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On the prognosis of failure coincidences in multi-system scenarios

Keywords

coincidence probability, coincidence patterns, multi-system scenario, integral products, staple graphs

Abstract

The objective of the approach is to calculate the probability of failure coincidences or maintenance conflicts, respectively, in an n-system-single-maintenance-unit scenario. Beside the operation of conventional systems, reliability-adaptive systems can be considered as well in this scenario. The approach applies multiple integrals over probability density functions, which are arranged Matryoshka-like. The contribution discusses permutations of coincidence patterns explicitly and general coincidence probabilities. So-called staple graph and staple graph coincidence permutation diagrams are introduced as graphical representations.

1. Introduction

At the first glance, this approach models a classical reliability, operational, and maintenance context defined on the following assumptions, conditions, restrictions, and statements:

Scenario

- The scenario includes $n \geq 2$ individually and independently operating systems
- and a single maintenance unit, which cares for all n systems.

Systems

- The reliability performance and the failure modes of every individual system are statistically independent from those of the other systems. (Required if integral products are applied, see 6.)
- The approach does not distinguish between different failure modes, i. e. all system failure modes are commonly and completely quantified by the system cumulative distribution function (CDF).
- It is assumed that only one system failure occurs at a time. At least an infinitesimal small time interval separates succeeding failures.
- Unlike as discussed in [12], all CDFs may vary in type, parameters, and t_0-offset (calendar time of restarting after restoration). Therefore, the assumption that the reliability properties of all systems can be characterised by the same CDF [12] is not required.

Maintenance and Maintenance Unit

- The maintenance unit has a capability of maintaining only one system at a time.
- The approach models instant maintenance access only. There is no delay considered between a system failure and the beginning of a maintenance action – insofar the unit is not busy with maintaining another system. In [13] it is shown how to model delayed maintenance access.
- The sequence of systems undergoing maintenance actions follows the sequence of system failures (first-fail-first-in-first-out).
- Once a system maintenance action is started, it will not be interrupted until it is completed.
- The time to restoration (*TTR*) is assumed to be constant for every system in all maintenance cases, representing the exchange of a faulty module for a functioning. For the sake of easy reading, the simplified notation $TTR \equiv M$ is applied pointing to the duration of a maintenance action.

Objectives

One objective of this approach is to support reliability modelling of a classical context: Many systems with fixed CDF parameters, but diverging system-individual t_0-offsets are operating in a common scenario with shortcoming maintenance capabilities.

In addition to this classical context, the systems considered may have dynamic reliability properties according to the concept of reliability-adaptive systems (RAS), see references, especially [16], meaning that their individual reliability properties are online estimated and permanently updated during operation, and (based on these results) that a prognosis on the time to failure is conducted.

A lot of papers on modern reliability engineering discuss “online estimated reliability”, “realtime reliability evaluation”, “realtime reliability assessment” and “online reliability prognosis”, “realtime reliability prediction” etc., refer to references. As stated by Xu, Ji & Zhou [24], the main ideas behind these approaches are

- to monitor the performance degradation signals of the item considered,
- and to model the tendency of the degradation process,
- then to predict future performance variation.

The objective of this approach is to go one step further and to set up on the results given from the approaches mentioned above. With that, a reliability-adaptive operation in an *n*-system scenario can be modelled, which considers failure coincidences and maintenance conflicts. Depending on reliability estimations, the system operation parameters can be changed and the system operation tasks can be rearranged to optimise the occupancy (or “booking”) of the maintenance unit. Hence, this approach can be helpful to develop a maintenance unit booking strategy.

In [14], [17], [18] it is postulated that the product of *n* (number of systems within the scenario considered) and *TTR* has to be non-negligible against the value of the mean time to failure ($MTTF_i$, refer to Section 0), otherwise reliability-adaptive operation is not effective or – even worse – counterproductive compared to regular operation. As shown in see Section 4, the coincidence probability depends on the three measures *n*, *TTR*, and $MTTR_i$. Intuitively, the higher the coincidence probability, the higher is the effectiveness of an RAS approach. Therefore, the following approach allows quantifying how the three measures affect the effectiveness of an RAS.

Moreover, the approach provides a methodological tool

- to calculate if the coincidence probability has exceeded a given threshold, which defines, when to put a RAS into derating mode
- and to get some understanding about the results gained in experiments, which are presently conducted by the author at the University of Wuppertal under working title *Reliabotix* – the combination of reliability and robotics.

