot \mu}{2} \cdot K} - 1 + 1/e} \right)^2 \tag{35}$$

A new value is introduced:

$$\psi = \frac{\mu \cdot N / \Delta V}{q} \tag{36}$$

Let us call it as *index of absorbability of energy of deformation*. Dimension of this value – $[J/m^4]$. The validity of the name follows from the analysis (36). This includes the dissipative factor $\mu = \mu' \cdot a$ (μ' from (21), a from (22), coefficient q from (16), and also $N/\Delta V$ – a volume density of defects in material).

Under unrestricted growth of K the expression in brackets approaches to 1, therefore, from the expression (35) we get:

$$S_\Sigma^{(\infty)} = \lim_{K \to \infty} S_\Sigma(K) = \left(\frac{E \cdot \bar{\Delta}^2_{\max}}{\psi} \right)^2 \tag{37}$$

The magnitude $S_\Sigma^{(\infty)}$ is an asymptotic value of the area of all N microcracks of rod's cross-section with unlimited growth of K.

As a result, the analytical formula for the estimation of the value of the accumulated area of microcracks in this cross-section of rod looks like:

$$S_{\Sigma}(K) = S_{\Sigma}^{(\infty)} \cdot \left(1 - \frac{1/e}{e^{\left(\frac{\lambda \cdot \mu}{2 \cdot \sqrt{S_{\Sigma}^{(\infty)}}} \right) \cdot K}} \right)^{-1+1/e} \right)^2 \qquad (38)$$

Let's agree, that transition from discrete variable k to the continuous value K (see (25)) can be considered, as a technical trick that, after a certain number of cycles k have no effect on the result. Therefore in the final expressions obtained value K can be safely interpreted as the number of cycles.

With the accumulation of damages load-bearing properties of the rod go down. It happens due to the decrease of its effective cross-sectional area by the amount of the accumulated area of microcracks $S_{\Sigma}(K)$. Assessment of acceptable tension $\sigma(K)$, which rod can withstand after K cycles, can be accomplished with the ratio:

$$\frac{\sigma(K)}{S - S_{\Sigma}} = \frac{\sigma_0}{S}, \quad \text{or} \quad \sigma(K) = \sigma_0 \cdot \left(1 - \frac{S_{\Sigma}(K)}{S} \right), \qquad (39)$$

Where σ_0 is permissible tension, that is before the action of cyclic loading, S – cross-sectional area. Substituting here the value $S_{\Sigma}(K)$ from (37), we obtain the evaluation analytical formula of fatigue curve for axial loading of the rod:

$$\sigma(K) = \sigma_0 \cdot \left[1 - A \cdot \left(1 - \frac{1/e}{e^{\left(B/\sqrt{A} \right) \cdot K}} \right)^{-1+1/e} \right)^2 \right], \qquad (40)$$

Where $A = \frac{S_{\Sigma}^{(\infty)}}{S} = \frac{1}{S} \cdot \left(\frac{E \cdot \bar{\Delta}_{\max}^2}{\psi} \right)^2$, $B = \frac{\lambda \cdot \mu}{2\sqrt{S}}$, $e \approx 2.718$. Dimensionless parameters A and B theoretically can change within the limits (0; 1).

Numerical value of parameter Ψ can be defined from (40), if the fatigue limit is known: $\sigma^{(\infty)} = \sigma(K) \big|_{K=\infty}$:

$$\psi = \frac{E \cdot \bar{\Delta}_{\max}^2}{\sqrt{S \cdot (1 - \sigma^{(\infty)} / \sigma_0)}}.$$

Calculations of $\sigma(K)$ with the change of parameters A i B are resulted below (Fig. 6)

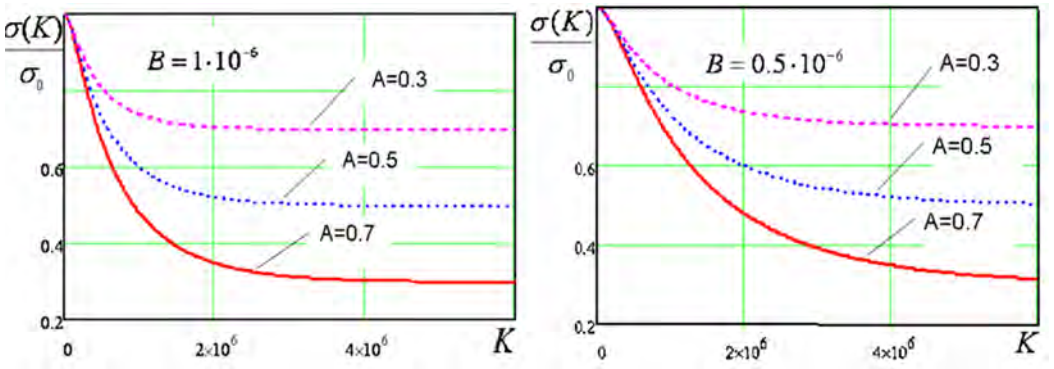

Fig. 6. The influence of the parameters A i B on the type of fatigue curve

Dependence (40), on which are these curves built, allows to estimate the influence on the character of the fatigue curve the number of factors.

