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The model of reusability of series system product

Keywords

reliability, reverse logistics, reuse

Abstract

The main goal of this paper is to create the reverse logistics model that uses reliability theory to describe reusability of product parts with assumption that recovered components are used in production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in a production and gives the base to optimize some of the process parameters: the threshold work time of returns or the warranty period for products containing reused elements.

1. Introduction

Reverse logistics understood as the process of managing reverse flow of materials, in-process inventory, finished goods and related information has become one of the logicians' key areas of interest. It enjoys ever-increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products can bring direct advantages because company uses recycled materials or recovered components instead of expensive raw materials.

Literature survey that has been done around the theme of the reverse logistics area, allowed to set out this article aims and objectives. In the reverse supply chain the issue of: timings, quantities and conditions of returned and reusable components make production planning difficult [4]. The majority of models assume that demand for new products and returns quantity are independent Poisson random variables [5] and very few use the reliability theory to estimate the number of reusable products. Murayama et al. [2]-[4] propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" [1]. Only few models use

the reliability theory to diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. [2]-[4]).

Literature review that have been done so far, allowed to define the main shortages of existing logistics models that deal with the reverse logistics problem:

- most of the created models assume single-component product,

- they are based on the assumption that recovered products are as good as new,

- they don't optimise reusability of the returns.

Main goal of this paper is to create the reverse logistics model that uses the reliability theory to describe reusability of product parts with assumption that recovered components are used in a production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in a production and gives the base to optimise some of the process parameters: the threshold work time of returns or the warranty period for products containing reused elements.

2. The model of the reusing policy

The model that is presented in the paper is based on the following assumptions:

- A company produces the object composed of two elements (A and B). The product fails when one of components fails – series reliability structure.
- A failure of each component occurs independently on other components' failures.
- If the product fails during the warranty period, it is returned to the manufacturer and he has to pay some penalty cost (e.g. the cost of a new product).
- The products are returned as soon as their lives are ended and reusable B components are stored in a stock until new production batch running, when they may be reused.
- The component B of the product may be reused in a new production, if it was not the cause of a product failure and its total work time up to this moment is not greater than some acceptable - threshold time T (Fig.1).
- Neither failed elements B can be reused in a new production (not repairable) nor any A element. All A components are new in a new production.
- Demand for the products is determined and fixed.
- New products are manufactured and sold periodically in established moments.

The process of reusing of the component B, dependently on its threshold age *T*, is shown in *Figure 1*.

Despite the fact that companies realize the potential of reusing products, the question "is it worth to do it", is not so simple to answer. Within main reasons for products reusing are: difficulties with raw material supplying, high cost of utilization of returned and damaged products or lower cost of reusing of products' components than buying new ones. The objective of the presented model is to estimate profitability of using returned and recovered elements in a new production, in the case when they are not as good as new.

According to the assumptions, before every production beginning, the manufacturer has to make the decision: which of returned elements B should be used in the new production. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period.
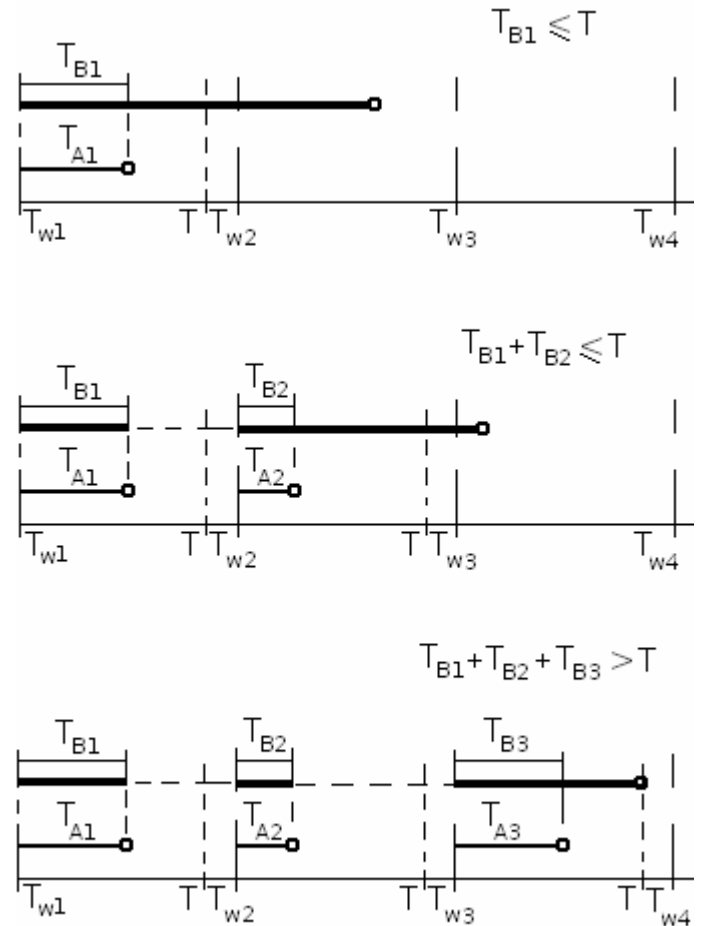