Analytical Approach versus Simulation

The author received many motivating proposals and hints from the reliability community to conduct a simulation of a scenario as described above. Simulation has proven to be an efficient and expressive tool in reliability engineering application, especially in quantitative reliability modelling.

However, putting the above mentioned RAS aspects step-by-step into a simulation of a scenario as defined above, may lead into a time paradox pitfall as discussed in [11]: The random *TTF* is based on a CDF, which includes a derating, which will be initialised later than the simulation-generated *TTF*.

Among other aspects, this work was motivated by the facts, that modelling of an analytical approach is feasible and RAS simulation presumably has some lacks in concept.

2. Fundamentals

The fault probability function F_i or cumulative distribution function (CDF) of the time to failure of system *i* is defined on $[0, \infty) \to [0, 1]$. The corresponding probability density function (pdf) f_i is defined on $[0, \infty) \to [0, \infty)$ with

$$\int_0^\infty f_i(t)\,\mathrm{d}t = 1. \tag{1}$$

According to the concept of reliability-adaptive systems, the following measures are introduced:

- the estimated value of $(1 - F_i)$ denoted as E_i, which is an a priori, forecasted, or prognosis measure, based on a priori given or online estimated CDF,
- the time to failure (so-called lifetime) *TTF*, which is a random number,
- the mean time to failure *MTTF*, which is an a posteriori statistical measure.

Note that the distinction between estimated value E_i and mean time to failure ($MTTF_i$) is essential in reliability-adaptive system modelling [11]. Generally, the overall scenario-covering multi-system performance depends on the number of systems and the individual system availabilities, which are quantifiable by the measures $MTTF_i$ and $MTTR_i$ respectively *TTR*. Again, $TTR \equiv M$ is a fixed number in this approach.

The interval diagrams (Figs. 1*ff*) may give the wrong impression that E_i, derived from

$$E_i = \int_0^\infty 1 - F_i(t)\,\mathrm{d}t = \int_0^\infty t \cdot f_i(t)\,\mathrm{d}t\,, \tag{2}$$

has a deterministic character. Note that E_i changes its value every instant of time depending on the results of changing prognoses.

Applying the maintenance condition of a fixed time to restoration yields the strict implication

$$E_{i'} < E_{i''} < E_{i'''} < \ldots \leftrightarrow E_{i'} + M < E_{i''} + M < E_{i'''} + M < \ldots, \quad (3)$$

describing the one-failure-at-a-time assumption on the left-hand side of the implication, representing the sequence of system failures, and the first-in-first-out character of the maintenance unit on the right-hand side. Both inequations are valid for any *n*-system scenario.

The contribution discusses two approaches to the prognosis of failure coincidences in a multi-system scenario:

- Modelling individual coincidence patterns and probabilities of defined failure sequences, see Section 3,
- Modelling general coincidence probabilities without explicit discussion of failure sequences (permutations), see Section 4.

The first aspect focuses and quantifies a certain situation within a dynamic environment and supports reasoning *what* to do in the given case. The second more general aspect gives quantitative hints *if* a proactive action is required.

3. Modelling individual coincidence patterns

This section shows a calculus how to model coincidence patterns and probabilities of defined failure sequences. The sub-sections show a collection of single-cluster coincidence patterns, which support e. g. reasoning in a multi-scenario context with individually given situations. The objective of these sub-sections is to illustrate the “baking” of integrals related to the given interval diagrams.

3.1 Two systems involved

The simplest case of system failure coincidence occurs in a two-system scenario if one system fails during maintenance of the other system. There is for example

$$E_1 < E_2 < E_1 + M < E_2 + M \quad (4)$$

as shown in the interval diagram (*Figure 1*).

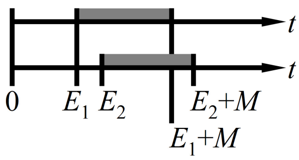

Figure 1. Interval diagram, grey blocks representing *TTR* (Note E_1 and E_2 are random numbers, *M* is fixed)

To simplify the diagrams of complex overlapping conflicts and to avoid denotation noise, staple-kind graphs are applied, see *Figure 2*. Please note that

- the left edge of a staple graph represents an estimated value,
- *M* is a fixed number; however, staple graphs do not provide a time-equidistant representation,
- overlapping *TTR*s indicating a conflict, but do not mean that more than one system is maintained at a time.

