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Instantaneous Frequency Estimation of Multi-Component Non- Stationary Signals using Fourier Bessel series and Time-Varying Auto Regressive Model

G. Ravi Shankar Reddy and Rameshwar Rao

Abstract—In this paper, we propose a novel technique for Instantaneous Frequency (IF) estimation of multi component non stationary signals using Fourier Bessel Series and Time–Varying Auto Regressive (FB-TVAR) model. In the proposed technique, the Fourier-Bessel (FB) expansion decomposes the multi-component non stationary signal into a number of mono-component signals and TVAR model is used to model each mono-component signal. In TVAR modeling approach the time varying parameters are expanded as a linear combination of basis functions. In this paper, the TVAR parameters are expanded by a discrete cosine basis functions. The maximum likelihood estimation algorithm for model order selection in TVAR models is also discussed. The Instantaneous Frequency (IF) is extracted from the time-varying parameters by calculating the angles of the estimation error filter polynomial roots. The estimation of the TVAR parameters of a multi-component signal requires the inversion of a large covariance matrix, while the projected technique (FB-TVAR) requires the inversion of a number of comparatively small covariance matrices with better numerical stability properties. Simulation results are presented for Multi component discrete Amplitude and Frequency modulated (AM-FM) signal

Keywords—basis function, Fourier-Bessel expansion, Instantaneous Frequency, multi component non stationary signal, Time-varying Auto Regressive model

I. INTRODUCTION

THE majority of real-life signals are non-stationary and often multi-component. Bat signal, sonar signal, seismic signals, whale sounds, bird songs, musical recordings are some examples of this kind of signals. Moreover, multi-component non-stationary signals are encountered in various man made signals such as radar, sonar, biomedical engineering and automotive emission. These signals also exist in multiple-signal environments [1].

A multi-component non-stationary signal is often expressed as a sum of individual mono-component signals, where each component has time-variant amplitude and frequency (or) Phase functions. The analysis of such signals involves estimation of instantaneous frequency (IF) function of each mono-component [1]

The spectrogram, the most commonly used TFD, describes the IF laws-IF trajectories in the time-frequency plane of the individual components of the observed multi-component

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signal. However, its time-frequency resolution, especially for closely spaced components, is often poor. The Wigner-Ville Distribution (WVD), another commonly used TFD, presents high amplitude cross terms for such signals. The Choi-Williams Distribution has a tradeoff between cross term suppression and time-frequency resolution. In general, a TFD used in analysis of multi-component non-stationary signals should be cross terms free and should have high resolution in the time-frequency domain.[1-5]

Spectral analysis of non-stationary signals, with high frequency-resolution is obtained by using the Time Varying Autoregressive (TVAR) Model [6].In the modeling of non-stationary signals by TVAR process, estimation of the TVAR parameters can be classified into one of the two categories as the adaptive technique and the basis function approach[7].The most popular approach to estimate time varying coefficients is to employ an adaptive algorithm and track the parameters. These methods work well with slowly varying signals but fail to track rapid variation. If the coefficients change fast enough, compared to the algorithm's convergence time, the adaptive algorithm will not be able to track the time varying parameters. Another common approach to estimate the parameters is to expand each time varying coefficient into a set of basis functions. If each coefficients time-evolution can be approximated by a combination of a small number of basis functions, then the estimation task is equivalent to the estimation of the weighting constants in this expansion[6 ,7].

In this basis function expansion, two issues need to be resolved. First, a general class of basis functions is to be chosen, and then, the significant basis functions need to be selected. Several classes of functions have been proposed including polynomial, wavelet and prolate spheroidal functions. However, no uniform rule exists to indicate which class should be adopted. Moreover, the approach of choosing the significant basis functions is based on trial and error[12].

Non stationary signal is modeled using TVAR process of order p. In the process of modeling, the estimation of the TVAR parameters require the inversion of a covariance matrix of size $p(q+1) \times p(q+1)$, where q is the required number of basis functions to represent each TVAR parameter. For modeling multi-component non stationary signal using TVAR process, large number of basis functions (q) and high model order (p) are essential to estimate TVAR parameter, which requires the inversion of a large covariance matrix [12].

It is therefore suggested that in such cases of modeling by a TVAR process it would be wiser to perform a component separation before doing estimation of the process parameters. This enables us to decompose a complex estimation problem into a set of sub problems, each much simpler and more favorable from a numerical-computation point of view.

In this paper, we develop a method of signal analysis based on the Fourier-Bessel (FB) expansion and the TVAR process. The multi-component Non stationary signal has been expanded into the FB series. Each component has non overlapping cluster of FB coefficients. The components can be reconstructed directly from the real FB coefficients of the separated clusters. Then, we model each reconstructed component by a lower order TVAR process. In this way it requires the inversion of a number of relatively small correlation matrices with better numerical stability properties.

The paper is organized as follows. In section II it explains the Fourier-Bessel Expansion. It explains the Time-varying Autoregressive modeling in section III. In section IV it explains the selection of basis function and TVAR model order determination by means of Maximum likelihood estimator. In section V it gives the steps for proposed algorithm. The investigational results of analysis of AM-FM signal is presented in section VI. Concluding remarks are given in section VII.

II. FOURIER-BESSEL EXPANSION

Fourier-Bessel expansions decompose the multi component signal in to mono component Signal without any prior information about the frequency band of the signal.

Fourier-Bessel (FB) series is a particular kind of generalized Fourier series (is an infinite series expansion on a finite interval) based on Bessel functions. Consider discrete – time signal $x(n)$ over an interval (0, N) the zero-order FB series expansion of this signal is expressed as [9]

$$x(n) = \sum_{m=1}^{M} C_m J_0\left(\frac{\lambda_m}{N} n\right) \qquad (1)$$

The FB coefficients C_m are computed by using the relation

$$C_m = \frac{2 \sum_{n=0}^{N} n x(n) J_0\left(\frac{\lambda_m}{N} n\right)}{N^2 \left[J_1(\lambda_m) \right]^2} \qquad (2)$$

Where M is the order of the FB expansion and J_0 (.), J_1 (.) are the zero-order of the Bessel function of the first kind and the first-order of the Bessel function of the first kind, respectively and can be used as basis functions for FB series[9]. $\{\lambda_m; m = 1, 2, ..., M\}$ are the ascending order positive roots of $J_0(\lambda) = 0$

It has been demonstrated in [9,12] that if the center frequency and the bandwidth of a test signal are varied then the order and range of nonzero coefficients of the FB series expansion of the signal are also changed .In particular, it is

shown that the order increases with higher center frequency and range widens with larger bandwidth .An explanation of the above properties of the FB series expansion can be obtained by analytically deriving the coefficient C_m for the signal

$$x(n) = \cos(\omega n) \qquad (3)$$

The FB coefficient C_m for the above is given by [10]

$$C_m = \frac{2\lambda_m \cos(\omega N - \alpha)}{J_1(\lambda_m) \left[\left(\lambda_m^2 - \omega^2 N^2 \right)^2 + \omega^2 N^2 \right]^{\frac{1}{2}}} \qquad (4)$$

Where $\alpha = \sin^{-1}\left(\dfrac{\omega N}{\left[\left(\lambda_m^2 - \omega^2 N^2 \right)^2 + \omega^2 N^2 \right]^{\frac{1}{2}}} \right)$ (5)

From (4) It can be verified that the peak value of C_m is attained for the order m where the root $\lambda_m \approx \omega N$ and the peak value is given by

$$C_{m,peak} \approx \frac{\sin(\omega N)}{J_1(\omega N)} \qquad (6)$$

The magnitude of C_m decreases quickly away from the order where the peak value is taking place and the value becomes irrelevant at far-away orders. It should be noted that like the Fourier series coefficients are unique for a given signal, the FB series coefficients C_m are also unique for a given signal. Nevertheless, unlike the sinusoidal basis functions in the Fourier series, the Bessel functions are having damped sinusoidal function and will decay over time. This characteristic of the Bessel functions makes the FB series expansion appropriate for non-stationary signals [9, 12].

