

MODELING OF IR PROXIMITY SENSORS WITH THE USE OF INTERVAL MITTAG-LEFFLER FUNCTION

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Abstract:

In the paper new, interval models of IR proximity sensors are presented. The dependence between distance and signal from sensor is described with the use of exponential function and two parameter Mittag-Leffler function with interval parameters. Identification method for was also proposed. Results of experiments show, that two parameter Mittag-Leffler function most accurate describes a behaviour of proximity IR sensor, than exponential function.

Keywords: IR proximity sensors, IR reflex sensors, interval models, Mittag-Leffler function

1. Introduction

IR reflex proximity sensors are available for a wide range of applications as ready-to-use components known as couplers, transmissive sensors (or interrupters), reflex couplers and reflex sensors. Increased automation in industry in particular has heightened the demand for these components. One of main areas of applications of these devices is mobile robotics, where they are applied to short range obstacle detecting. Advantages of application this kind of sensors are obvious: they are easy in use and no to expensive, they can measure both ambient and reflective light.

A proper model of IR sensor is also necessary to construct a robust control system for each device using it, for example a mobile robot.

However it is important to notice that measurements done with the use of IR sensors are strongly influenced by ambient conditions: light, temperature, reflectivity of detected object. An example of output signal vs distance from object for different reflecting surfaces is shown in the Fig. 1.

Notice that the set of characteristics shown in the Fig. 1 can be shown as a sector limited by borders obtained for white and black objects.

Consequently, a mathematical model should describe all situations during work of sensor. This implies, that reasonable is to propose an interval model.

The goal of this paper is to propose new, interval parameter models using Mittag-Leffler function describing IR proximity sensors. Proposed models are expected to properly describe the work of sensor for different reflecting surfaces and ambient conditions

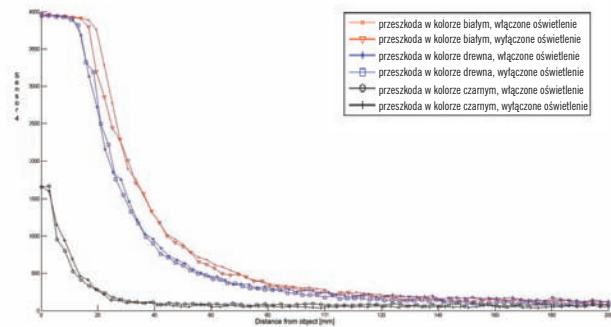


Fig. 1. Collection of characteristics IR sensor for different materials and ambient light: white object (red), natural wood (blue) and black (black)

(temperature, light). The paper is organized as follows: at the beginning one- and two parameter Mittag-Leffler functions are presented. Next interval models using two-parameter Mittag-Leffler function are proposed and verified with the use of experimental results.

2. Mittag-Leffler Functions

The one parameter Mittag-Leffler function is a generalization of exponential function e^x and it is broadly applied in non-integer order calculus, but of course it can be also used to another goals. It is expressed as follows:

$$E_{\alpha}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k\alpha + 1)} \quad (1)$$

Where $\alpha \in \mathbb{R}$, $\Gamma(x)$ denotes the Gamma function:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad (2)$$

The Mittag-Leffler function turns to exponential function for order $\alpha=1$: $E_1(x) = e^x$.

Next the two parameter Mittag-Leffler function is expressed as underneath:

$$E_{\alpha, \beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k\alpha + \beta)} \quad (3)$$

For $\beta=1$ the two parameter Mittag-Leffler function (3) turns to one parameter function, described by (1).