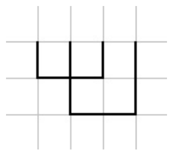

Figure 2. Simplification of Figure 1: Staples are indicating overlapping restoration intervals of length *M* each

Additionally,

$$E_2 < E_1 < E_2 + M < E_1 + M \quad (5)$$

must be considered. Then the coincidence probability $p_{\text{coin}\_k,c}$ of k = 2 involved and c = 2 conflicting systems yields

$$\begin{aligned} p_{\text{coin}\_2,2} &= \int_0^\infty f_1(t') \int_{t'}^{t'+M} f_2(t'')\,\mathrm{d}t''\,\mathrm{d}t' \\ &+ \int_0^\infty f_2(t') \int_{t'}^{t'+M} f_1(t'')\,\mathrm{d}t''\,\mathrm{d}t' \\ &= \int_0^\infty f_1(t')\big(F_2(t'+M) - F_2(t')\big)\,\mathrm{d}t' \\ &+ \int_0^\infty f_2(t')\big(F_1(t'+M) - F_1(t')\big)\,\mathrm{d}t' \\ &= \frac{1}{\lambda_1 + \lambda_2} \cdot \mathrm{e}^{-(\lambda_1+\lambda_2)\cdot(M+t)} \cdot \left(1 - \mathrm{e}^{(\lambda_1+\lambda_2)\cdot t}\right) \cdot \\ &\left(\lambda_1 \cdot \mathrm{e}^{\lambda_1 \cdot M} + \lambda_2 \cdot \mathrm{e}^{\lambda_2 \cdot M} - (\lambda_1 + \lambda_2) \cdot \mathrm{e}^{(\lambda_1+\lambda_2)\cdot M}\right). \end{aligned} \quad (6)$$

Equations (4) and (5) both describe mutually exclusive system failures sequences, so the + between both terms in equation (6) is justified.

If the scenario includes $n \geq 2$ systems, the probability for any coincidence involving two components is

$$p_{\text{coin\_2-of-}n} = \sum_{n^{\underline{2}}} \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'')\,\mathrm{d}t''\,\mathrm{d}t', \tag{7}$$

where the sum over $n^{\underline{2}}$ includes all permutations of i with

$$n^{\underline{k}} \equiv \frac{n!}{(n-k)!} \tag{8}$$

3.2. Three systems involved

Involving one more system (k = 3) results in two coincidence patterns following

$$E_{i'} < E_{i''} < E_{i'''} < E_{i'} + M < E_{i''} + M < E_{i'''} + M\,, \tag{9}$$

$$E_{i'} < E_{i''} < E_{i'} + M < E_{i'''} < E_{i''} + M < E_{i'''} + M\,. \tag{10}$$

The coincidence probability $p_{\text{coin\_3,3}}$ with c = 3, refer to *Figure 3*, yields

$$p_{\text{coin\_3,3}} = \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t'+M} f_{i'''}(t''')\,\mathrm{d}t'''\,\mathrm{d}t''\,\mathrm{d}t'. \tag{11}$$

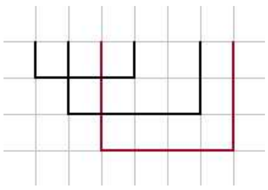

Figure 3. Coincidences of 3 system failures with maximum 3-fold overlapping of *TTR*

The coincidence probability $p_{\text{coin\_3,2}}$ of three involved and two conflicting systems, refer to *Figure 4*, yields

$$p_{\text{coin\_3,2}} = \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t'+M}^{t''+M} f_{i'''}(t''')\,\mathrm{d}t'''\,\mathrm{d}t''\,\mathrm{d}t'. \tag{12}$$

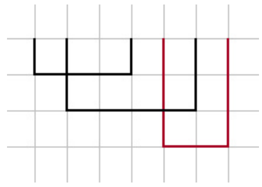

Figure 4. Coincidences of 3 system failures with maximum 2-fold overlapping of *TTR*

In case of 3-out-of-n systems and $n^{\underline{3}}$ failure sequences, the probability

$$p_{\text{coin\_3-of-}n} = \sum_{n^{\underline{3}}} \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t''+M} f_{i'''}(t''')\,\mathrm{d}t'''\,\mathrm{d}t''\,\mathrm{d}t' \tag{13}$$

holds for any coincidence involving three components.