Figure 1. Process of element reusing in a new production.

The objective is to find the threshold work time *T* for returned element that equalizes potential cost and profits of the reusing policy:

$$E\left(C_{WO}(T_W,T)\right) - E\left(C_{WN}(T_W)\right)C_O =$$

$$= C_B - C_R - C_M \qquad (1)$$

$$E\left(C_{WO}(T_W,T)\right) = \left[1 - \frac{R_B(T_W+T)R_A(T_W)}{R_B(T)}\right]C_O \quad (2)$$

$$E\left(C_{WO}(T_W,T)\right) = [1 - R_B(T_W)R_A(T_W)]C_O \quad (3)$$

where C_{WO} = the cost of warranty services if an "old" element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; C_{WN} = the cost of warranty services if a "new" element is used in production; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of:

decomposition, cleaning, preparing of the returned B element to reusing in a production; C_O = penalty cost resulting from a product failure during warranty period (e.g. the total cost of production of a new object); $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; n = production batch percent of reusable B elements, that return during warranty period; E(x) = the expected value of variable x.

The left side of the equation 1 specifies the increase in expected costs of product warranty services (during a single warranty period) caused by the reusing in a production element that is not as good as new. This part of the expression depends on: both elements' reliability (A and B), the length of warranty period and the length of the acceptable total work time T of the returned B element. The increase in expected cost of warranty services is calculated for the case, when all reused elements are in the age of T. The real age of reused elements is equal or lower than T and this way the left side of the expression 1 estimates the maximum possible growth in warranty cost when "old" elements are reused in the production. The direction of changes in the expected cost of reusing elements is not so obvious. In special cases (low value of time T and short warranty period) it can happen that reused product is more reliable than a new one.

The right side of the equation 1 determines the potential cost savings resulting from using cheaper, recovered components instead of new elements.

On the basis of this expression the threshold value T of total work time of the element B can be found, above which reusing the component is not economical. An analytical solution of the equations can't be achieved for the majority of probability distributions describing components' time to failure, but the value of T can be easily found by applying numerical calculations for given vector of T.

The practical application of the proposed model is limited because of the mentioned simplification that all reused elements are in the age of T. According to model assumptions, the demand for the products is determined and fixed. It means that a new production is usually mix of new and reused elements, dependently on their accessibility. The threshold work time T (for which savings from components' reusing are equal losses) can be specified on the base of Equation 1, but practical questions require more precise data: how many reusable elements will return before the new production batch, how many new components must be kept in a stock or what is the expected profit/cost when mix of new-old elements is used in the production?

To answer this questions, the model should consider the percent of returns used in the production. The number of returns that can return between two moments of a production beginning and may be reused depends on the number of the products that were sold earlier, the length of the period between two consecutive production batch, the length of the warranty period and threshold work time T. The number of returns (calculated as a percent of the production batch size) may be estimated as follows [5]:

$$n(t_a, t_b, T_W, T) = n_1 + n_2 + n_3 \tag{4}$$

$$n_1 = \int_{max(t_a - T_M, 0)}^{t_b - T_M} D(t) F(t_a - t, T_M) \, dt \tag{5}$$

