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# Implications of loss of stability of deformation sequences of reinforced concrete sections

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## ABSTRACT

**Purpose:** The paper analyses the strain of reinforced concrete sections of flexural and eccentrically compressed sections up to and including failure.

**Design/methodology/approach:** The paper shows that taking into account realistic $\sigma$-$\varepsilon$ diagrams for concrete, which should be considered curves with descending branches, it is possible to determine the state of exhaustion of the load-bearing capacity of the section without the need to introduce the limiting strain $\varepsilon_{cu}$ assigned only to the concrete class.

**Findings:** It was pointed out that the physical law $\sigma$-$\varepsilon$ describing the behaviour of concrete should include the range of material weakening, expressed by the falling branch in the physical law.

**Research limitations/implications:** It was proposed to study the stability of the deformation process of the compressive zone of concrete and the entire critical section based on Drucker's postulate and, on such a basis, to infer the nature of reinforced concrete strain states - the state of exhaustion of load-bearing capacity incipient destruction.

**Practical implications:** The formulation of the bearing capacity problem is then complete because there is no need to introduce an apriori limit strain value $\varepsilon_{cu}$ to determine the bearing capacity.

**Originality/value:** It is shown that it is furthermore possible to distinguish a certain covering condition, occurring after the load-bearing condition is reached, in which the process of rapid, avalanche-like destruction begins. The deformation accompanying the state can be considered as the failure deformation of the reinforced concrete section, $\varepsilon_{cf}$.

**Keywords:** Construction, Bearing capacity and failure of reinforced concrete sections, Material weakness, Material stability, Drucker's postulate, Safety and health protection

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**PROPERTIES**

## 1. Introduction

An important part of the design procedures for reinforced concrete structures is the proper determination of the load-carrying capacity of critical sections. The bearing capacity can be predicted based on two qualitatively different theoretical formulations.

The basis of the first formulation is the theory of boundary resistance. Based on it, we look for the condition that determines the plastic flow of the section using the equations of limit equilibrium of the section. The essential assumptions of the analysis are the rigid-perfectly plastic strain models of concrete and reinforcing steel, in addition to the assumption of free accumulation of deformation in the section. The latter assumption is expressed by the hypothesis of a flat cross-section written in strain rates. The hypothesis defines the plastic flow mechanism of a reinforced concrete section. From those assumptions, the stress distribution in the normal section of a reinforced concrete element was derived. Theoretically, the result of the rigid-plastic analysis is an overestimation of the true load-carrying capacity of the section due to the kinematic nature of the formulation [1-7]. In applications, one usually starts by defining the stress distribution in the section and makes some modifications to analyse partial plastic flow mechanisms. The modified formulation is the basis of the simplified method in the current standard [8].

The second formulation adopts more realistic models of material deformation by limiting the deformation capacity of both reinforcing steel and concrete in compression. characterised. A bilinear, elastic model with plastic reinforcement usually characterises the behaviour of reinforcing steel. It is possible to introduce a limitation on the deformation of tensile reinforcement. Compressed concrete is assumed to be physically nonlinear over the entire deformation range. Two deformation models of compressive concrete are used.

In the first model, after the average compressive strength $f_{cm}$ and the associated strain $\varepsilon_{c1}$, the further behaviour of the compressed concrete is determined by the ideal plastic shelf. Such a model approximates the physical law by a Madrid parabola, Figure 1. The plastic shelf must have a limited length, which expresses the limited deformation capacity of the compressed concrete. Those capacities are defined by the limit strain $\varepsilon_{cu}$ related to the outermost layer of the reinforced concrete section. It can be defined as a criterion value for determining the load-carrying capacity of a reinforced concrete section, which was determined based on experiments.