3.3. Four systems involved

Four systems are involved in the next modelling steps. The coincidence probability $p_{\text{coin\_4,4}}$ of four conflicting systems, refer to *Figure 5*, yields

$$p_{\text{coin\_4,4}} = \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t'+M} f_{i'''}(t''') \int_{t'''}^{t'+M} f_{i''''}(t'''')\,\mathrm{d}t''''\,\mathrm{d}t'''\,\mathrm{d}t''\,\mathrm{d}t'. \tag{14}$$

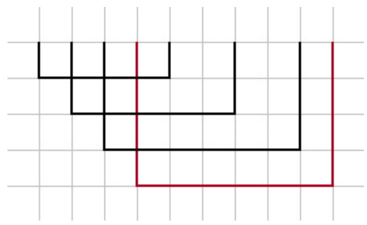

Figure 5. Coincidences of 4 system failures with maximum 4-fold overlapping of *TTR*

The coincidence probability $p_{\text{coin\_4,3}}$ of three conflicting systems, refer to *Figure 6*, yields

$$p_{\text{coin\_4,3}} = \int_0^\infty f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t'+M} f_{i'''}(t''') \int_{t'+M}^{t''+M} f_{i''''}(t'''')\,\mathrm{d}t''''\,\mathrm{d}t'''\,\mathrm{d}t''\,\mathrm{d}t'. \tag{15}$$

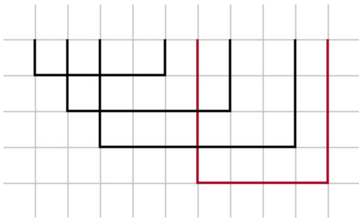

Figure 6. Coincidences of 4 system failures with maximum 3-fold overlapping of *TTR*

The coincidence probability $p_{\text{coin\_4,2}}$ of two conflicting systems, refer to *Figure 7*, yields

$$p_{\text{coin\_4,2}} = \int_0^{\infty} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t'+M} f_{i'''}(t''') \int_{t''+M}^{t'''+M} f_{i''''}(t'''') \, dt'''' \, dt''' \, dt'' \, dt' . \tag{16}$$

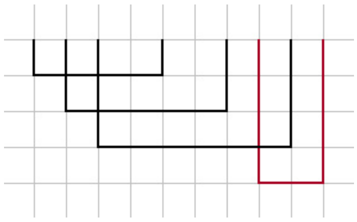

Figure 7. Coincidences of 4 system failures with maximum 2-fold overlapping of *TTR*

The coincidence probability $p_{\text{coin\_4,3'}}$ of the pattern as given in *Figure 8* , yields

$$p_{\text{coin\_4,3'}} = \int_0^{\infty} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t'+M}^{t''+M} f_{i'''}(t''') \int_{t'''}^{t''+M} f_{i''''}(t'''') \, dt'''' \, dt''' \, dt'' \, dt' . \tag{17}$$

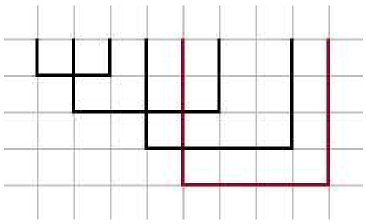

Figure 8. Coincidences of 4 system failures with maximum 3-fold overlapping of *TTR*

Figure 9 shows another pattern of four coinciding failures. The assigned probability $p_{\text{coin\_4,2'}}$ is

$$p_{\text{coin\_4,2'}} = \int_0^{\infty} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t'+M}^{t''+M} f_{i'''}(t''') \int_{t''+M}^{t'''+M} f_{i''''}(t'''') \, dt'''' \, dt''' \, dt'' \, dt' . \tag{18}$$

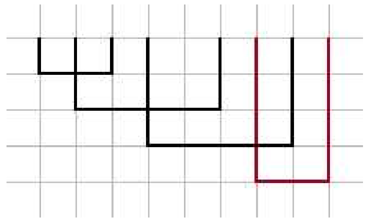

Figure 9. Coincidences of 4 system failures with maximum 2-fold overlapping of *TTR*

