OKREŚLENIE OPTYMALNEJ POJEMNOŚCI PARKINGU NA PODSTAWIE ANALIZ SYMULACYJNYCH

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Streszczenie: *W artykule przedstawiono model symulacyjny funkcjonowania parkingu wykorzystujący zorientowane obiektowo podeście. W zaproponowanym modelu uwzględniono stochastyczny charakter popytu na usługi parkowania w mieście. W artykule omówiono wyniki symulacji funkcjonowania parkingu oraz przedstawiono dyskusję praktycznych rezultatów. Przedstawiono również model regresji określający optymalną pojemność parkingu otrzymany na podstawie wyników z modelu symulacyjnego.*

Słowa kluczowe: *pojemność parking, symulacja model regresji*

1. Introduction

Parking lots are the necessary element of contemporary urban transport systems. Use of parking lots eliminates traffic jams caused by decrease of road capacity due to vehicles parked on a roadway. Interceptive parking lots allow minimizing the traffic intensity in central part of the cities, which significantly reduce air and noise pollution. Parking lots are obligatory element of any entertainment, office or shopping center; they enhance the attractiveness of the commercial objects for the clientele.

On the other hand, parking lots are the sources of water pollution because of their extensive impervious surfaces [10]. As a rule, parking lots need more land area than the respective office or shop buildings, this leads to covering of large areas with asphalt and results the excessive accumulation of heat. According to research results, shown in [11], the heat from paved areas in urban zones could even change the weather locally. It also should be mentioned, that parking lots functioning demands a certain amount of funds; as objects of commercial activities, they should be profitable.

Thus, the capacity of parking lot should be sufficient to ensure the implementation of main purpose of the parking lot as an element of transport system. At the same time, it should not be excessive in order to eliminate the negative effects of parking lots functioning.

2. Literature review

Contemporary research projects in the field of parking systems optimization deal with the problems of parking lots design, sustainable functioning of parking lots, safety of pedestrians at parking areas, use of informational technologies in parking lot management, etc.

Parking lots design is usually discussed from the qualitative positions, not quantitative ones. R. Porter, the author of the research [8], investigates methods for maximizing the number of car parking spaces that can be placed within a car park. In his report R. Porter assumes that parking lots design is particularly important for basement car parks in residential apartment blocks or offices, where parking spaces are being characterized by a high value. Other researcher, Ben-Joseph Eran, in his book [2] underlines an esthetic significance of parking lots design and discusses the ways to make them a natural part of sustainable living areas. In the research [9] John A. Stark examines the physical design components of parking lots through the lenses of safety and environmental protection; he concludes his report with the statement, that while designing parking lots, it is important to consider the experience of the drivers as well as pedestrians to facilitate a safe and welcoming shared space. This conclusion has been made in the report [1] as well. The authors emphasize that every motorist must be a pedestrian before and after every trip; therefore, parking lots should be planned not just for motorists but for all users, including pedestrians.

Authors of the report [1] on the example of the Minnetonka city (Minnesota, United States) propose the reduction or even the elimination of restrictions on parking minimums (low bound restriction). It's established that parking minimums tend to be rigid, and this often provides too much parking. The research team suggests to lower minimum requirements on parking, and where applicable, to add maximums (high bound restriction). This is supposed to prevent parking underutilization while providing an alternative to meeting peak demand. It should be mentioned that the recommendations on parking restrictions, obtained by the authors of [1], are not grounded by numerical calculations, they are based mainly on literature review.

Parking demand and parking lots parameters are interdependent values. Thus, in order to determine the optimal parking strategy it's necessary to create a model of demand forming first. The problem of travel demand modeling, where parking demand is considered as a key factor, is examined in researches [4] and [3]. The authors of the paper [4] deal with enhancement of the London Transportation Studies model, which is used as the major multi-modal strategic modeling tool. The proposed in [4] model contains a stand-alone component which interacts with mode and destination choice models to directly modify car demand to different zones; this model treats parking choice and capacity constraints. A mechanism, developed by authors, takes account of explicit supplies of parking spaces in all model zones in London, allowing the modeling of charge- and supply-based parking policies, as well as increasing the realism of the parking-related behavior of travelers. The research [3] also proposes a new approach to estimation of parking demand on the base of numerical results for the central part of Kharkiv (Ukraine); use of the proposed approach in transport systems planning process allow enhancing the precision of forecasted demand parameters.