A. Spectrum Representation

Similar to the Fourier expansion, the FB expansion can also be used for spectrum representation. The benefit lies in the fact that unlike a Fourier coefficient which represents only a single frequency, each FB coefficients represents a band of frequencies. Thus only fewer FB coefficients are adequate to represent a band-pass signal compared to the number of Fourier coefficients. For a given sinusoidal signal $x(n) = \cos(\omega n)$ the FB coefficients will have peak at $\lambda_m \approx \omega N$. consequently the relation between the order of the FB expansion and the frequency is given by the following equation [9, 12]

$$\lambda_m = 2\pi f N \qquad (7)$$

Where N is the length of the signal, and f is the frequency in Hz .It is obvious from the above that there is one-to-one correspondence between the order (M) of the FB expansion where the coefficients attain peak magnitude and frequency

content of a signal [9, 10]. The FB expansion represents the spectrum of a signal like the Fourier expansion. Since the Fourier transform of the Bessel function

$$s(n) = J_0\left(\frac{\lambda_m}{N}n\right) \text{ is given by}$$

$$s(\omega) = \frac{1}{\sqrt{\left(\frac{\lambda_m}{N}\right)^2 - \omega^2}} \quad \text{for} \quad |\omega| < \frac{\lambda_m}{N} \tag{8}$$

Each term $C_m J_0\left(\frac{\lambda_m}{N}n\right)$ of (1) has an approximate

bandwidth $\omega_B \cong \frac{\lambda_m}{N}$ in the spectral domain.

Consequently, the reconstruction of $x(n)$ using the first M

terms has a maximum bandwidth of $\omega_{\max} \cong \frac{\lambda_M}{N}$.

When $x(n)$ is the sum of L cosine signals $\cos(\omega_i n)$ i.e.

$$x(n) = \sum_{i=1}^{L} \cos(\omega_i n) \tag{9}$$

The FB coefficients of the above signal is

$$C_m = \sum_{i=1}^{L} \frac{2\lambda_m}{J_1(\lambda_m)} \left(\frac{\cos(\omega_i N - \alpha_i)}{\left[\left(\lambda_m^2 - \omega_i^2 N^2\right)^2 + \omega_i^2 N^2\right]^{1/2}} \right) \tag{10}$$

Where

$$\alpha = \sin^{-1}\left(\frac{\omega_i N}{\left[\left(\lambda_m^2 - \omega_i^2 N^2\right)^2 + \omega_i^2 N^2\right]^{1/2}} \right) \tag{11}$$

It can be shown that for well-separated frequencies ω_i every expression on the right-hand side of (10) will represent a region of the FB coefficients where the coefficients are nonzero corresponding to a sub-signal of the composite signal $x(n)$. Since coefficients are real, each cosine components can be directly reconstructed from the FB coefficient plot [9, 11]

B. Applications of Fourier-Bessel decomposition

1) One can obtain different band-limited signals by selecting appropriate FB coefficients in re-synthesizing signals. Further by means of each band-limited signal one can perform AM-FM analysis for different speech analysis tasks.
2) For decomposition of a signal into its constituent components, the FB series based technique is beneficial over the technique based on the filter-bank approach, because we do not need any prior information about the frequency band of the signal.

C. Requisite Conditions for Fourier-Bessel Decomposition

1) The FB expansion order M *must* be known a priory. Since the interval between successive zero-crossing of the Bessel function $J_0(\lambda)$ increases slowly with time and approaches π in the limit. When order M *is* not known, then for covering full signal bandwidth, that is, the half of the Sampling frequency, M *must* be equal to the length of the signal
2) Because the FB decomposition basically represents only the oscillatory nature of a signal, the DC component of the signal, if any must be removed prior to the decomposition.

In the present study, we presume that the non stationary signals are well separated in the frequency domain, and the signals will be associated with various different non-overlapping clusters of the FB coefficients. Consequently, each signal of the multi-component non stationary signal can be reconstructed separately by identifying and separating the corresponding FB coefficients

III. TVAR MODELING

The non stationary discrete-time stochastic process x_n is represented by pth order TVAR model as [19]

$$x_n = -\sum_{k=1}^{p} a_{k,n} x_{n-k} + v_n \tag{12}$$

Here $a_{k,n}$ are time-varying coefficients and v_n is a stationary white noise process with mean is zero and variance is σ_v^2. According to the time-varying coefficients evolution, TVAR model is likely to be categorized in to two group's i.e. adaptive method and basis function approach[16,18].

TVAR model based on the basis function technique is able to trace a strong non-stationary signal that is why this model is focused in the present study[19].In this technique, each of its time-varying coefficients are modeled as linear combination of a set of basis functions. The purpose of the basis is to permit fast and smooth time variation of the coefficients. If we denote $u_{m,n}$ as the basis function and consider a set of (q + 1) function for a given model, we can state the TVAR coefficients in general as

$$a_{k,n} = \sum_{m=0}^{q} a_{km} u_{m,n} \tag{13}$$

In the basis function approach not only the model order p, but also the basis functions $u_{m,n}$, and the expansion dimension q must be chosen. From (13) we examine that, we have to calculate the set of parameters a_{km} for $\{k = 1, 2, ..., p; \ m = 0, 1, 2, ..., q; a_{0m} = 1\}$ in order to compute the TVAR coefficients $a_{k,n}$,and the TVAR model is absolutely specified by this set. The estimation by the basis function approach is to calculate not the time-varying parameters $a_{k,n}$, but the unknown constant coefficients a_{km}.

The TVAR coefficients are designed as follows, we consider single realization of the process x_n.For a given realization of x_n we can analyze (12) as a time-varying linear

prediction error filter and consider v_n to be the prediction error[19]

$$v_n = x_n - \hat{x}_n \tag{14}$$

Where

$$\hat{x}_n = -\sum_{k=1}^{p} a_{k,n} x_{n-k} \tag{15}$$

The prediction error can be written as

$$v_n = x_n + \sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k} \tag{16}$$

The total squared prediction error, is now specified by

$$\varepsilon_p = \sum_{\tau} |v_n|^2$$

$$\varepsilon_p = \sum_{\tau} \left| x_n + \sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k} \right|^2 \tag{17}$$

Here, τ is the interval over which the summation is performed .In covariance technique for modeling the non stationary stochastic process x_n we formulate no assumptions on the information outside [0, N-1] and set $\tau = [p, N-1]$. The time-varying parameters a_{km} are estimated by minimizing the mean squared prediction error in (17) can be found by means of setting the gradient of ϵ_p with respect to a_{lg}^* zero

$$\frac{\partial \varepsilon_p}{\partial a_{1g}^*} = \sum_{\tau} \frac{\partial v_n v_n^*}{\partial a_{1g}^*} = \sum_{\tau} v_n \frac{\partial v_n^*}{\partial a_{1g}^*} = 0 \tag{18}$$

$$\{l = 1, 2, \cdots, p; g = 0, 1, \cdots, q\}$$

Where

$$v_n^* = x_n^* + \sum_{l=1}^{p}\sum_{g=0}^{q} a_{1g}^* u_{g,n}^* x_{n-l}^* $$

And the derivative of v_n^* with respect to a_{lg}^* is $u_{g,n}^* x_{n-l}^*$.consequently, (18) become,

$$\sum_{\tau} v_n u_{g,n}^* x_{n-l}^* = 0 \tag{19}$$

The above condition is corresponding to the orthogonality law encountered in stationary signal modeling. Substituting for v_n in (19) from (16) we have

$$\sum_{\tau} \left(x_n + \sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k} \right) u_{g,n}^* x_{n-l}^* = 0 \tag{20}$$

Now we define a function $c_{mg}(l,k)$ as shown below,

$$c_{mg}(l,k) = \sum_{\tau} u_{m,n} x_{n-k} u_{g,n}^* x_{n-l}^* \tag{21}$$

Using the above definition in (20) we have,

$$\sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} c_{mg}(l,k) = -c_{0g}(l,0) \tag{22}$$