3.4. *k*-out-of-*n* systems involved

The probability for a *k*-out-of-*n*-system ($2 \le k \le n$) coincidence is

$$p_{\text{coin\_}k\text{-of-}n} = \sum_{n^{\underline{k}}} \int_0^{\infty} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \int_{t''}^{t''+M} f_{i'''}(t''') \ldots \int_{t^{(k-1)}}^{t^{(k-1)}+M} f_{t^{(k)}}(t^{(k)}) \, dt^{(k)} \ldots dt''' \, dt'' \, dt' , \tag{19}$$

where the sum over $n^{\underline{k}}$ includes all permutations of *i*. Replacing the first upper integral limit (infinity) by *t* yields a time-dependent coincident probability function $p_{\text{coin\_}k\text{-out-of-}n}$ with

$$p_{\text{coin\_}k\text{-of-}n}(t) = \sum_{n^{\underline{k}}} \int_0^{t} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'') \ldots \ldots dt'' \, dt' , \tag{20}$$

which is helpful in reliability-adaptive operation. The function allows calculating, when a given threshold is exceeded and e. g. component derating has to be initialised.

3.5. Coincidence permutations

The *staple graph coincidence permutation diagram* (see *Figure 11*) shows the permutations of coincidence patterns. This diagram gives an impression of a 6-system scenario failure coincidence permutation if single-clusters are considered only.

The first column shows $k = 2$ coincidences developed for $n = 6$ systems. The second column involves a third system introducing two variants of overlapping (refer also *Figure 3* and *Figure 4*). The third to the fifth column continues evolving permutations.

3.6. Coincidence clusters

Failure coincidences of $k \le n$ systems can occur cluster-wise. Every system in a cluster has an overlapping *TTR* to at least one other cluster member. The smallest multi-cluster configuration consists of four systems with overlapping *TTR* divided into two two-system clusters as shown in *Figure 10.*

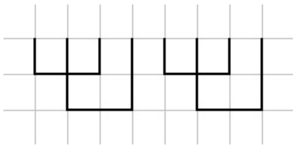

Figure 10. Coincidences of four system failures in two cluster

The coincidence probability $p_{\text{coin\_2+2}}$ of two involved and two conflicting systems yields

$$p_{\text{coin\_2+2}} = \left(\int_{0}^{\infty} f_{i'}(t') \int_{t'}^{t'+M} f_{i''}(t'')\, \mathrm{d}t''\, \mathrm{d}t' \right) \cdot \left(\int_{t''+M}^{\infty} f_{i'''}(t''') \int_{t'''}^{t'''+M} f_{i''''}(t'''')\, \mathrm{d}t''''\, \mathrm{d}t''' \right) \qquad (21)$$

Any type of multi-cluster coincidence can be calculated analogously.

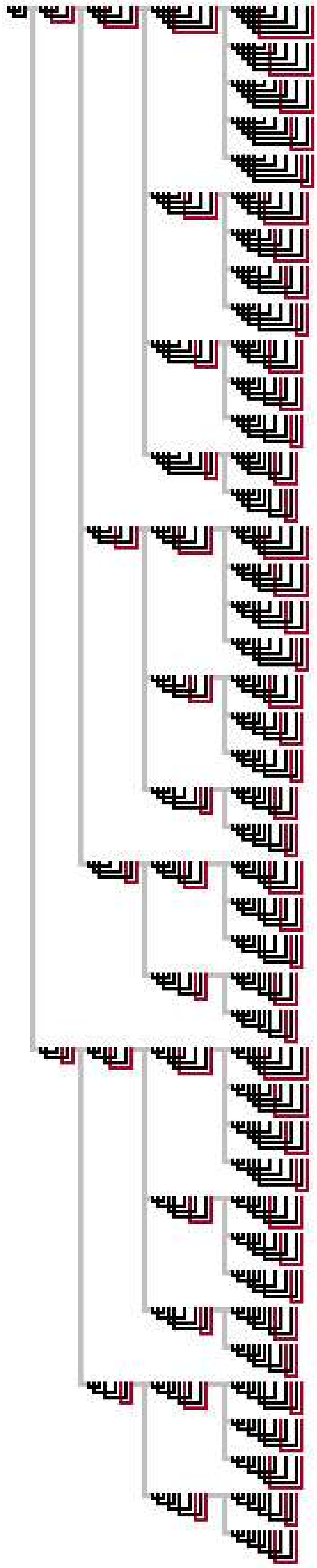

Figure 11. Staple graph coincidence permutation diagram of a 6-system scenario considering single-clusters

4. Modelling general coincidences

This section discusses a more general approach to the prognosis of failure coincidences. The general probability for a coincidence in an *n*-system scenario serves as a measure if reliability-adaptive operation can be more effective than regular operation. Intuitively, the probability depends directly on the number of systems in the scenario *n*, the pdf of system failures f_i, and the time to restoration *M*.