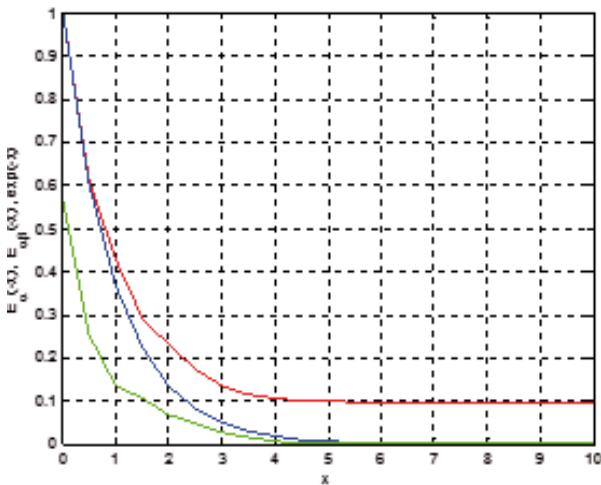


Fig. 2. Mittag-Leffler functions: $E_{0.5,0.5}(-x)$ (red) $E_{0.5,0.5}(-x)$ (green) vs $\exp(-x)$ (blue)

Elementary properties of Mittag-Leffler functions are presented by many Authors, for example in [2]. Exemplary diagrams of Mittag-Leffler functions (1) and (3) vs exponential function are given in the Fig. 2

3. Interval Expansion of Estimating Functions

Mathematical model of the sensor, dedicated to apply in robotics is required to properly describe a level of output signal from sensor as a function of distance between sensor and obstacle. It is important to notice, that this model should be correct for different conditions (temperature, light, reflecting properties of object, etc.). This implies that a sensible option is the use of an interval model. Elementary information about interval analysis are given for example in fundamental book [7].

The shape of function shown in the Figure 1 suggests the use an exponential function or Mittag-Leffler function with interval coefficients. The diagram of function with interval parameters has the form of sector, limited by border values of applied function.

Firstly the exponential function will be proposed. It has the following form:

$$U_{exp}(l) = U_0 e^{-bl} \quad (4)$$

Where $U_0 = [U_0; \bar{U}_0]$ denotes the level of signal in minimal distance, $b = [b; \bar{b}]$ denotes interval parameter and l denotes a distance between sensor and object. The border values of interval function (4) are expressed as follows:

$$\begin{aligned} \underline{U}_{exp}(l) &= \underline{U}_0 e^{-\bar{b}l} \\ \bar{U}_{exp}(l) &= \bar{U}_0 e^{-bl} \end{aligned} \quad (5)$$

Next, the model using the two parameter Mittag-Leffler function can be proposed. It is expressed as underneath:

$$U_{MLf2}(l) = U_0 E_{\alpha,\beta}(-al) \quad (6)$$

Where $U_0 = [U_0; \bar{U}_0]$, $a = [a; \bar{a}]$, $\alpha \in [\underline{\alpha}; \bar{\alpha}]$, $\beta \in [\underline{\beta}; \bar{\beta}]$.

Relation analogical to (5) in the case of Mittag-Leffler functions is not so obvious, as for exponential function. Limits of Mittag-Leffler function with interval parameters can be uniquely assigned for certain combinations of parameters α , β and a with constant (not interval!) values of other parameters. They are given underneath.

For interval orders α , β and $a = \text{const}$ the borders are expressed as follows:

$$\begin{aligned} \underline{U}_{MLf2}(l) &= \underline{U}_0 E_{\alpha,\beta}(-al) \\ \bar{U}_{MLf2}(l) &= \bar{U}_0 E_{\alpha,\beta}(-al) \end{aligned} \quad (7)$$

For $\beta = 1$, $a = \text{const}$ and interval α equations (7) reduce to interval model with one parameter Mittag-Leffler function:

$$\begin{aligned} \underline{U}_{MLf1}(l) &= \underline{U}_0 E_{\alpha}(-al) \\ \bar{U}_{MLf1}(l) &= \bar{U}_0 E_{\alpha}(-al) \end{aligned} \quad (8)$$

For α , $\beta = \text{const}$ and interval a the borders of Mittag-Leffler function are defined analogically to (5):

$$\begin{aligned} \underline{U}_{MLf2}(l) &= \underline{U}_0 E_{\alpha,\beta}(-\bar{a}l) \\ \bar{U}_{MLf2}(l) &= \bar{U}_0 E_{\alpha,\beta}(-\underline{a}l) \end{aligned} \quad (9)$$

It is important to notice that estimation of borders, analogical to (5)–(9) for all interval parameters: α , β and a cannot be given. This implies that the another form of interval functions describing sensor indicating different objects should be proposed.