The above equation represents a system of p(q+1) linear equations. The above linear equations can be articulated efficiently in matrix form as follows, we first define a column vector a_m as follows

$$a_m = \begin{bmatrix} a_{1m} a_{2m} \cdots a_{pm} \end{bmatrix}^T \tag{23}$$

$where$
$$m = 0, 1, \cdots, q$$

We now use the function in (21) to find the following matrix for $0 \le (m, g) \le q$

$$C_{mg} = \begin{bmatrix} c_{mg}(1,1) c_{mg}(1,2) & \cdots & c_{mg}(1,p) \\ c_{mg}(2,1) & c_{mg}(2,2) & \ldots & c_{mg}(2,2) \\ \vdots & & \ddots & \vdots \\ c_{mg}(p,1) c_{mg}(p,2) & \cdots & c_{mg}(p,p) \end{bmatrix} \tag{24}$$

The above matrix is pxp and all the dissimilar values for m and g result in (q+1)x(q+1) such matrices, by means of these matrices, we can now describe a block matrix as shown below,

$$C = \begin{bmatrix} C_{00} & C_{01} & \cdots & C_{0q} \\ C_{10} & C_{11} & \ddots & C_{1q} \\ \vdots & \vdots & \vdots & \vdots \\ . & . & . & . \\ C_{q0} & C_{q1} & \cdots & C_{qq} \end{bmatrix} \tag{25}$$

Matrix C has (q+1)x(q+1) elements and each element is a pxp matrix follow-on in a C of size p(q+1)x p(q+1).Now we describe a column vector d_g as shown below

$$d_g = \begin{bmatrix} c_{0g}(1,0) c_{0g}(2,0) \cdots & c_{0g}(p,0) \end{bmatrix}^T \tag{26}$$

$where \ g = 0, 1, \cdots, q$

Using the definitions in (23)-(26) we can write down the linear system of equations (22) in matrix form as

$$\underbrace{\begin{bmatrix} C_{00} & \cdots & C_{0q} \\ \vdots & \ddots & \vdots \\ C_{q0} & \cdots & C_{qq} \end{bmatrix}}_{C} \underset{a}{\begin{bmatrix} a_0 \\ \vdots \\ a_q \end{bmatrix}} = -\underset{d}{\begin{bmatrix} d_0 \\ \vdots \\ d_q \end{bmatrix}} \tag{27}$$

$$Ca = -d \tag{28}$$

Note that as soon as q=0, the above equation reduces to the Yule-walker equations (YWE) for a stationary AR model. The set of TVAR parameters a_{km} are elements of a and can be calculated by solving the above matrix equation. The predictor coefficients $a_{k,n}$ that minimize the prediction error ϵ_p can now be calculated using (13).

IV. PARAMETER SELECTION

The TVAR parameter selection is basically depends on three degrees of freedom, such as the TVAR order p, the basis function order q, and the set of basis functions $u_{m,n}$.

A. Choice of the Basis Functions

The basis functions $u_{m,n}$ must be independent and non-zero for n=0,1,.....N-1,and $u_{m,n}=1$,if n=0.If a priori information about the signal variation is known, the basis functions should be chosen such that the trends in parameter change is retained. In case, when a priori information is unavailable selection of basis is trial and error[16, 19].

According to equation(13),no particular constraint is imposed on the basis $u_{m,n}$ consequently; one will be able to track only variations which are approximable by this set of functions. Numerous solutions have been projected, in the literature such as time basis functions, Legendre polynomial, Chebyshev polynomial, Discrete prolate spheroidal (DPSS) sequence, Fourier basis, Discrete cosine basis, Walsh basis, Multi wavelet basis, none of these solutions seems to be perfect, since the selection of $u_{m,n}$ desires some priori information upon the time variations present in x_n[16 ,19]. Then again, basis such as prolate spheroidal functions are extremely tough to generate. For AM-FM Analysis Discrete cosine basis functions is best suitable basis function [25]

Discrete Cosine basis function

$$u_{m,n} = \alpha \, (\text{m})\cos\cos\left(\frac{\pi\text{m}(2\text{n}+1)}{2\text{N}}\right) \qquad (29)$$

Where

$$\alpha \, (\text{m}) \; = \begin{cases} \sqrt{\dfrac{1}{N}}; m=0 \\ \sqrt{\dfrac{2}{N}}; m=0,1,2.......q \end{cases} \qquad (30)$$

$$\text{n}=1, \; 2.....\text{N}$$

B. Order Selection

In the presence of noise The TVAR model can distinguish several time-varying spectral peaks well. However it is sensitive to model order change. False spectral peaks may be produced by the TVAR modeling approach, when an erroneous model order is chosen. Thus, the determination of right model order in TVAR modeling is a significant issue. There are few techniques in choice of TVAR model order. For instance, Bayesian technique [16] and Akaike information criterion (AIC) [17] are used for the determination of model orders in TVAR models. In this article, we consider the choice of the model order as a Maximum-likelihood (ML) estimation [18] technique. In this technique, by maximizing the likelihood function we can determine the model order

Maximum likelihood estimation (MLE)

The TVAR Model for the non stationary discrete-time stochastic process x_n is

$$x_n = -\sum_{k=1}^{p}\sum_{m=0}^{q}a_{km}u_{m,n}x_{n-k} + v_n$$

The above equation can be represented in compact form as

$$x_n = -Z^T[n]a + v_n \qquad (31)$$

Where $Z[n]$ is

$$Z[n] = \Phi[n] \otimes u[n] \qquad (32)$$

Here, $\otimes$ denote Kronecker multiplication

$$\Phi[n] = \left[x_{n-1}, x_{n-2}......x_{n-p}\right]^T \qquad (33)$$

$$u[n] = \left[u_{0n}, u_{1n}......u_{qn}\right]^T \qquad (34)$$

$$a = \left[a_1^T, a_2^T......a_p^T\right] \qquad (35)$$

Here

$$a_k^T = \left[a_{k0}, a_{k1}......a_{kq}\right] \qquad (36)$$

Step-1: compute

$$Z[n] = \Phi[n] \otimes u[n]$$

Step-2: calculate

$$C = -\left(\sum_{n=p}^{N}Z[n]Z^T[n]\right)^{-1}\left(\sum_{n=p}^{N}Z[n]x[n]\right) \qquad (37)$$

Step-3: Estimate

$$\hat{\beta} = \frac{1}{N}\sum_{n=0}^{N}\left[x_n - C^T Z[n]\right]^2 \qquad (38)$$

Step-4: Obtain the cost function

$$J(p,q) = \frac{p(q+1)+2-N}{2}\log\left(2\pi\hat{\beta}\right)$$

$$-\frac{1}{2}\log\left|\sum_{n=0}^{N}Z[n]Z^T[n]\right| \qquad (39)$$

Step-5: Maximize the above cost function to select the expansion dimension $q = q_{opt}$ and the model order $p = p_{opt}$

Where $p_{opt} \in \{1, 2, 3, 4 \cdots\cdots p_{max}\}$

$$q_{opt} \in \{0, 1, 2, 3, 4 \cdots\cdots q_{max}\} \quad (40)$$

V. PROPOSED ALGORITHM

Our procedure based on the FB expansion and the TVAR model for modeling a multi-component non stationary signal requires the following steps:

Step 1: Component separation

Calculate the FB coefficients C_m for a given signal using (2). Every component of the multi-component non stationary signal has non-overlapping cluster of FB coefficients. Since coefficients are real; each component is directly reconstructed by using (1) from coefficient versus order plot. For every mono component signal apply TVAR model

Step 2: TVAR Based IF estimation

1) Compute TVAR model order p and q using MLE algorithm, choose the basis function u_{mn}

$$m = 1, 2 \ldots\ldots q, \quad n = 1, 2, \ldots\ldots, N.$$

2) For covariance technique of signal modeling set $\tau = [p, N-1]$ and compute $c_{mg}(l, k)$ by means of equation (21) to find the matrix C_{mg} in (24), subsequently, set up the matrix C in (25), as well, use $c_{mg}(l, k)$ to calculate d_g in (26).

3) Calculate the TVAR parameters $a_{k,m}$ by solving $Ca = -d$, in (27) and form the coefficients $a_{k,n}$ using (13).