Without introducing the strain, the load-carrying capacity of the section cannot be clearly determined theoretically. It can depend, for example: on the largest plastic strain in the section. With its increase, it tends asymptotically to the value of the load capacity determined according to the

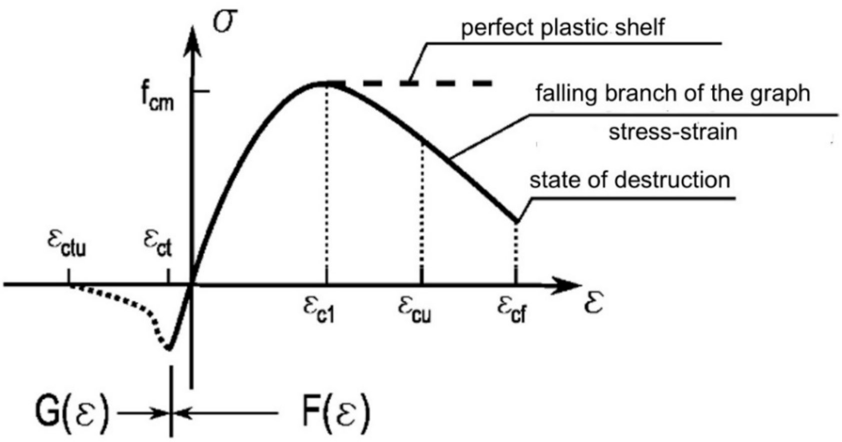


Fig. 1. $\sigma$-$\varepsilon$ diagrams for concrete: with an ideal plastic shelf or with a falling branch

approach of the proper theory of limit load capacity based on the assumption of rigid-plastic behaviour of the material.

An approximation procedure is proposed to determine the limiting strain $\varepsilon_{cu}$ for different eccentric loading cases. The relevant value of the strain for different eccentric compressive forces is interpolated from the values considered appropriate for the cases of axial compression ($\varepsilon_{cu} = \varepsilon_{c1}$) and pure bending ($\varepsilon_{cu} = 0.0035$). The procedure testifies to the dependence of the ultimate strain of concrete on the stress state of the reinforced concrete section. The section capacity thus determined is not an analytical maximum but a value of the acceptable supremum type.

In the second deformation model, the deformation behaviour of concrete $\varepsilon > \varepsilon_{c1}$ is described by a descending branch. It expresses the progressive material weakening. The slope of the branch reflects the development of deterioration of the concrete structure. If we use a model with a descending branch in theoretical analyses, it is possible to determine the theoretical value of strain $\varepsilon_{cu}$, without the need to determine the value apriori. In such a way, the formulation of the problem of the bearing capacity of a reinforced concrete section becomes complete. The bearing capacity of the section becomes a maximum value. Reaching the load-carrying capacity means the start of a destructive process but generally with the character of a mildly intensifying process. It is also possible to indicate during the process such a state of deformation and strain of the section, which can be considered as a state of beginning of the avalanche process of destruction. The state should be associated with destructive deformation $\varepsilon_{cf}$ , greater than the value of the limiting strain $\varepsilon_{cu}$.

An example of such a complete formulation and solution of the carrying capacity problem was presented in the work [9]. However, the problem of destructive deformation was not analysed there. In the work, the physical law of concrete was assumed to be a complete parabola of the second degree with a descending branch. The bending resistance was determined from the conditional maximum moment. A theoretical formula was obtained for the value of the ultimate strain $\varepsilon_{cu}$ inherent bending resistance.

$$\varepsilon_{cu} = \sqrt{3}\left(\sqrt{3} - 1\right)\varepsilon_{c1}. \quad (1)$$

Detailed results of the analyses are provided in the monograph [4].

It will be noted that, by its nature, the magnitude of the boundary strain is not a magnitude that characterises the material, but a magnitude that determines the deformation capacity of the section under a certain state of stress. They should depend not only on the eccentricity of the force loading the section but also on the degrees of reinforcement. The latter factor is not accentuated in interpretations of experimental results. Investigation of the problem requires subjecting the loading path of the section to analysis and tracing it in terms of either stable or unstable evolution of the stress states. The paper presents a suitable criterion for studying the stability of the loading process of a reinforced concrete section. It allows a more complete interpretation of the load states as the maximum values of the loading forces and the determination of deformations at which the avalanche process of failure of the reinforced concrete section begins.