$$n_2 = \int_{t_b - T_M}^{t_a} D(t) F(t_a - t, t_b - t) \, dt \tag{6}$$

$$n_3 = \int_{t_a}^{t_b} D(t) F(0, t_b - t) \, dt \tag{7}$$

$$T_M = \min(T_W, T) \tag{8}$$

$$F(t_1, t_2) = F(t_2) - F(t_1) \tag{9}$$

$$F(t) = (1 - R_A(t)) R_B(t) \tag{10}$$

$$F(t) = (1 - R_A(t)) R_B(t + T - T_M) \tag{11}$$

where n = production batch percent of reusable B elements, that return during warranty period; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; T_M = minimal value of threshold time T and warranty period T_W; $D(t)$ – size of the production sold in t moment; $F(t_1, t_2)$ = the increase of product unreliability (caused by A component when B element is still working) in period between t_1 and t_2 moments; $R_A(t)$ = reliability of the element A in t moment; $R_B(t)$ = reliability of the element B in t moment; min, max (t_1, t_2) = minimum/maximum value of variables t_1 and t_2.
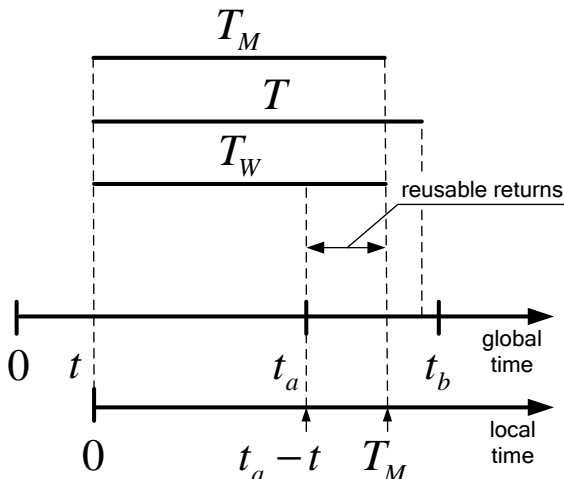
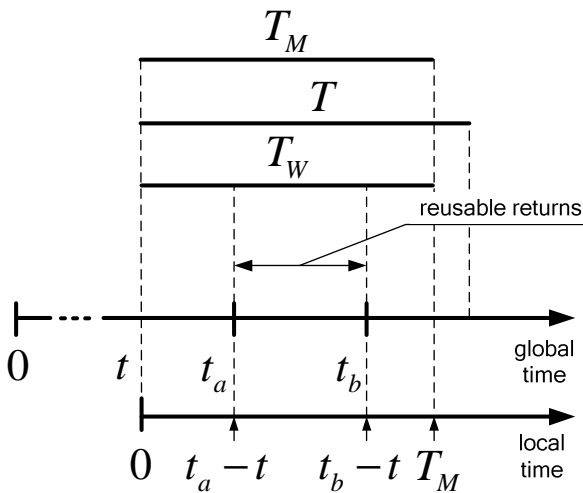*Figure 2.* The period when reusable components may be returned to a producer according to equation 5.

Figure 3. The period when reusable components may be returned to a producer according to equation 6.
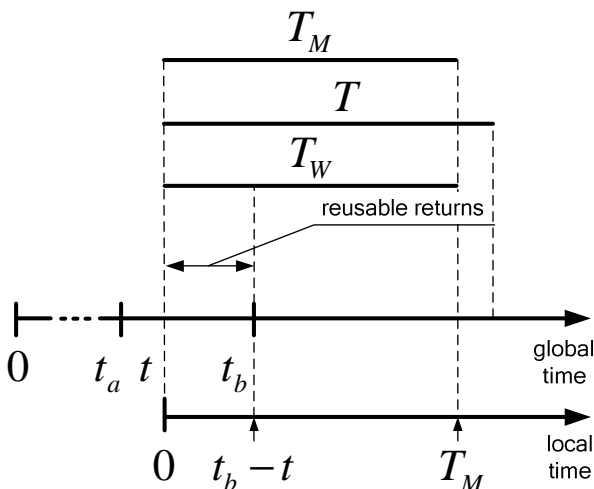

Figure 4. The period when reusable components may be returned to a producer according to equation 7.

The expression 4 allows to calculate the percent of B elements that can be returned to a manufacturer between two consecutive production batch and may be reused for the case when $t_2 - t_1 < T_M$. Value T_M is the minimal value of warranty period and threshold time lengths. It limits the possibility of returns: component B may be returned only during the product warranty period and reused only if it is not older then T. The expression may estimate maximal or minimal number of reusable B elements dependently on the form of the reliability function of B component. If the formula 10 is used, the maximum number of reusable elements is estimated according to the assumption that all products were as good as new when they were sold. The minimal number of reusable returns may be calculated when the expression 11 is considered in the equation 4 because it assumes that all components B in new products are in the maximal allowed age of $T - T_M$. Other cases (when $t_2 - t_1 > T_M$) are presented in detail in literature [5].