Existing approaches to estimation of a parking lot capacity (including the approaches, used in [4] and [3]), as a rule, assume that the parking demand for the specific lot is determined and stable. Thus, the parking lot capacity in transportation planning models is usually accepted as a constant value, which does not depend on stochastic demand parameters. Authors of [5] propose to simulate a parking area in the city of Rijeka (Croatia) as a queue model with the use of MS Excel spreadsheets. They demonstrated that by applying the queuing theory methods the optimal number of servers and the required capacity in closed parking areas could be estimated.

3. Conceptual model of a parking lot functioning

In this paper a parking lot is considered as an element of city transport system on the one hand; on the other hand, a lot is treated as a commercial object (as far as drivers pay a fee for the parking services). Developing of a new model aims to consider stochastic nature of demand on parking services while solving problems of parking lot parameters optimization. A problem of estimation of parking lot optimal capacity is being resolved with the use of proposed conceptual model; however, it's not the only optimization task where the model could be used.

Demand on parking services could be characterized by a number of vehicles, parked on a lot during the certain time period, and by duration of parking per one vehicle. For the given time period instead of a number of parked vehicles it's proposed to use an interval between arrivals of two cars in a row. Thus, parking demand D_{PL} could be described by a pair of stochastic values – parking interval and parking duration:

$$D_{PL} = \tilde{\zeta}_{PL}, \tilde{t}_{PL} \tag{1}$$

Where:

$\tilde{\zeta}_{PL}$ – random variable of time interval between arrivals of two vehicles in a row, sec./veh.;

$\tilde{t}_{PL}$ – random variable of parking duration per a vehicle, sec./veh.

As an efficiency criterion to measure a parking lot functioning, it is proposed to use a profit of an enterprise which provides the parking services. Profit as a numerical measure accounts an income and total costs of functioning, so it makes possible to consider factors of different origin – economic, social, ecological, etc.

In a common form profit P_{PL} from a parking lot functioning could be presented as follows:

$$P_{PL} = \frac{1}{3600} \cdot N_{sv} \cdot \overline{t}_p \cdot T_{1hr} - E_{PL} \quad (2)$$

Where:

N_{sv} – total quantity of vehicles serviced at the parking lot during given time period, veh.;
$\overline{t}_p$ – average parking duration per a vehicle, sec./veh.;
T_{1hr} – tariff per 1 hour parking per a vehicle, \$/hr;
E_{PL} – total costs for a parking lot functioning during given time period, \$.

It's obvious, that an average parking duration of a vehicle, serviced at the specific parking lot, is equal to mathematical expectation of a parking duration (one of numeric parameters of demand on parking services):

$$\overline{t}_p = \mu_t \quad (3)$$

Where: μ_t – mathematical expectation of a random variable $\tilde{t}_{PL}$ sec./veh.

Total quantity N_{sv} of serviced vehicles could be estimated as a multiplication of total quantity of vehicles, which arrive during given time period T_m, hrs, to a specific parking lot, and a probability of an event, that vehicles were serviced at the lot. Given that total quantity of vehicles could be defined as a ratio between duration of time period T_m and mathematical expectation of time interval between two vehicles arrived to a parking lot, a value of N_{sv} is calculated with the use of formula (4):

$$N_{sv} = \frac{3600 \cdot T_m}{\mu_\zeta} \cdot p_s \quad (4)$$

Where:

μ_ζ – mathematical expectation of a random variable of time interval between arrivals of two vehicles in a row, sec./veh.;
p_s – probability of the event, that a vehicle, arrived at a parking lot, would be serviced (because the parking lot has enough free space).

Total costs of a parking lot functioning it's proposed to present in a generalized form, as a linear function from a parking lot capacity:

$$E_{PL} = c_0 + c_s \cdot C_{PL} \quad (5)$$

Where: c_0 – constant component of parking lot functioning costs, \$; c_s – costs per one parking space maintenance, \$/unit; C_{PL} – parking lot capacity, units.

The elements c_0 and c_s of total costs, in addition to operating costs, may include other components, such as such as social costs, environmental losses, the cost of ensuring road safety, etc.