The simple idea is to identify sets of interval parameters for both border measurements (black and white object) separately and next use they to the whole sector estimation. More details about it will be given in the next section.

4. Identification of Interval Models

In a real situation characteristics generated by a model is compared to real measurements done in discrete, separated distance points. Assume that the maximal range of distance is equal L and K_s measurements can be done. Then the k -th point of measurement is done for distance: kL/K_s . These values will be written as $U(k)$ and $U_m(k)$, respectively. Furthermore, these values in each step are expressed by intervals to describe a broad spectrum of environmental conditions:

$$\begin{aligned} U(k) &= [U(k), \bar{U}(k)] \\ U_m(k) &= [U_m(k), \bar{U}_m(k)] \end{aligned} \quad (10)$$

Where $U(k)$ denotes the measurement in k -th distance point. The lower value $U(k)$ describes the characteristics for black object and the upper value $\bar{U}(k)$ describes the characteristics for white object.

Next, $U_m(k)$ denotes the response of model in the same point and for the same object, described by (5), (6) or (8). The interpretation of both border values is the same, as for real measurements (lower value for black object and upper value for white object).

Denote the difference between measurement and model response in k -th point as $e(k)$:

$$e(k) = U(k) - U_m(k) \tag{11}$$

Both measurement and model are described by intervals (10), what causes that the error (11) is also interval, expressed as underneath:

$$e(k) = [e(k), \overline{e(k)}] \tag{12}$$

Where borders of interval (12) are equal:

$$\begin{aligned} \underline{e(k)} &= \underline{U(k)} - \overline{U_m(k)} \\ \overline{e(k)} &= \overline{U(k)} - \underline{U_m(k)} \end{aligned} \tag{13}$$

An identification of the all considered models can be done with the use of typical MSE cost function:

$$MSE = \frac{1}{K_s} \sum_{k=1}^{K_s} (e(k))^2 \tag{14}$$

The cost function (14) is dedicated to describe models with known parameters. In the considered case it will be applied to estimate borders of all interval functions $U_m(k)$. This will be shown in the next section.

5. Experiments

Experiments were done with the use of IR proximity sensor *Vishay TCRT5000* mounted at Khepera III mobile robot. Description of this sensor is given by [3]. The analog signal from sensor is converted to 12 bit number and next it is transferred to computer via USB link. Set of signal vs distance characteristics is given in the Fig. 1. From Fig. 1 it can be concluded, that borders of interval function (9) are drawn by diagrams for white and black materials. They are shown in the Fig. 3. The number of samples during experiments was equal: $K_s=72$.

Diagrams shown in Fig. 3 were applied to estimate borders of interval functions (4) and (6). Results are given in Tables 1 and 2. Sectors estimated with the use both proposed models are shown in Figures 4 and 5.

Based on figures 4 and 5 it is visible, that two-parameter Mittag-Leffler function describes characteristic of IR proximity sensors much better than exponential function- because of its specific shape.