Real values of costs and savings coming from the reusing policy is proportional to the variable *n*:

$$\left[E\left(C_{WO}(T_W, T) \right) - E\left(C_{WN}(T_W) \right) \right] \cdot n \cdot C_O =$$

$$= (C_B - C_R) \cdot n - C_M \qquad (12)$$

where C_{WO} = the cost of warranty services if "old" element is used in a new production; T_W = the warranty period of the product; T = the threshold age of the element B, after that the further exploitation isn't continued; C_{WN} = the cost of warranty services if "new" element is used in a production; *n* = production batch percent of reusable B elements, that return during warranty period; C_B = the purchase cost of a new element B; C_R = the total cost of all activities of preparing of returned B element to reusing in a production; C_O = penalty cost resulting from a product failure during warranty period; E(*x*) = the expected value of variable *x*.

The cost of elements' identification C_M is independent on the number of returns, because if a company decides to apply the reusing policy, all elements used in a production have to be identified.

3. Research results

Some example of analytical model results are presented in figures 5-17 (for process parameters: $C_O = 5$, $C_B = 1$, $C_R = C_M = 0$, $R_A = R_B = \exp(-(t/100))^2$.
Figure 5 presents the number of reusable returns assuming that all new products sold earlier:

− were as good as new (maximal number),
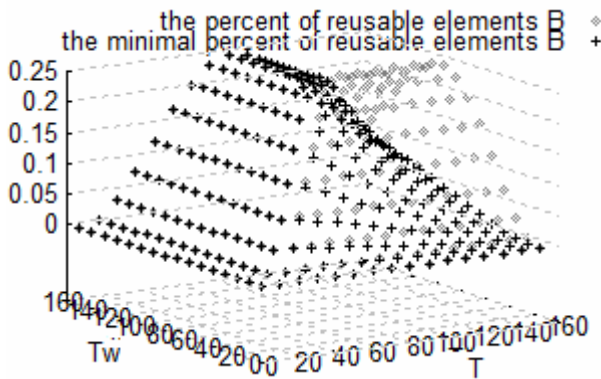− include reused B elements (minimal number)

Figure 5. Maximal and minimal production batch percent of reusable B elements, that may be used in new production.

The minimal and maximal number of reusable elements are similar for low values of threshold time T and warranty period T_W. When an acceptable time T exceeds the average life time of the element B, the minimal number of reusable elements decreases very fast – second reusing of "old" elements is little probable.
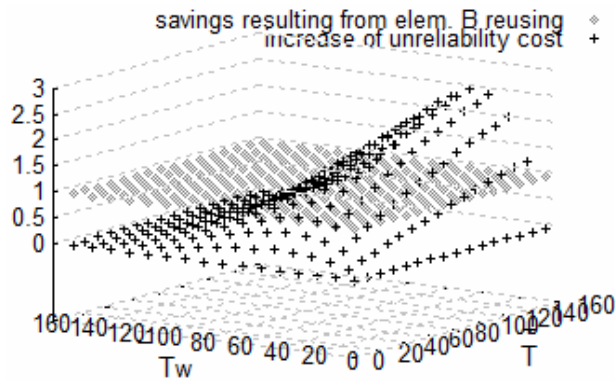
Figure 6. Increase of unreliability costs and savings resulting from the reusing policy according to Eqn1.

The results in *Figures 6-8* are obtained for the same process parameters, but are different because of the influence of returned elements amount, used in a new production batch. Savings coming from element reusing in Figure 6 (without considering the number of returns) are constant for all analysed values of T and T_W, but they have irregular shape in *Figure 7,8*. Considering two alternative values of the total work time T, sometimes it is more economical to get lower unit profit but coming from greater quantity of reused elements. Potential savings and costs are proportional to the number of reusable elements (*Figure 5*).
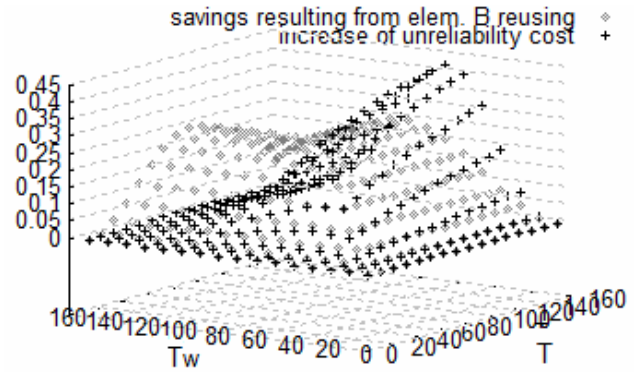