CONCLUSIONS

There were two basic results obtained in the article. The first result - dependence (9) for the distribution of microcracks from the absorbed energy of deformation $n_i(\varepsilon_i)$. Given dependence was obtained based on the variational principle of maximum entropy, and has hyperbolic character with a rapidly decreasing exponential factor. The authors called it an *extremely hyperbolic distribution law.* This law explains the known phenomenon of non-Gaussian nature of the empiric curves.

The second result is a withdrawal of ratios (28) and (29) for the growth rate $V_i(K)$ and accumulated microcracks area $S_i(K)$, based on the understanding of the energy balance, which is formed in the process of their development. These two results allow us to calculate (39), the fatigue curve $\sigma(K)$.

The authors are aware that the given calculations have evaluative nature only. The obtained results are reflection of the outcome of the assumptions about the nature of the energy balance which has been accepted in this paper. Nevertheless the value of these results, in our opinion, is following: starting from the explicit phenomenological statements, it was succeeded to obtain estimations for the important values that are currently defined, as a rule, only empirically.

REFERENSES

[1] Botvina L. R., Berenblatt G. I. (1985). Selfsimilarity of accumulation damageability [Automodalnost nakopleniya povregdaemosty]. *Problemy prochnosty*, No.12, pp. 17-24.

[2] Khoroshun L. P. (1998). Basics of micromechanics damageability of the material. 1. Short-term damaging [Osnovu micromehanyky ponregdaemosty materiala. 1. Kratkovremennaia povregdaemost]. *Prikladnaia mehanika*, No. 10, pp. 120-127.

[3] Delas N. I., Kasyanov V. A. (2012). Extremely hyperbolic law of self-organized distribution systems [Predelno giperbolicheskiy zakon raspredeleniya v samoorganizovanyh sistemah]. *Vostochno-evropeyskiy zhurnal peredovyh tekhnologiy*, No 4/4, pp. 13-18.

[4] Delas N. I., Kasyanov V. A. (2012). Extremely hyperbolic law of distribution [Predelno giperbolicheskiy zakon raspredeleniya]. *Sbornik nauchnykh trudov Mezhdunarodnoy nauchnoy konferentsii «Intellektualnye sistemy prinyatiya resheniy i problemy vychislitelnogo intellekta»* (International conference «Intellectual systems for decision making and problems of computational intelligence»). *Evpatoria*, pp. 67-68.
[5] Delas N. I., Kasyanov V. A. (2012). Nongauss distribution as a property of complex systems that are organized by type of cenoses [Negaussovo raspredelenie kak svoystvo slozhnykh sistem, organizovannykh po tipu tsenozov]. *Vostochno-evropeyskiy zhurnal peredovyh tekhnologiy*, No. 3/4, pp. 27–32.
[6] Shanyavskiy A. A. (2003). Bezopasnoe ustalostnoe razrushenie elementov aviakonstruktsiy. Sinergetika v inzhenernykh prilozheniyakh [Safe fatigue failure of elements of aircraft structures. *Synergetics in engineering applications*]. *Ufa*, 803 p.
[7] Bannikov M. V., Terekhina A. I., Plekhov O. A. (2011). Experimental studies of the generation of heat in the top of the fatigue crack [Eksperimentalnoe issledovani osobennostey protsessa generatsii tepla v vershine ustalostnoy treschiny]. *Vestnik PGTU. Mekhanika*, No. 2. pp. 14-27.
[8] Plekhov O. A., Neymark O. B. (2009). Theoretical and experimental study of energy dissipation in the process of localization of deformation in iron [Teoreticheskoe i eksperimentalnoe issledovanie dissipatsii energii v protsesse lokalizatsii deformatsii v zheleze]. *PMTF*, T. 50, vup. 1, pp. 153-164.

Nikołaj DELAS, Władimir KASJANOW

ENTROPIA ENERGII JAKO MODEL DEFEKTÓW ZMĘCZENIOWYCH

Streszczenie

Pojawianie się defektów zmęczeniowych określana jest metodą maksymalnej entropii dla rozkładu liczby mikropęknięć zmęczeniowych według klas pochłoniętej energii. Wyznaczony w publikacji wskaźnik nie jest rozkładem gausowskim i nazwany został "prawem ekstremalnego rozkładu hiperbolicznego", które staje się asymptotycznie hiperboliczne wraz ze wzrostem argumentu funkcji. Krzywa ta pozwala określić mikropęknięcia wg "rozkładu" pochłoniętej energii w najbardziej korzystny sposób. Bilans energii pochłoniętej przez mikropęknięcia wykonano wychodząc z założenia, że zwiększenie jego powierzchni jest proporcjonalne do energii pęknięć międzykrystalicznych i energii dyssypacji w obszarze plastyczności, oraz jest proporcjonalnie do wielkości jego granic. Dało to możliwość uzyskania współczynników dla wskaźników wzrostu obszaru mikropęknięć. Te rezultaty z uwzględnieniem mikropęknięć w zależności od kategorii pochłoniętej energii, pomagają uzyskać formułę dla krzywej zmęczeniowej.