The time-dependent probability function $p_{\text{coin\_}n}$ shall quantify the occurrence of *any* coincidence in the interval [0, *t*]. This function supports reasoning in dynamic environments if a pro-active action has to be considered or is required, respectively – or if not.

To minimise calculation effort, the probability of non-occurrence is calculated and then inverted. Non-occurrence of coincidences means that every failure must occur outside the fixed *TTR* intervals, see *Figure 12*.

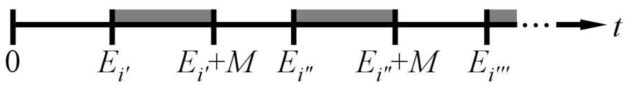

Figure 12. Non-overlapping failure sequences as applied here to calculate the probability that no (resp. any) coincidence occurs

On a first sketch, the general probability function for a coincidence in an *n*-system scenario is

$$p_{\text{coin\_}n}(t) = 1 - \sum_{n!} \int_0^\infty f_{i'}(t') \int_{t'+M}^\infty f_{i''}(t'') \int_{t''+M}^\infty f_{i'''}(t''') \ldots \int_{t^{[n-1]}}^\infty f_{t^{[n]}}(t^{[n]})\, \mathrm{d}t^{[n]} \ldots \mathrm{d}t'''\, \mathrm{d}t''\, \mathrm{d}t' , \qquad (22)$$

where the sum over *n*! includes all permutations of *i*. The non-occurrence of coincidences in the interval between *t* = 0 and the first system failure is covered by the *n*! permutations of failure sequences. This equation applies ∞ as upper integration limit and, unfortunately, equals zero.

The equation includes only single system failures and restorations. Succeeding failures and restorations of the same system are not considered here. After the first failed system has been restored and put into service at *t'* + *M* again, left sub-intervals of CDF domains of the *n* – 1 remaining *and* functioning systems are cut and set to $t' + M = t_0$. With that, the pdf of forward recurrence time has to be discussed in the framework of alternating renewal processes, refer to [22] and to Future Work in Section 7.

5. Illustration

The objective of this illustration is to demonstrate, how the relations between the measures *n*, *TTR*, and $MTTF_i$ affect the coincidence probabilities and – with that – the effectiveness of an RAS approach, as addressed in Section 1.

5.1. Two-system scenario

The simplest case is a two-system scenario, where the coincidence probability is given by equation (6). In case of constant failure rates, $MTTF_1$ = 100, $MTTF_2$ = 80, and *TTR* = 45 yields to the graph as given in *Figure 13* with the limitation value

$$\lim_{t\to\infty} p_{\text{coin\_2,2}}(t) = 1 - \frac{4}{9\cdot e^{9/16}} - \frac{5}{9\cdot e^{9/20}} \approx 0.39 . \qquad (23)$$

The graph describes the following situation: At time *t* = 0 both system start operation with $F_i(0) = 0$.

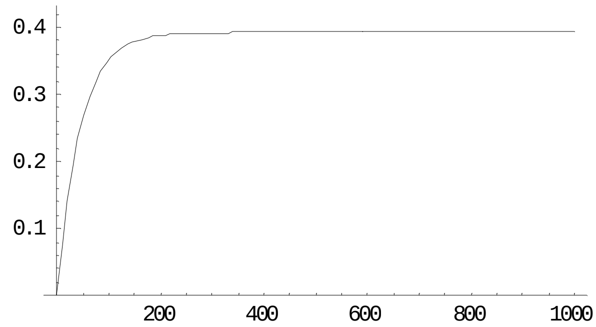

Figure 13. Graph of a 2-system coincidence probability function

5.2. Four-fold coincidence

Four systems are given in this scenario with $MTTF_1$ = 100, $MTTF_2$ = 80, $MTTF_3$ = 60, $MTTF_4$ = 40, and *TTR* = 45. The graphs of the resulting pdfs are shown in *Figure 14*

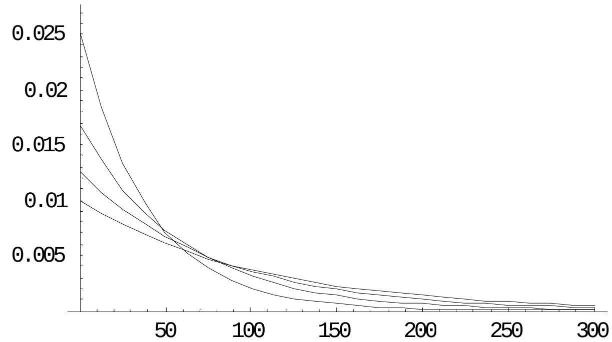

Figure 14. Graphs of the f_1, f_2, f_3, and f_4

Considering the initial conditions *t* = 0 and $F_i(0) = 0$ with 4! = 24 permutations result in 24 terms of coincidence probabilities for a four-fold coincidence, see *Figure 15*.