Taking into account (3)-(5), the efficiency criterion of a parking lot functioning gets the following form:

$$P_{PL} = T_m \cdot T_{1hr} \cdot \frac{\mu_t}{\mu_\zeta} \cdot p_s - c_0 - c_s \cdot C_{PL} . \tag{6}$$

Considering, that the efficiency criterion (6) has its maximum value in the range of a parking lot capacity values, the optimal parking lot capacity could be determined as a root of the following equation:

$$\frac{\partial P_{PL}}{\partial C_{PL}} = 0 . \tag{7}$$

It should be mentioned, that to make the expression (6) complete, a functional dependence $p_s = f \;\; C_{PL}, \mu_\zeta, \mu_t$ should be determined. That could be achieved by analyzing statistical data on empirical observations results, or on the base of computer simulations.

4. Simulation model of a parking lot

To provide a tool for identification of consistent patterns of a parking lot functioning, the simulation model was developed in bounds of this research. The simulation model was implemented on the base of object-oriented approach with the use of C# programming language; its programming code could be downloaded at [7].

Proposed simulation model of a parking lot functioning contains three classes, which represent base entities: *ParkingModel* (a parking lot), *Vehicle* (a vehicle intended to be parked) and *VehiclesFlow* (demand on parking services).

Class *Vehicle* has two fields (*InParkingTime* and *ParkingDuration*) and one property (*OutParkingTime*). *InParkingTime* contains a value of a time moment when a vehicle arrives to a parking lot, *ParkingDuration* represents a value of a time interval during which a vehicle supposed to be parked in a lot. *OutParkingTime* returns a sum of *InParkingTime* and *ParkingDuration* values and shows a time moment when a vehicle leaves a parking lot.

Class *VehiclesFlow* contains a collection *Vehicles* of *Vehicle* objects and a property *VehiclesNumber*. Collection *Vehicles* is a list of objects of *Vehicle* type, where those objects represent vehicles intended to be parked in the parking lot. Property *VehiclesNumber* returns a number of objects in the *Vehicles* list. Generation of vehicles flow is implemented in a constructor of the *VehiclesFlow* class. The constructor arguments are duration of model period and stochastic variables of vehicles arrival intervals and vehicles parking duration. Stochastic variables presented as entities of the *Stochastic* class (the *Stochastic* class implementation code is presented and could be downloaded at [6]).

Class *ParkingModel* contains numeric fields *ModelTime* and *Capacity*, a field *Demand* of *VehiclesFlow* type, collections *vehicles*, *CurrentlyParkedVehicles*, *ServicedVehicles* and *RejectedVehicles*, a property *Occupancy* and a method *Simulate*. *ModelTime* is a private field (used only for inner class procedures) and contains current model time. *Capacity* represents a value of parking spaces of the parking lot. The *Demand* field describes a demand on services of the parking lot. The *vehicles* collection contains a list of the *Vehicle* objects from the *Vehicles* field of *Demand*; it is used for inner procedures in the *Simulate* method. The *CurrentlyParkedVehicles* private list contains a list of the *Vehicle* objects, which describe vehicles parked at the parking lot at the *ModelTime* moment; this list is used for purposes of simulations as well. Lists *ServicedVehicles* and *RejectedVehicles* contain the *Vehicle* objects, which represent vehicle services at the parking lot and rejected ones respectively. The *Occupancy* property returns a number of vehicles, parked at the parking lot at the *ModelTime* moment.

Method *Simulate* of the *ParkingModel* class directly implements the procedure of simulation of the parking lot functioning. The method has a single argument – time step for iterations of model time. While iterations model time (value of the *ModelTime* private field) is being incremented on a value of the method's argument. Those iterations are held, if one of conditions is true: all the vehicles from the *Demand* field (initially contained in the *vehicles* collection) are reviewed by the simulation procedure, or the parking lot occupancy is greater than zero. If not all the vehicles were reviewed (the *vehicles* collection has objects in it), for the first object in the *vehicles* collection the following condition is being verified: if the vehicles arrival moment is less than the current model time and the parking lot occupancy is less than its capacity, then the vehicle is being added to the *CurrentlyParkedVehicles* list and to the *ServicedVehicles* list and is being removed from the *vehicles* collection; if the vehicles arrival moment is less than the current model time and the parking lot occupancy is equal to its capacity, then the vehicle is being added to the *RejectedVehicles* list and is being removed from the *vehicles* collection. If the *vehicles* collection is empty and current occupation of the parking lot is greater than zero, the all the vehicles, which should leave the lot before the current model time, are being removed from the *CurrentlyParkedVehicles* collection.