Figure 7. Increase of unreliability costs and savings resulting from the reusing policy according to Eqn12 with maximal possible number of reusable returns.
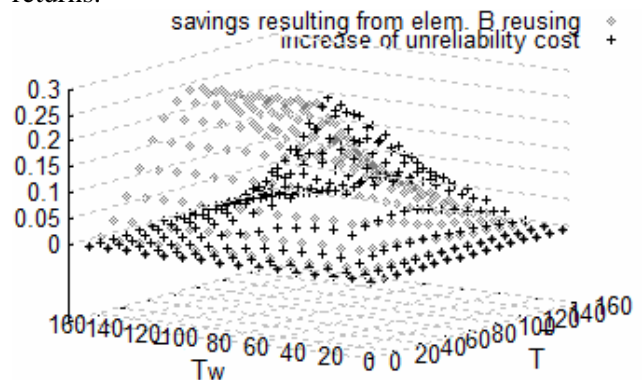

Figure 8. Increase of unreliability costs and savings resulting from the reusing policy according to Eqn12 with minimal possible number of reusable returns.

Their real values will take place between plates presented in *Figure 9* and it is seen that incorrect estimation of T time (too long) causes great costs of the reusing policy. Irregular shape of reusing policy benefits and costs shows that the plane may have more than one local maximum or minimum. The research conducted for various process parameters haven't given any general rules where the optimal value of work time T may be found.

The estimation of threshold and optimal work time T for various warranty periods proves that they are the same for the minimal and maximal number of returns (*Figure 10-11*).
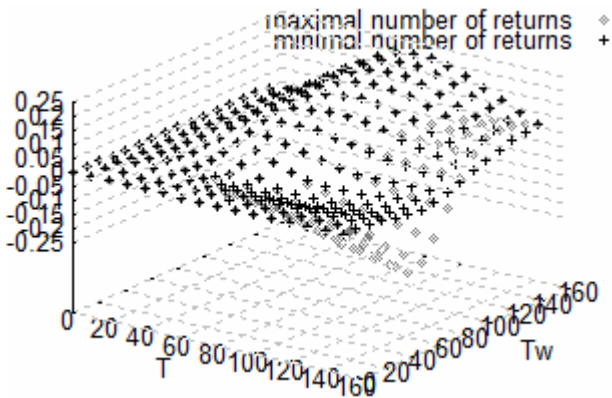
Figure 9. Summary profit resulting from reusing of elements B in new production.
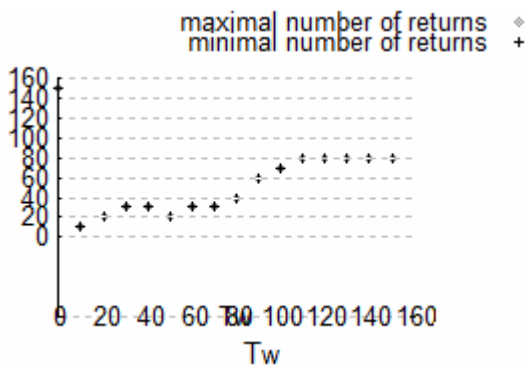

Figure 10. Optimum value of *T* (savings resulting from the reusing policy get maximum).
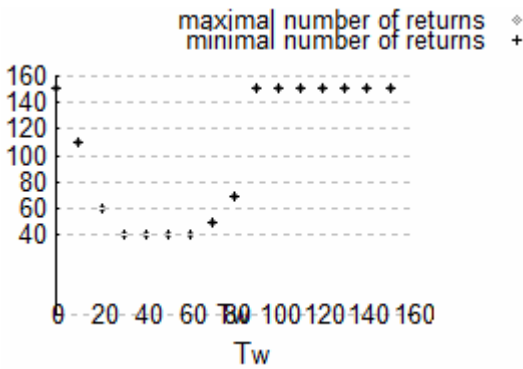

Figure 11. Threshold value of *T* (reusing is not economical above this value).