## 2. The criterion of stability of the deformation process of the reinforced concrete section

We analyse the formulation of the problem of bearing capacity of a reinforced concrete section, in which the deformation properties of the concrete in compression are described by a curve with a branch of weakening. We assume that concrete is characterised by an average compressive strength $f_{cm}$. The study finds that the value corresponds to the strain $\varepsilon_{c_1}$, depending on the concrete class. Until the strength is reached the highest stress in the loading path, concrete is a stable material in the sense of Drucker [4,5,10].

$$d\sigma \cdot d\varepsilon \geq 0. \quad (2)$$

Once strength is reached, condition (2) is not satisfied locally. The deformation response of the concrete becomes unstable at a point, or layer, of the section. In the case of a section with a spatially homogeneous stress distribution, a rapid failure process of a dynamic nature would then begin if the load on the section is indelible. In cases of stressed sections with bending moment contribution, the course of the process to failure will show varying intensity. The occurrence of local instability at the edge region of the largest deformations, exceeding, does not necessarily imply a loss of stability of the entire compressed zone of the concrete, nor globally of the entire section. The remainder of the section, with actively stressed concrete according to the ascending branch of the physical law and reinforcing steel, can determine the continued stable behaviour of the entire section. As a measure of the stability of a reinforced concrete section, take the global quantity resulting from the integral of condition (2) over the entire heterogeneous section.

$$\chi = \int_{A_c} d\sigma \cdot d\varepsilon \cdot dA + A_{S1} \cdot d\sigma_{S1} \cdot d\varepsilon_{S1} + A_{S2} \cdot d\sigma_{S2} \cdot d\varepsilon_{S2}. \quad (3)$$

In (3), the active, concrete part of the reinforced concrete section is denoted by $A_c$. It consists of a compressed zone

$A_{cc}$ and a tensile zone $A_{ct}$. The latter is located below the neutral axis, above the apex of the normal crack. The quantities $d\sigma$ and $d\varepsilon$ are the differentials of the stress and strain prevailing in the concrete. The other measure members (3) refer to tensile and compressive reinforcement, denoted by quantities with subscripts *S1* and *S2,* respectively. The first member of the right-hand side of (3) is a measure of the stability of the actively stressed concrete zone of the section.

$$\chi_c = \int_{A_c} d\sigma \cdot d\varepsilon \cdot dA. \tag{4}$$

The proposed measures expressed in stresses can be represented in the section stability analysis in internal forces $K$ and generalised deformations $(\varepsilon_0, \kappa)$. Internal forces $K = (F, F_c, M, M_c)$ denote, respectively, the total axial cross-sectional force, the resultant stresses occurring on the active surface of the concrete $A_c$ and the corresponding values of moments calculated with respect to the central axis of the concrete section. We can write the following differential relations for the mentioned forces:

$$dF = dF_c + \sum_{i=1}^{i=2} A_{Si}\, d\sigma_{Si},$$
$$dM = dM_c + \sum_{i=1}^{i=2} A_{Si} \cdot (-1)^{i-1}(0.5 - \delta_{Si}) \cdot h \cdot d\sigma_{Si} \tag{5}$$

where: the quantities $\delta_{Si} = (\delta_{S1}, \delta_{S2})$ are dimensionless coordinates defining the position of the centres of gravity of the reinforcements $A_{S1}$ and $A_{S2}$ with respect to adjacent edges of the section of height $h$.