4) Solve the roots of the time-varying autoregressive polynomial formed by TVAR linear prediction filter.

$$A(z; n) = 1 + \sum_{k=1}^{p} a_{k,n} z^{-k}$$ at each instant n to find the time-varying poles: $P_{i,n}$, $i = 1, 2 \ldots p$

5) The IF of the non stationary signal, for each sample instant n can be estimated from the instantaneous angles of the poles

using $f_{i,n} = \dfrac{\arg[P_{i,n}]}{2\pi}$ for $|P_{i,n}| \cong 1$

VI. SIMULATION RESULTS

We have considered a two-component Amplitude modulated and Frequency modulated (AM-FM) signals to study the performance of the proposed FB-TVAR method.

FB Expansion separates multi-component AM-FM signal in to mono-component AM-FM signal, then apply TVAR model to each mono component signal. The Instantaneous frequency (IF) is extracted from the time-varying

parameters by calculating the angles of the estimation error filter polynomial roots
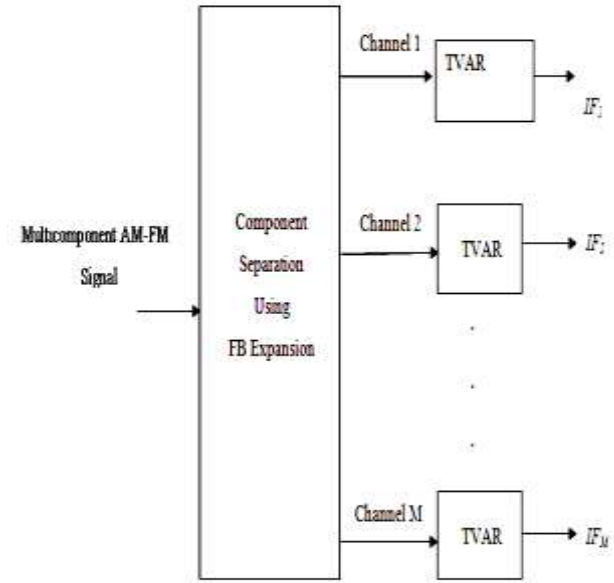
FB-TVAR Method

1) Multi component AM-FM sinusoidal signal

The sequence $y(n)$ consisting of M single-tone AM-FM sinusoidal signals is represented by [9]

$$y(n) = \sum_{i=1}^{M} A_i \left(1 + \mu_i \cos(v_{ai} n)\right) \cos\left(\omega_i n + \beta_i \sin(v_{fi} n)\right) \quad (41)$$

For simulation, the signal consisting of two (M =2) single tone AM-FM signals is sampled at N = 512 points. Two sets of parameters in (41) are chosen as

$$A_1 = 1, \ \mu_1 = 0.8, \ v_{a1} = 2\pi \times 0.0063, \omega_1 = 2\pi \times 0.1250,$$
$$\beta_1 = 2, v_{f1} = 2\pi \times 0.0094 \quad (42)$$

$$A_2 = 1, \ \mu_2 = 0.6, \ v_{a2} = 2\pi \times 0.0125, \omega_2 = 2\pi \times 0.2500,$$
$$\beta_2 = 1.5, v_{f2} = 2\pi \times 0.0125 \quad (43)$$

IF law for first component signal is given by

$$f_{1n} = \frac{\omega_1 + \left(\beta_1 * v_{f1}\right)\cos(v_{f1} n)}{2\pi} \quad (44)$$

IF law for second component signal is given by

$$f_{2n} = \frac{\omega_2 + \left(\beta_2 * v_{f2}\right)\cos(v_{f2} n)}{2\pi} \quad (45)$$

The plots of the two-component AM-FM signal, the first component, and second component AM-FM signals are shown in Figs.1–3 respectively. The composite signal is expanded into FB expansion and the plot of FB coefficients for the composite signal is shown in Fig.4.The first

component of the multi-component signal is separated and reconstructed using 50-190(From Fig.4.) significant non-overlapping FB coefficients. The reconstructed signal is shown in Fig.5. The second component of the multi-component signal is separated and reconstructed using 210-315 (From Fig.4.) significant non-overlapping FB coefficients. The reconstructed signal is shown in Fig.6.

Each of the two reconstructed AM-FM signals from the clusters of non-overlapped FB coefficients is modeled by TVAR process. By using step 2 in the proposed algorithm we can estimate Instantaneous Frequency (IF) of each reconstructed AM-FM signal.

For separated first component and second component of two component AM-FM signal apply TVAR model with discrete cosine basis. Using Maximum likely hood estimation algorithm we have computed TVAR Model order p=2 and q=22, for first component signal, p=2and q=28for second component signal. TVAR coefficients $a_{1,n}$ $a_{2,n}$ for the first component and second component signals are shown in Fig.7.andFig.10.respectively.Trajectory of time varying poles of the first component and second component are shown in Fig.8.and Fig.11.respectively.From Fig.8. and Fig.11.we observe that poles are complex conjugate to each other and also half of the poles are lie in the upper half of Z-plane and remaining half of the poles are lower half of the Z-plane. For IF estimation consider the poles that lie either upper half (or) lower half of the Z-plane.Fig.9.and Fig.12.shows the IF estimation of first component of two component AM-FM signal and the IF estimation of second component of two component AM-FM signal, respectively. From Fig.9.and Fig.12.we observe that estimated IF is close to true IF. The mean square error (MSE) among the true IF and estimated IF for n=2,3,………,512 is calculated to be -118.3282dB. and -106.693dB respectively.
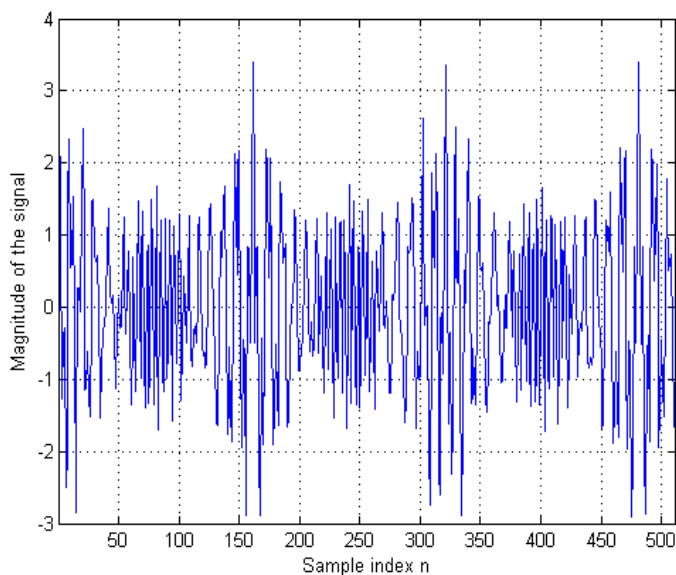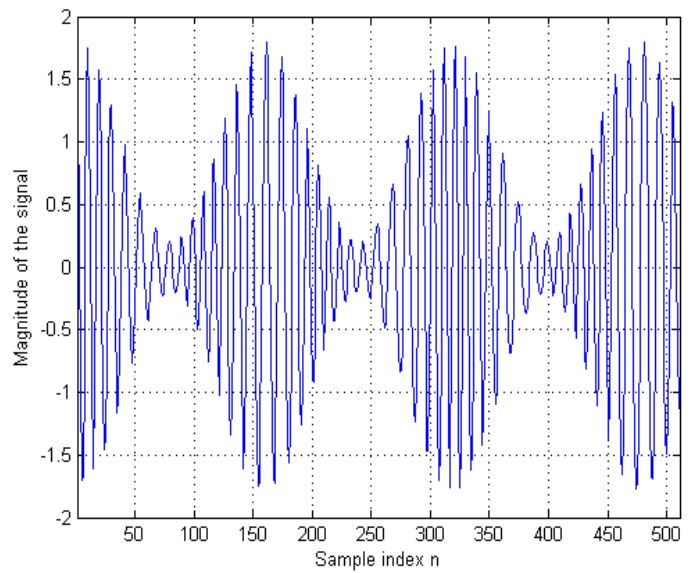
Fig.2. First component of Multi-Component AM-FM Sinusoidal Signal