5. Results of experimental studies

With the use of developed simulation model, the full factorial experiment was conducted in order to determine the $p_s = f\ \ C_{PL}, \mu_\zeta, \mu_t$ dependence. The probability value was measured as a ratio of a number of objects in the *ServicedVehicles* list and a value returned by *Demand.VehiclesNumber* property.

In the experiment the following bounds of input parameters were used:

- parking lot capacity (10 levels): minimum value – 50 units, maximum value – 500 units, step – 50 units;
- mathematical expectation of the vehicles arrival interval (6 levels): minimum value – 30 sec., maximum value – 1830 sec., step – 300 sec.;
- mathematical expectation of the parking duration (12 levels): minimum value – 600 sec., maximum value – 7200 sec., step – 600 sec.

Thus, according to the defined factor levels, the simulation experiment consists of 840 series. To ensure the statistical significance of the experiment, 100 runs were carried out in each series of the experiment.

The experiment numerical results show, that there exists a non-linear dependence between probability of servicing p_s and input factor values. Some results for mathematical expectation of the parking duration in 2400 sec. are shown at Fig. 1.

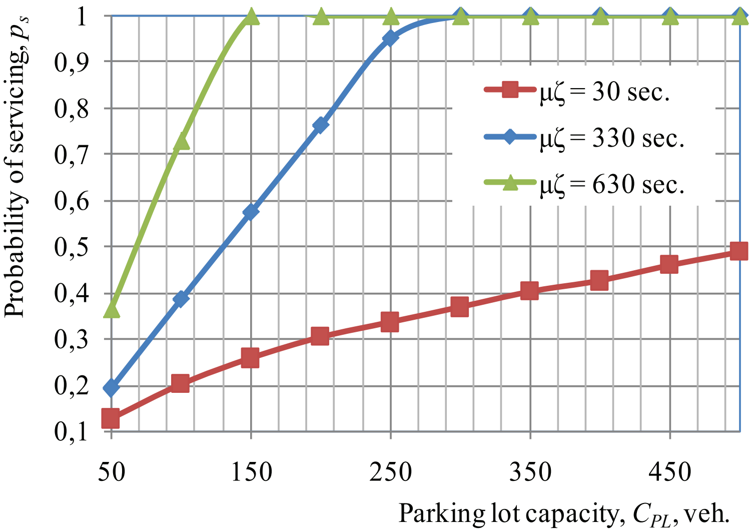

Fig. 1. Dependence of servicing probability from a parking lot capacity (parking duration: 40 min.)

In order to define a form of the $p_s = f\ C_{PL}, \mu_\zeta, \mu_t$ functional dependence, the following hypotheses on form of the dependence were examined:

- linear form with non-zero free coefficient: $H_1 : p_s = a_0 + a_C \cdot C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t$,
- linear form with zero free coefficient: $H_2 : p_s = a_C \cdot C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t$,
- power form with non-zero free coefficient: $H_3 : p_s = a_0 \cdot C_{PL}^{a_C} \cdot \mu_\zeta^{a_\zeta} \cdot \mu_t^{a_t}$,
- power form with zero free coefficient: $H_4 : p_s = C_{PL}^{a_C} \cdot \mu_\zeta^{a_\zeta} \cdot \mu_t^{a_t}$,
- linear-logarithmic form with zero free coefficient:

$$H_5 : p_s = a_C \cdot \ln C_{PL} + a_\zeta \cdot \ln \mu_\zeta + a_t \cdot \ln \mu_t,$$

– linear-logarithmic form with zero free coefficient:

$$H_6 : p_s = a_C \cdot C_{PL} + a_\zeta \cdot \ln \mu_\zeta + a_t \cdot \ln \mu_t ,$$

– linear-logarithmic form with zero free coefficient:

$$H_7 : p_s = a_C \cdot \ln C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t ,$$

where: a_0, a_C, a_ζ, a_t – coefficients of regression models.

With the use of regression analysis tools of Microsoft Excel the coefficients for the proposed hypotheses were estimated. The results of regression analysis are presented in tab. 1.