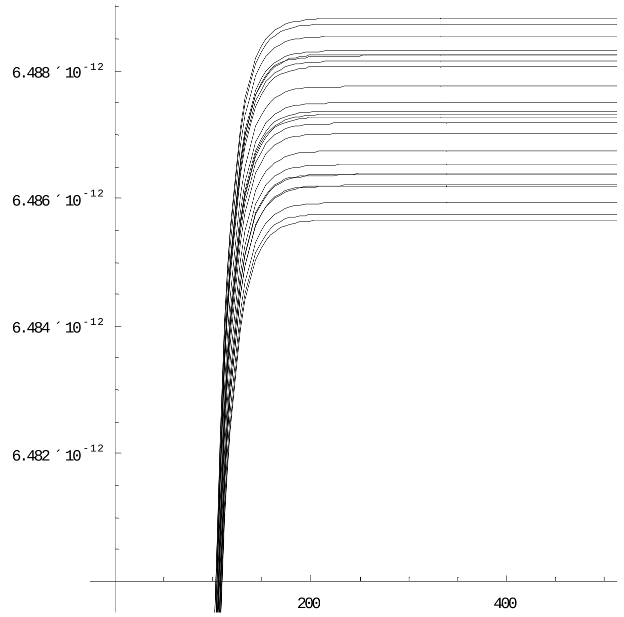

Figure 15. Graphs of 4! coincidence probabilities terms.

The limiting value of approximately $1.56{\cdot}10^{-10}$ (refer to *Figure 16*) indicates that a 4-fold coincidence has no significant influence on the scenario operation.

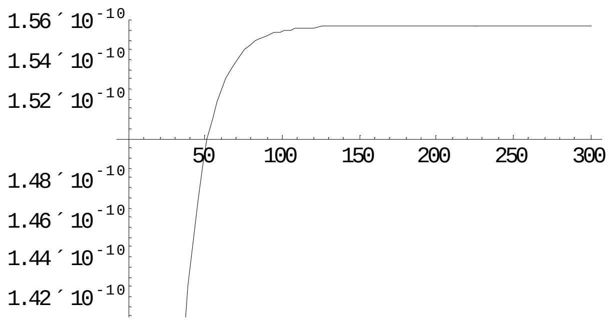

Figure 16. Graph of a 4-system coincidence probability function

6. Some notes

Modelling

It should be noted that convolution approaches as applied in modelling standby configurations (cold redundancies, cold spare) may not be confused with the application of Matryoshka-like integral products as shown in this contribution. The author would like to give a hint on the work of Schneeweiss [22] on *Renewal Processes for Reliability Modeling*. Especially the minimal digest for practical work (Chapter 9) is very helpful in application. The advice on page 154 addresses the "baking" of multiple integrals over products of pdfs, which is applied here. Additional fundamentals can be found in [23].

Note that the two systems coincidence probabilities as given in [13], [14], [17] correspond with equation 6.

The properties of t', t'' … $t^{[k]}$ and t

Variables t', t'' … $t^{[k]}$ represent system-individual time-axes, where $t^{[k]}$ is assigned to the k^{th} system. Variable t represents the calendar time of the scenario and (as a consequence of the integral products) the time axis of the complete ensemble.

Snapshot character

If the approach is embedded in a reliability-adaptive systems concept, then coincidence probabilities, pdf, CDF, and estimated values change their values every instant of time. Again, all E_i do not have a deterministic character, e. g. a system may fail directly after restoration. Such an event changes a prognosis completely; i. e. pdf, CDF, estimated values have a so-called snapshot character, valid at the moment they were calculated. Thus, the snapshot character puts a lot of uncertainty into the model.