Fig.3.Second-Component of Multi-Component AM-FM Sinusoidal Signal
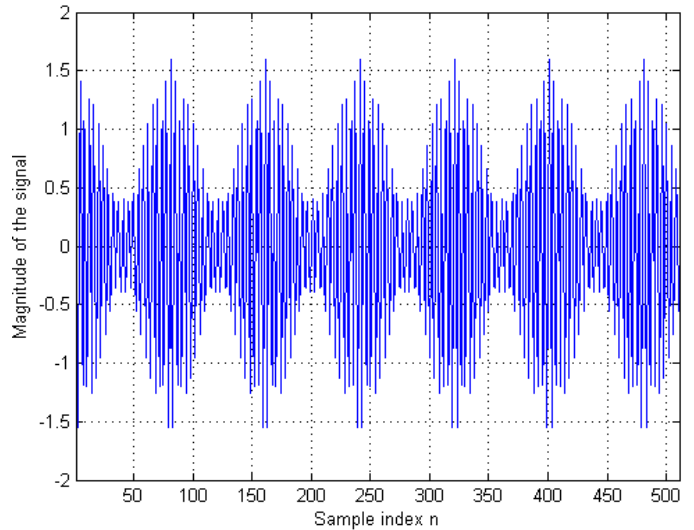
Fig.1.Multi component AM-FM Sinusoidal signal
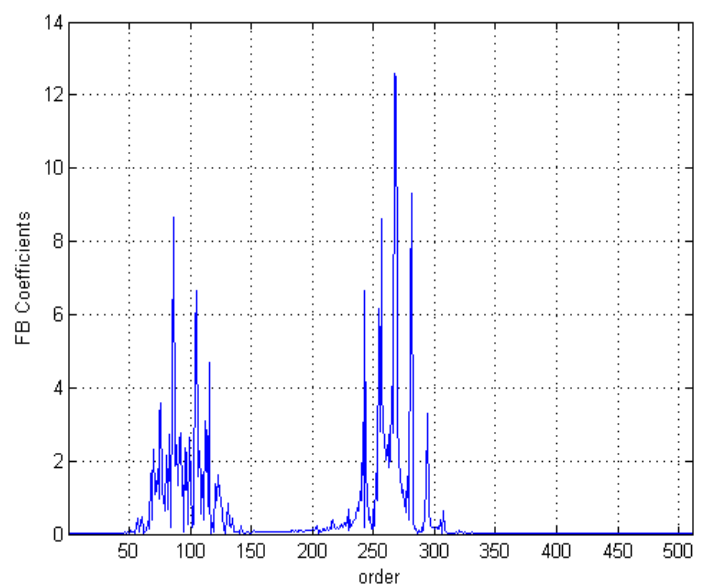
Fig.4.FB Coefficients of the Multi components AM-FM Sinusoidal Signal
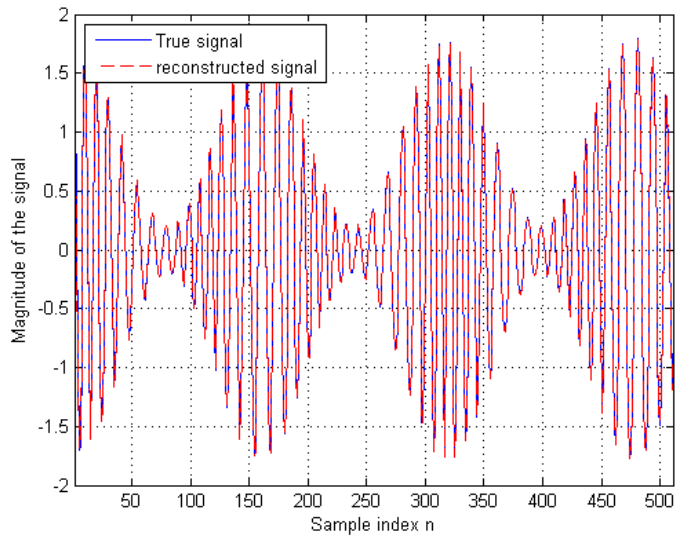
Fig.5.Estimated first component of Multi-Component
AM-FM sinusoidal signal using FB expansion
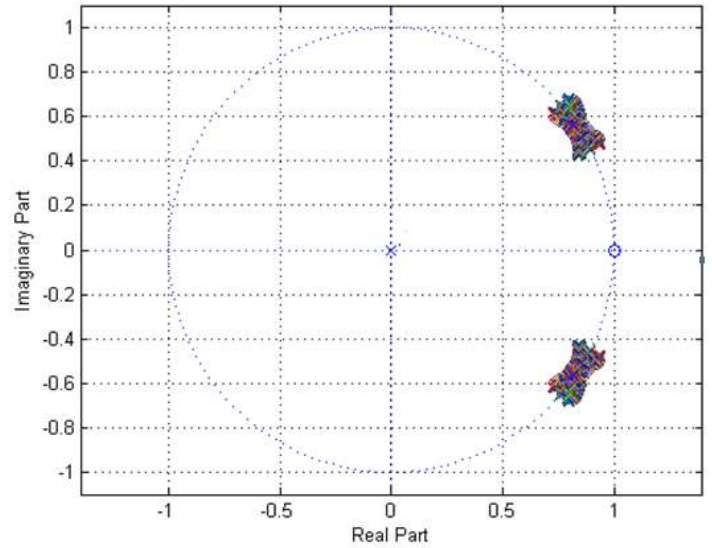
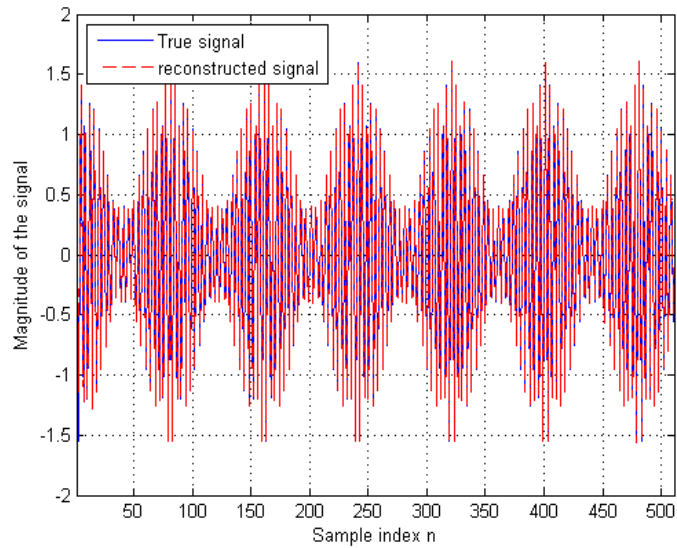Fig.8.Trajectory of Time Varying poles of first component of
Multi Component AM-FM Signal

Fig.6.Estimated second component of Multi-Component
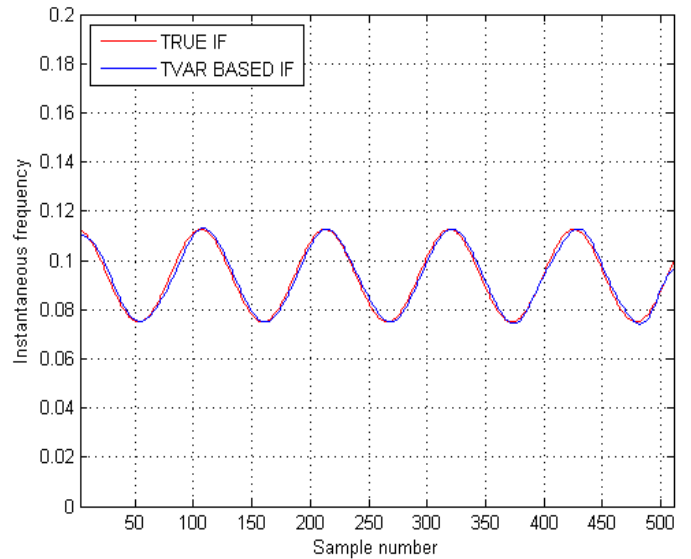AM-FM sinusoidal signal using FB expansion