The components of generalised deformations in the cross-section of a reinforced concrete element are the relative elongation $\varepsilon_0$ of the element's central axis and the curvature $\kappa$ of the axis. Generalised strains define the distribution of strains at the height of the section. We will assume that the increment of deformations at the height of the section is consistent with the assumption of a flat section, i.e.

$$d\varepsilon = d\varepsilon_0 + d\kappa \cdot z. \tag{6}$$

Relationship (6) gives the corresponding increments of strain in the reinforcing steel $A_{S1}$ and $A_{S2}$

$$d\varepsilon_{S1} = d\varepsilon_0 + d\kappa(0.5 - \delta_{S1})h,$$
$$d\varepsilon_{S2} = d\varepsilon_0 - d\kappa(0.5 - \delta_{S2})h. \tag{7}$$

At the level of generalised magnitudes, sectional forces and deformations of the element axes, measures of stability of the entire section and the active zone of the concrete can be represented as

$$\chi = dF d\varepsilon_0 + dM d\kappa,$$
$$\chi_c = dF_c d\varepsilon_0 + dM_c d\kappa. \tag{8}$$

By the theoretical carrying capacity of the section, we will understand such a system of internal forces $(F, M)_n$, for which, in the process of loading, we find a change in the sign of the stability measure from positive to negative. The necessary condition for achieving the load carrying capacity is $\chi = 0$ .

The occurrence of the theoretical load-bearing condition is associated with the loss of stability of the reinforced concrete section in the process of loading. The value of the corresponding edge strain accompanying the occurrence of the load-carrying capacity so defined is then the limit value $\varepsilon_{cu}$. It can be determined from the theoretical analysis of the problem and need not be taken a'priori as an assumption for formulating the problem. The value depends mainly on the assumed weakening law of compressive concrete expressed by the descending branch of the curve $\sigma$-$\varepsilon$.

The second of the proposed stability measures $\chi_c$ makes it possible to infer the behaviour of the active zone of stressed concrete during the loading process of a reinforced concrete section. The phase of the loading process prior to reaching the load condition, which is characterised by $\chi_c > 0$ indicates a stable redistribution of stresses in the zone $A_c$. In particular, it may be the compression zone of concrete $A_{cc}$. Reaching the end of the stable behaviour $\chi_c = 0$ is a theoretical symptom of the exhaustion of the deformation capacity of the concrete part of the reinforced concrete section in the loading process. There are situations in which we simultaneously find $\chi_c = \chi = 0$. Such situations correspond to the cases of sections without excessive reinforcement, that is, reaching the load capacity according to full plastic flow mechanisms. In sections reinforced or reinforced with steel showing the effect of plastic reinforcement, the loss of stability of the active zone of concrete will precede the loss of global stability of the section. Similarly, it will also be the case in cases of small eccentricity, where the load-bearing capacity is achieved according to partial plastic flow mechanisms. The resolution of the mechanism of exhaustion of the load-carrying capacity of a reinforced concrete section in a particular loading process and its subsequent subcritical behaviour can be made on the basis of the analysis of the loading path of the section.

## 3. Configuration of states along the load path in a bent section

We assume that the assumption of a flat section determines the distribution of deformations in the section. The corresponding stress distribution is derived from the physical laws of concrete and reinforcing steel. The physical law for concrete in compression, according to [6,11] is assumed,

$$\sigma = f_{cm} \frac{\eta(k-\eta)}{1+(k-2)\eta}, \tag{9}$$

where tagged:

$$\eta = \frac{\varepsilon}{\varepsilon_{c1}},\ \varepsilon_{c1} = 0.7 f_{cm}^{0.31},\ f_{cm} = f_{ck} + 8\ \text{(MPa)},$$
$$k = 1.1 \frac{\varepsilon_{c1} E_{cm}}{f_{cm}},\ E_{cm} = 22 \cdot (0.1 f_{cm})^{0.3}\ \text{(GPa)}. \tag{10}$$

Relationship (9) captures the weakening effect of concrete as it maps in the $\{\varepsilon, \sigma\}$ coordinate system as a curve having a descending branch. We will not introduce a limitation of the range of the relation to the value of $\varepsilon_{cu}$ known a'priori. The value of the strain will be sought by analysing the stability of the section in terms of the adopted measure (2). Theoretically, the relation (9, 10) can be applied in the range up to the complete relaxation of the concrete, which occurs if the strain in the concrete reaches the value $\varepsilon = k\varepsilon_{c1}$.