Applicability

The applicability is addressed to swarm, fleet, or groups of independent operating systems. The approach is especially tailored to swarm robotics as it complies with the requirements postulated by Sahin & Spears [21]:

- Large Number
  The approach is applicable to large robotic swarms if the number of maintenance units is smaller than the number of robots.
- Homogeneity
  The requirement for few homogeneous groups of robots within a swarm supports the RAS applicability: Performance diagnosis and reliability prognosis are easier to compare and it is easier to estimate the characteristics.
- Simplicity and incapability
  It is required that the robots should be relatively simple and incapable so that the tackled tasks require the cooperation of the individual robots. (This requirement does not have an effect on the reliability-adaptive operation.)
- Sensing and communication abilities
  The robots should only have localised and limited sensing and communication abilities. Here, it is required that each robot transmits instantaneous performance characteristics and that it receives performance command values.

Moreover, classical applications as booking wharves in harbour or air traffic control operation can be supported by this approach. The results are helpful

- to make a proper predictive maintenance policy for the maintenance unit,
- to make a proper predictive operation and maintenance policy for every individual system,
- and for the complete swarm in the scenario.

Again, it should be emphasised that the value of the (mean) time to restoration should not be negligible against the mean time to failure in terms of the number of systems involved in the scenario. Otherwise reliability-adaptive operation is not effective or – even worse – counterproductive compared to regular operation.

Presently, experiments are conducted at the University of Wuppertal under working title *Reliabotix* to validate the results of this approach and to show the applicability as addressed above.

7. Conclusions

A multi-system scenario including dynamically operating systems in a dynamic environment is a very interesting and appealing approach to proof the effectiveness of reliability-adaptive systems. The core of reliability modelling of such a scenario and the objective of the approach given here is the calculation of the probability of a failure coincidence or maintenance conflict, respectively, in an n-system scenario. With that, the following measures can be quantified:

- The probability function of a maintenance conflict depending on the values of n, *MTTF*, and *MTTR*
- The time, when the probability for the next system failure coincidence will be greater than a given threshold

Further Work

The following way of reliability modelling is shown by this approach: Individual coincidence patterns and probabilities of defined failure sequences are modelled including an explicit discussion of failure sequences (permutations), see Section 3. The modeling requires the discussion of combining individual coincidence probabilities of k-out-of-n patterns with k = 2, 3, …, n. A simple addition as proposed in [12] can not be justified yet.

Modelling general coincidence probabilities as sketched in Section 4 considers only single system failures and restorations. Succeeding failures and restorations of the same system are neglected here. Hence, further work shall include a more detailed modelling, which applies the pdf of forward recurrence time in the framework of alternating renewal processes [22].

Moreover, the approach applies forecasted estimated values E_i for maintenance unit booking and reasoning if a system is taken out of service or not. However, the probability that a system with an exponential CDF fails before E_i is ~63%. Applying E_i is just an assumption, whereas the effectiveness has not been proven yet, but shall be discussed in a further approach and shall be validated by *Reliabotix* experiments as presently conducted. (Note that it may be helpful to have a look on maintenance strategies of streetlights.) Summarising, the following questions are still open:

- A system does not exactly fail at time E_i. Thus, the ranking of systems in maintenance booking changes more frequently with higher n. Does any discrepancy occur after re-arranging the ranking, e. g. oscillating system performances?
- Does the change of ranking in combination with the snapshot character have a significant influence on the effectiveness of RAS approaches?

Finally, the assumptions as given in the introduction are easy to justify. But the assumptions on single system failures and restorations; and on E_i require further work.

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Acronym and Notation

Acronym

The singular and plural of an acronym are always spelled the same.

CDF cumulative distribution function
coin coincidence
pdf probability density function
RAS reliability-adaptive system

Notation

E estimated value of the CDF; a priori, forecast, or prognosis measure
f differential of the system fault probability function
F system fault probability function (CDF)

M time to system restoration, random number (short notation of TTR, "maintenance")
$MTTF$ mean time to failure; mean value, a posteriori measure
p probability (measure)
$p_{\text{coin}\_k,c}$ coincidence probability of k involved and c conflicting systems
t time
TTF time to failure, random number
TTR time to restoration, fixed number short notation: M

Integers

c maximal number of (concurring, coinciding, conflicting) systems with overlapping TTR in a cluster
i system index with $i', i'', i''', \ldots \in \{1, 2, \ldots, n\}$
k number of systems in a certain subset of all systems
l number of coincidence clusters considered
n number of systems within the scenario considered