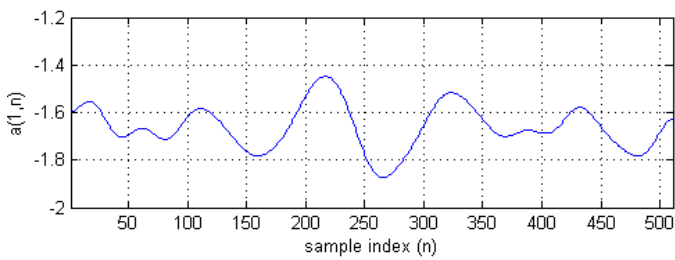Fig.9.IF Estimation of first component of multi component
AM-FM Signal

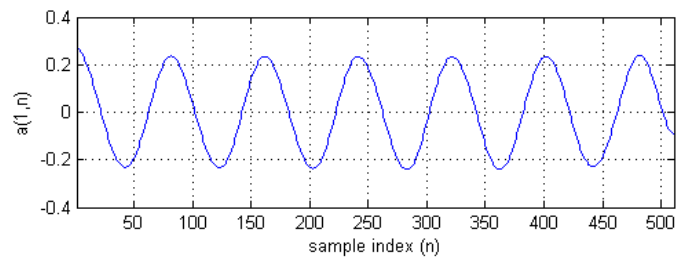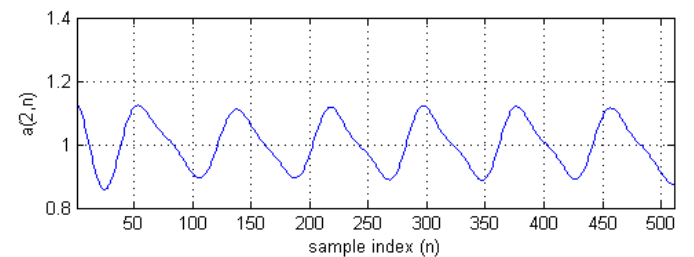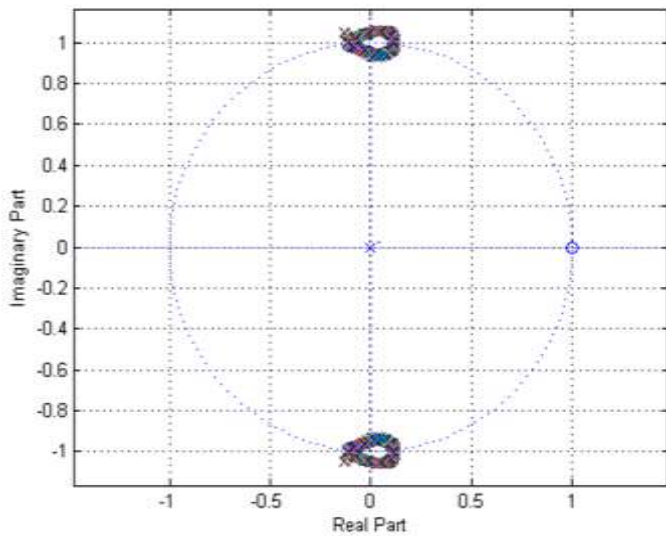Fig.7.Estimated TVAR Coefficients of first component of
Multi-Component AM-FM Signal

Fig.10.Estimated TVAR Coefficients of second component of
Multi-Component AM-FM Signal
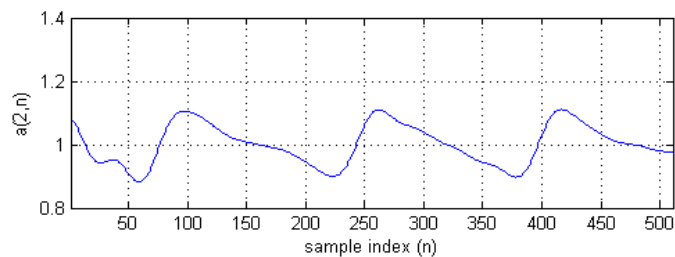
Fig.11.Trajectory of Time Varying poles of second Component of multi component AM-FM Signal
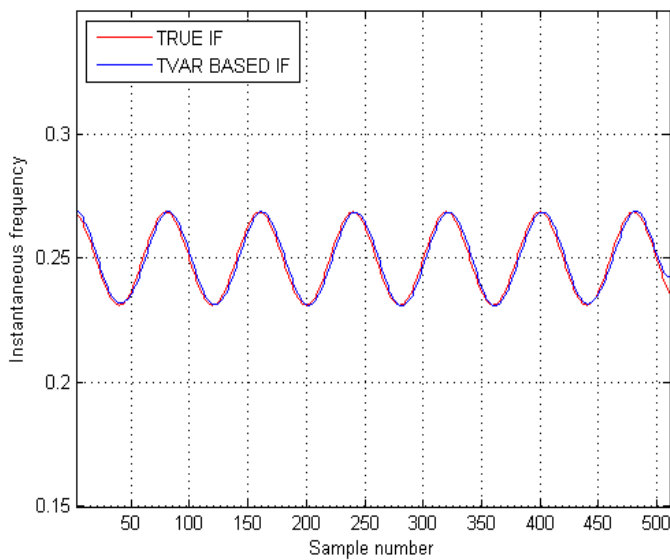
Fig.12.IF Estimation of Second component of Multi -Component AM-FM Signal

2) Multi-component AM-FM signal

$$x(t) = \sum_{k=1}^{2} \exp(\alpha_k) \exp\left[j2\pi\left(\beta_{k,3}t^3 + \beta_{k,2}t^2 + \beta_{k,1}t \right) \right] (46)$$

The parameters of the first and the second components are[21]

$$\alpha_1 = -2, \beta_{1,1} = 10^3, \beta_{1,2} = 5 \times 10^3, \beta_{1,3} = -20 \times 10^3$$

and $\alpha_2 = -5, \beta_{2,1} = 5 \times 10^3, \beta_{2,2} = -3 \times 10^3, \beta_{2,3} = 0$

The two following IF s are embedded in this AM-FM signal:

$$f_1(t) = -60t^2 + 10t + 1kHZ$$

$$f_2(t) = -6t + 5kHZ \qquad (47)$$

The signal in (46) is sampled at T_s =50μs intervals, so the sampling frequency is f_s =20 kHz .Since the absolute values

of the instantaneous frequencies increase over time, a limited span of signal is observed to avoid aliasing. The Algorithm is run over N=2,000samples.

The plots of the two-component AM-FM signal, the first component, and second component AM-FM signals are shown in Figs.13–15 respectively. The composite signal is expanded into FB expansion and the plot of FB coefficients for the composite signal is shown in Fig.16.The first component of the multi-component signal is separated and reconstructed using 150-400(From Fig.16.) significant non-overlapping FB coefficients. The reconstructed signal is shown in Fig.17.The second component of the multi-component signal is separated and reconstructed using 800-1100(FromFig.16.) significant non-overlapping FB coefficients. The reconstructed signal is shown in Fig.18.

Each of the two reconstructed AM-FM signals from the clusters of non-overlapped FB coefficients is modeled by TVAR process. By using step 2 in the proposed algorithm we can estimate Instantaneous Frequency (IF) of each reconstructed AM-FM signal.