The optimum work time *T*, after that the further exploitation isn't continued, rises when a warranty period gets longer, but does not exceed the average lifetime of the component B.

The shape of threshold values of *T* time may also be interesting (*Figure 11*). For very short warranty periods costs are usually equal to savings and are very close to zero. It is the effect of very low changes of reliability of the product in very short time. In such cases reusing even "very old" (high values of threshold time *T*) component is profitable.

In order to assess the influence of the reusing process parameters on its cost results, the sensitivity analysis of the proposed model was conducted. The parameters that was concerned as the most meaningful were tested and the results obtained during the research are shown in *Figures 12 - 17*.

When a returned component that was working some time in the past is used in the product offered as new for a customer, the element's intensity of failures is one of the determinant of possible reusing. For this reason the impact of the reusable component failure rate on production process costs and profit was tested.
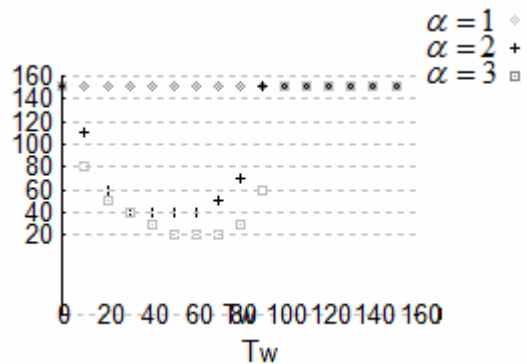

Figure 12. Threshold value of *T* for various failure intensity of the component B: $R_B = \exp(-(t/100))^{\alpha}$.
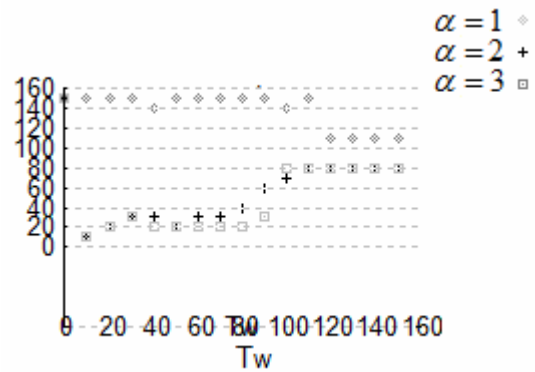

Figure 13. Optimal value of *T* for various failure intensity of the component B: $R_B = \exp(-(t/100))^{\alpha}$.

Exponentially distributed time to failure has a constant failure rate and reusing of such elements always is profitable (*Figure 12,14*). The optimal value of work time *T*, for which the profit is the highest, gets maximum possible value for all tested warranty periods (*Figure 13*). The sudden decrease in optimal *T* value for warranty periods two times longer than co-component's (A) expected time to failure is the effect of lower number of possible returns of the product. Only in the case when A element fails during a warranty period and is returned to the manufacturer, reusing of B

components is possible. The optimum *T* takes value of double A lifetime.
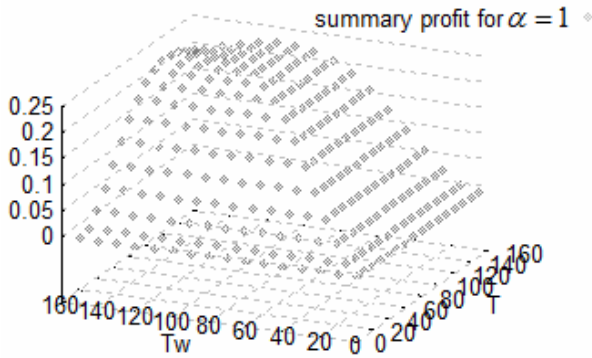

Figure 14. Summary profit resulting from reusing of elements B in new production for $R_B = \exp(-(t/100))^1$.
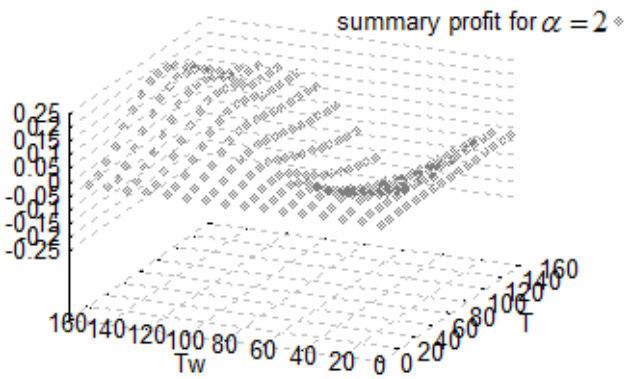