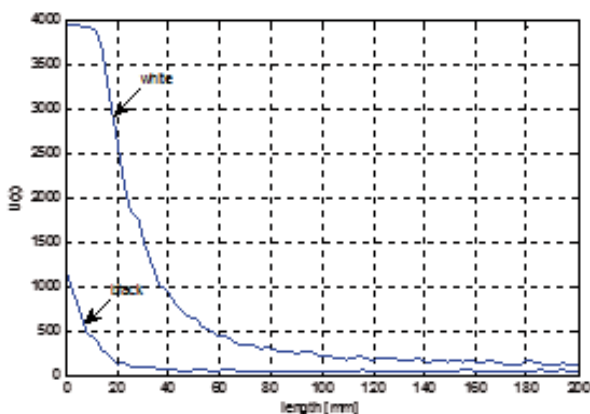


Fig. 3. Border characteristics of sensor

Table 1. Parameters of interval exponential function (4)

U_0	a	MSE
[1154;	0.0290	1.9398e+05
3944]	0.0911	6.8454e+06

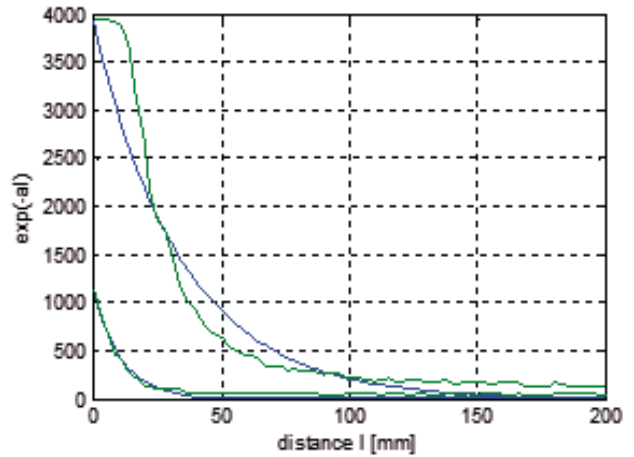


Fig. 4. Sector limited by exponential function

Table 3. Parameters of interval two-parameter Mittag-Leffler function (6)

U_0	α	β	a	MSE
[1154;	0.2347	1.8192	0.1775	828.1
3944]	0.0622	1.2317	0.0394	6851.3

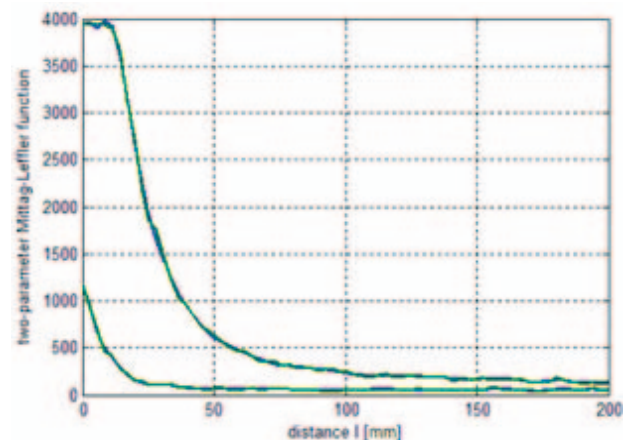


Fig. 5. Sector limited by the two-parameter Mittag-Leffler function

6. Conclusions

Final conclusions from the paper can be formulated as underneath:

Using of interval models of IR proximity sensors enable construct universal model of sensors characteristic. This universal model is necessary to collision-free tracking of Khepera III mobile robot in various workspace. Normally, value of measurements

of IR proximity sensors depend on material features of obstacles such color, kind of material and illumination intensity in the room. For example, the black obstacles are visible for IR proximity sensors much later than the white ones. So, it is necessary to improve universal application to detect obstacles independently of its features. Using of Mittag-Leffler function enabled to create satisfactory model of IR proximity sensors. Especially, the two-parameter Mittag-Leffler function describes the characteristic of IR proximity sensors much better than exponential function and can be used in collision-free path planning applications in unknown environment. For example, in [9] path planning algorithms with obstacle avoidance based on IR proximity sensors was presented. During a lot of experiments it turned out to propose algorithms insensitive to for example color of obstacles. In this case using of two-parameter Mittag-Leffler function to describe a behavior of IR proximity sensors was successful.

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