Table 1. Results of regression analysis

Hypothesis on form of dependence	*Values of regression coefficients*				*Value of the coefficient of determination*
	a_0	a_C	a_ζ	a_t	
$H_1 : p_s = a_0 + a_C \cdot C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t$	0,4592	$5{,}79\cdot10^{-4}$	$3{,}12\cdot10^{-4}$	$-1{,}69\cdot10^{-5}$	0,4793
$H_2 : p_s = a_C \cdot C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t$	0	$1{,}16\cdot10^{-3}$	$4{,}23\cdot10^{-4}$	$2{,}24\cdot10^{-5}$	0,9166
$H_3 : p_s = a_0 \cdot C_{PL}^{a_C} \cdot \mu_\zeta^{a_\zeta} \cdot \mu_t^{a_t}$	0,0317	0,3129	0,4505	–0,1799	0,6809
$H_4 : p_s = C_{PL}^{a_C} \cdot \mu_\zeta^{a_\zeta} \cdot \mu_t^{a_t}$	exp(0) = 1	0,1280	0,3901	–0,4338	0,6545
$H_5 : p_s = a_C \cdot \ln C_{PL} + a_\zeta \cdot \ln \mu_\zeta + a_t \cdot \ln \mu_t$	0	0,1096	0,1676	–0,1008	0,9686
$H_6 : p_s = a_C \cdot C_{PL} + a_\zeta \cdot \ln \mu_\zeta + a_t \cdot \ln \mu_t$	0	$5{,}79\cdot10^{-4}$	0,1783	–0,0557	0,9672
$H_7 : p_s = a_C \cdot \ln C_{PL} + a_\zeta \cdot \mu_\zeta + a_t \cdot \mu_t$	0	0,1169	$3{,}06\cdot10^{-4}$	$-1{,}90\cdot10^{-5}$	0,9446

As we could see, according to estimated value of the coefficient of determination, the H_5 hypothesis fits the better of all the considered hypotheses ($R^2 = 0{,}9686$). Thus, functional dependence of the vehicle servicing probability from a parking lot capacity and parking demand parameters must be defined as follows:

$$p_s = 0{,}1096 \cdot \ln C_{PL} + 0{,}1676 \cdot \ln \mu_\zeta - 0{,}1008 \cdot \ln \mu_t . \tag{8}$$

Taking into account (8), the efficiency criterion of a parking lot functioning could be presented in a form

$$P_{PL} = T_m \cdot T_{1hr} \cdot \frac{\mu_t}{\mu_\zeta} \cdot \ 0{,}1096 \cdot \ln C_{PL} + 0{,}1676 \cdot \ln \mu_\zeta - 0{,}1008 \cdot \ln \mu_t \ - c_0 - c_s \cdot C_{PL} . \tag{9}$$

Differentiating the expression (9), on the basis of (7) we obtain the following equation:

$$\frac{\partial P_{PL}}{\partial C_{PL}} = 0{,}1096 \cdot \frac{T_m \cdot T_{1hr}}{C_{PL}} \cdot \frac{\mu_t}{\mu_\zeta} - c_s = 0 . \tag{10}$$

A root of the equation (10) in regard to C_{PL} is the optimal value of a parking lot capacity:

$$\hat{C}_{PL} = 0{,}1096 \cdot \frac{T_m \cdot T_{1hr}}{c_s} \cdot \frac{\mu_t}{\mu_\zeta}, \tag{11}$$

Where: $\hat{C}_{PL}$ – optimal parking lot capacity, units.

It should be noted, that obtained formula for estimation of optimal parking lot capacity is statistically valid only for demand parameters bounds used in the simulation experiment.

6. Summary

Contemporary approaches to estimation of a parking lot capacity, as a rule, assume that the parking demand for the specific lot is known and stable. Thus, the parking lot capacity in transportation planning models is usually accepted as a constant value, which does not depend on stochastic demand parameters. Proposed simulation model of a parking lot functioning, based on object-oriented approach, allows modeling and estimation of stochastic characteristics of parking process. The optimal parking lot capacity formula, obtained on the basis of computer simulations, is recommended to be used for estimation of a parking lot capacity. This expression, however, could be clarified and supplemented with the help of presented simulation model.

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