Figure 15. Summary profit resulting from reusing of elements B in new production for $R_B = \exp(-(t/100))^2$.
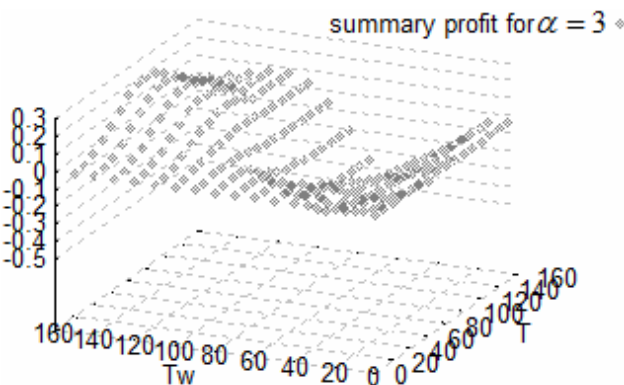

Figure 16. Summary profit resulting from reusing of elements B in new production for $R_B = \exp(-(t/100))^3$.

Figures 15,16 present cost results of the reusing policy for the growing failure rate of the reusable component. The faster growth of this parameter

shifts the threshold and the optimal *T* to lower values – reusing is less profitable (*Figure 12,13*).

The second most meaningful parameter of the reusing process is the relationship between possible costs of warranty services and savings coming from the lower number of elements in the production process. The research results are presented in *Figures 17-18*.
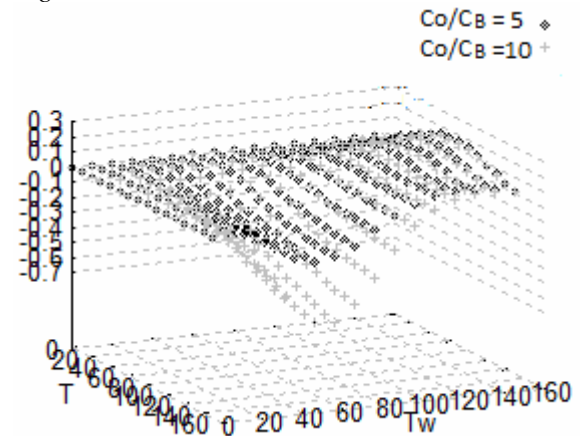

Figure 17. Summary profit resulting from reusing of elements B for $c_O = 5$ and $c_O = 10$, $c_B = 1$, $c_R = c_M = 0$.
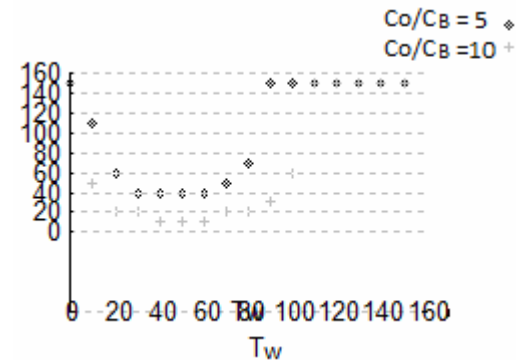

Figure 18. Threshold value of *T* for various relationship between possible costs ratios.

The results of the research are quite obvious – higher cost of a product failure during warranty period causes lower profitability of reusing policy.

4. Conclusion

The model presented in this paper is the continuation of wide researches in the reverse logistics area. The majority of models deal with single – element system or with the assumption that reused elements are as good as new. The proposed model develops the previous ones by releasing both assumptions and gives the base to determine some of reusing policy parameters such as: the threshold work time of returned element that can be used again, the warranty period for the product containing elements which have some history of

work, the size of new elements' stock necessary to fulfil production planes. The model is presented and tested for two-element, series system but it is very simple to be developed it to the case of *x*-element, series system. From the point of view of possible product returns during a warranty period this assumption is usually real because only a failure of a one product component allows to reuse others.

Although the majority of presented expressions is difficult to solve analytically, their analytical form enables easy implementation to a numerical search of optimal process parameters.

Practical application of the model is possible in companies which are interested in the reusing policy because the decision must be supported by the calculation of possible cost and benefits. Further development of the model should release the assumptions about deterministic demand for the product and constant moments of production.

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