For separated first component and second component of two component AM-FM signal apply TVAR model with discrete cosine basis. Using Maximum likely hood estimation algorithm we have computed TVAR Model order p=2 and q=8, for first component signal ,p=2and q=12 for second component signal. TVAR coefficients $a_{1,n}$ $a_{2,n}$ for the first component and second component signals are shown in Fig.19.andFig.22.respectively.Trajectory of time varying poles of the first component and second component are shown in Fig.20.and Fig.23.respectively.Fig.21.and Fig.24.shows the IF estimation of first component of two component AM-FM signal and the IF estimation of second component of two component AM-FM signal, respectively .From Fig.21.and Fig.24.we observe that estimated IF is close to true IF. The mean square error (MSE) among the true IF and estimated IF for n=2,3,………,2000 is calculated to be -93. 282dB. and -86. 932dB respectively.
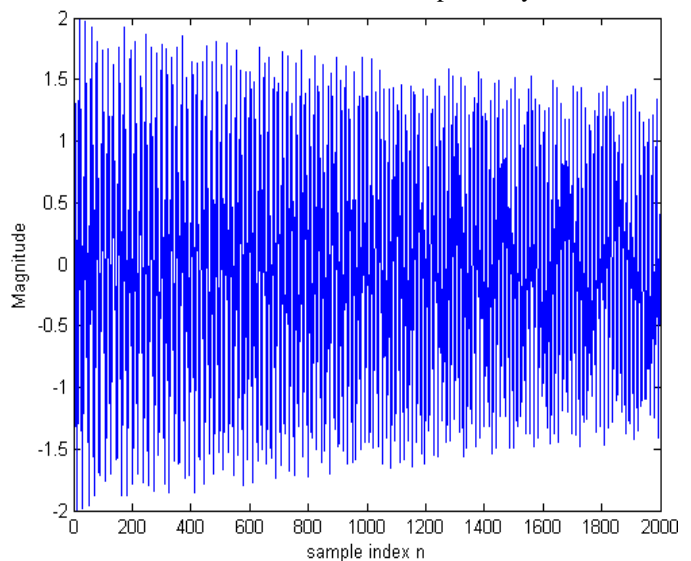

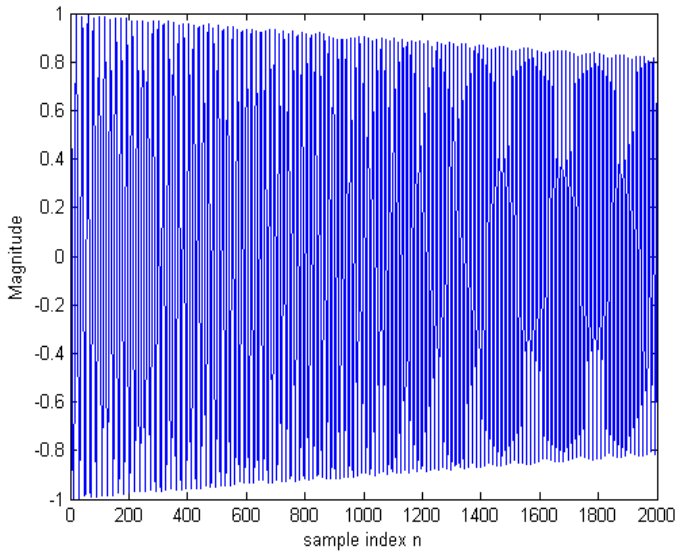Fig.13.Multi component AM-FM signal

Fig.14.First component of Multi component AM-FM signal
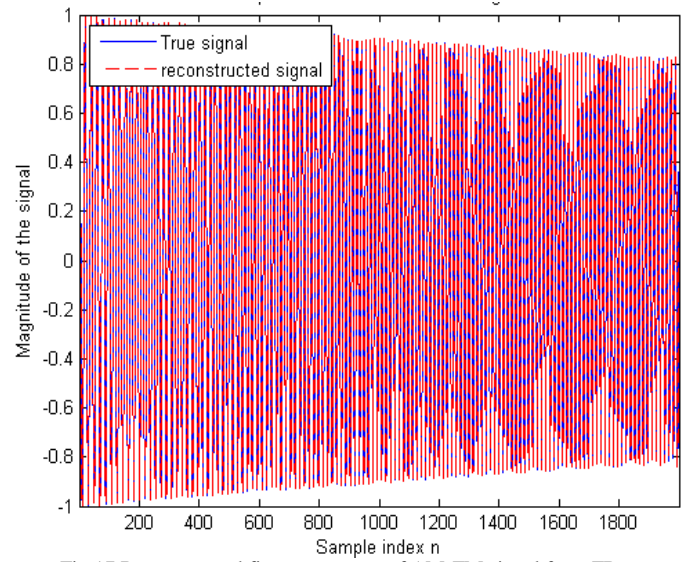

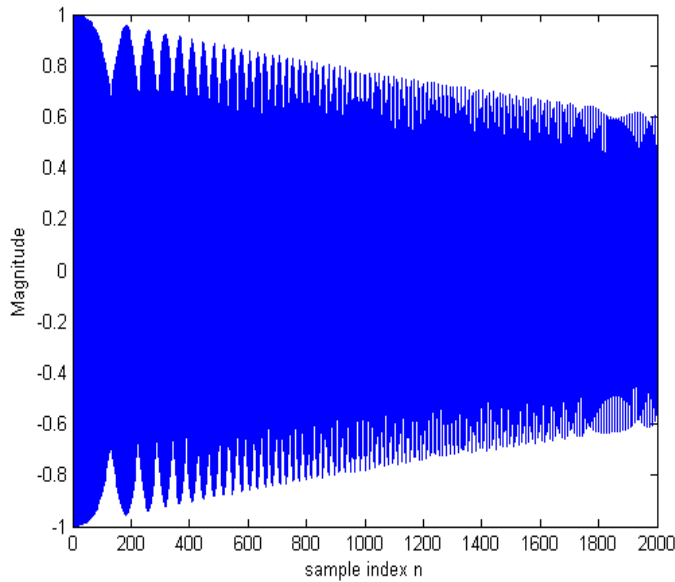Fig.17.Reconstructed first component of AM-FM signal from FB coefficients

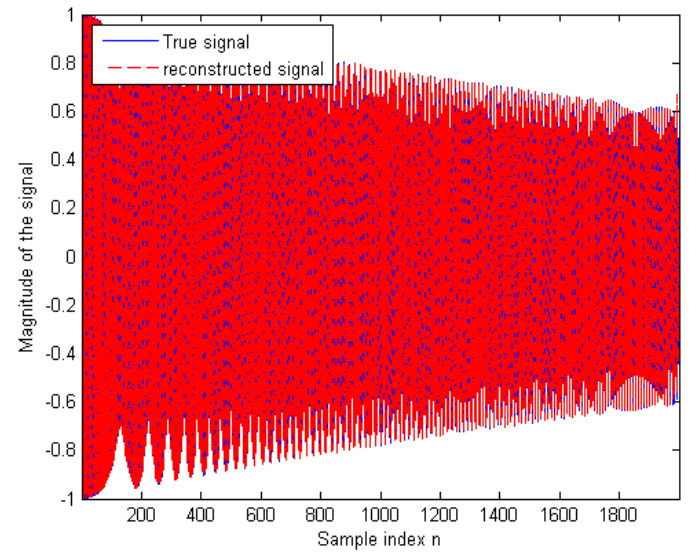Fig.15.Second component of Multi component AM-FM signal

Fig.18.Reconstructed second component of AM-FM signal from FB coefficients
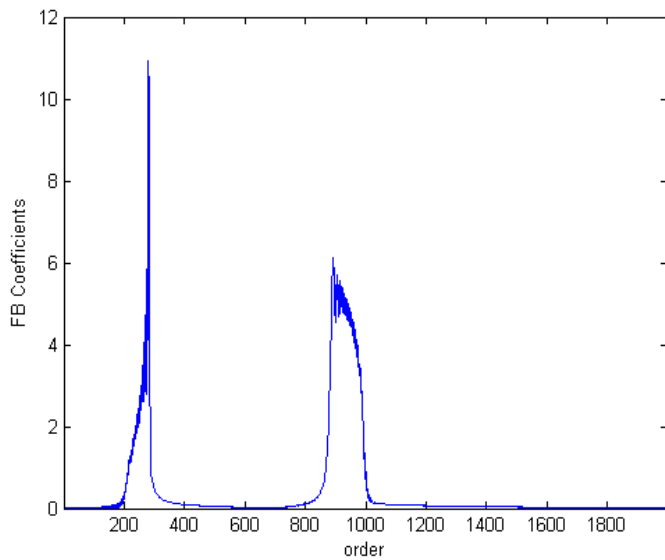

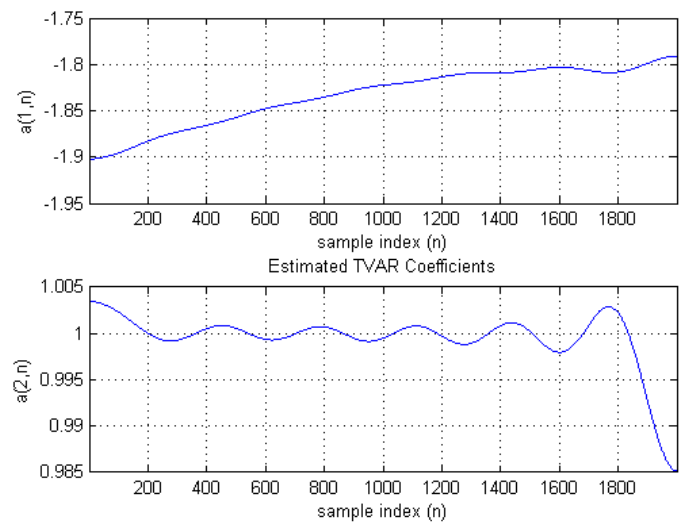
Fig.16.FB coefficients of Multi component AM-FM Signal

Fig.19.Estimated TVAR coefficients of First component of a multi component AM-FM signal
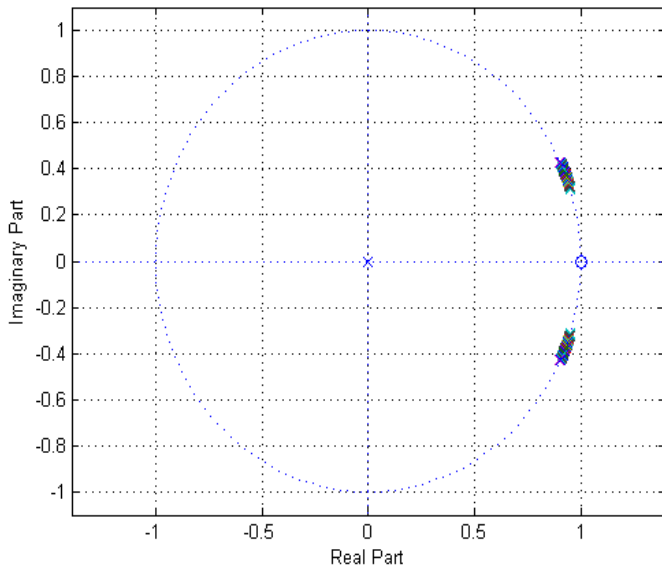
Fig.20.Trajectory of Time Varying of First component of multi component AM-FM signal
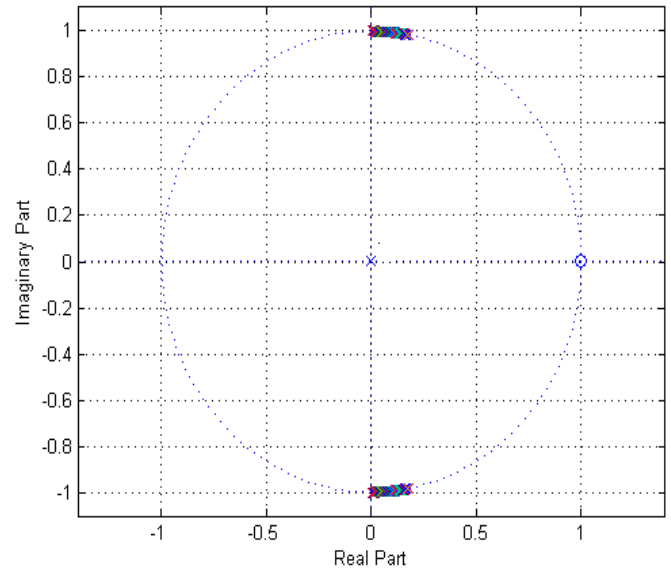

Fig.23.Trajectory of Time Varying of second component of multi component AM-FM signal
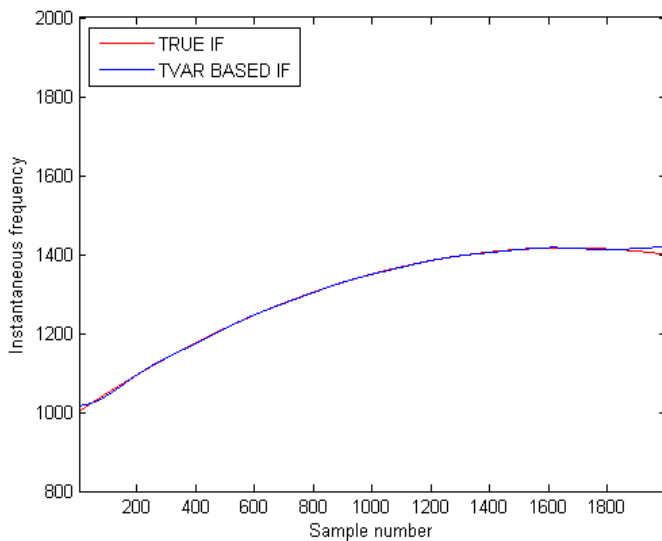

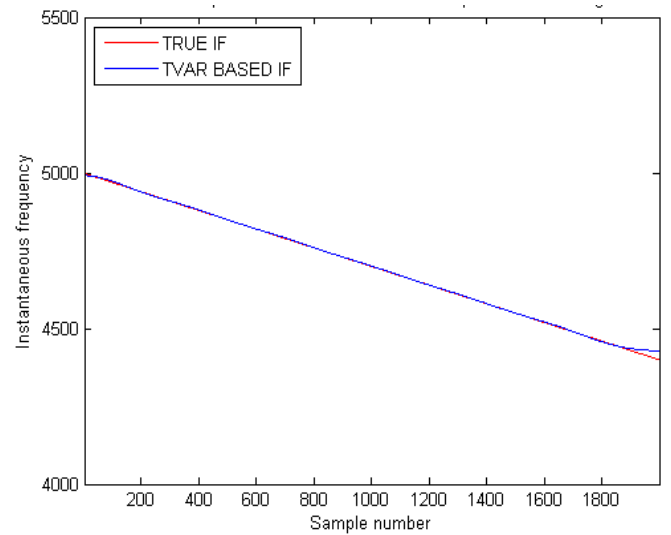Fig.21.First component IF estimation of Multi component AM-FM signal

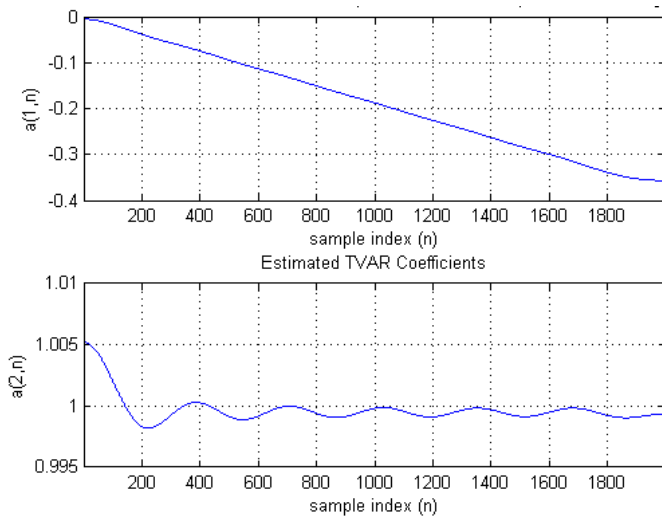Fig.24.second component IF estimation of Multi component AM-FM signal

Fig.22.Estimated TVAR coefficients of second component of a multi component AM-FM signal

VII. CONCLUSIONS

A new method FB-TVAR, for Instantaneous frequency estimation of multi component AM-FM signals is presented in this paper. The FB expansion decomposes a multi-component AM-FM signal into a number of mono component signals, and each component signal is modeled using TVAR Model. In this way, the estimation of the model parameters of a multi-component signal is done accurately. The proposed technique does not need any information a priori about the frequency-bands of the multi component AM-FM signals. It is demonstrated that the performance of proposed FB-TVAR method is good for estimation of the frequency functions of an AM-FM signal

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