In the environment of deformation $\varepsilon = 0$, the adopted relationship is quasi-linear. We will take advantage of the fact and assume, moreover, that it can correctly estimate the behaviour of concrete, also in the range of tensile deformations limited to scratching. Therefore, we will assume that the range of the physical law (9) is $\varepsilon_{ctm} \leq \varepsilon < k\varepsilon_{c1}$. The value of $\varepsilon_{ctm}$ can be chosen so that the value of $f_{ctm}$ appropriate for the concrete class under consideration is derived from (9).

We will assume the following data characterising the concrete: $f_{ck} = 7{,}0\ MPa$, $E_{cm} = 28{,}0\ GPa$, $\varepsilon_{c1} = 0{,}002$, $\varepsilon_{ctm} = 0{,}0001$.

We assume that reinforcing steel is a perfectly elastic-plastic material with elastic modulus $E_S$. We neglect the plastic strengthening of the steel. The elastic limit is $f_{yk} = 350\ MPa$. Because of the need to consider in the analysis the strain relief of tensile reinforcement, previously plasticised, we assume the law of linear strain relief with a load-specific modulus, $E_S$.

Based on the assumptions made, the loading path of a reinforced concrete rectangular section subjected to curvature forcing $\kappa$ of the beam axis was constructed. The concrete relief law was not formulated because the reliefs occur in terms of a quasi-linear relationship $\sigma$-$\varepsilon$ and locate in the middle part of the section. Also, the descending branch for tension was not introduced, $G(\varepsilon) \equiv 0$. Single reinforcement of the section with a degree of $\rho$=0.01 was assumed. The results illustrating the loading path are provided in Figure 2.

At the initial stage of the strain, the instability generated by the scratching of the section can be seen on it. The plasticisation of the tensile reinforcement corresponds to the state marked with the signature YS1. The state initiates pronounced plastic deformation of the cross-section, which is made visible by a quasi-plastic shelf. On the shelf, it is possible to distinguish states:

- loss of stability of the active zone of concrete,
- load capacity – maximum bending moment max $M = M_u$,
- identical to the with loss of stability of the entire section $\chi = 0$,
- relieving previously plasticised tensile reinforcement, $\Delta\varepsilon_{S1} < 0$.

When the load-bearing capacity exhaustion state is reached (max $M = M_u$), the internal force arm decreases, and the flexural response of the section decreases. The thickness of the compressed zone continues to decrease, but the redistribution of compressive stresses causes the internal force arm to decrease. Then the thickness of the compressive zone of the concrete $A_{cc}$ begins to increase, which causes a further decrease in the arm of forces. At the same time, the plastic flow of the tensile reinforcement slows down. Eventually, the increments of plastic deformation in the reinforcement disappear, and it experiences elastic relief, Figure 3. It results in an exponential decrease in the section's resistance to bending. It is reasonable to associate the beginning of the process with the value of the strain in the outermost compressive layer of the section and refer to it as the failure strain $\varepsilon_{cf}$.

## 4. Configuration of states on the loading path of an eccentrically compressed section

We will study the behaviour of a reinforced concrete section doubly reinforced in the process of forcing the reaction of the section of the kinematic type. The reinforcement of the section is given by the degrees $\rho_{S1}$ =0.0178 and $\rho_{S2}$ =0.00178. It is assumed that the section is subjected to an increasing longitudinal force, the variation of which is determined by a linear function with respect to the curvature $\kappa$ of the element axis. The consequence of the assumption is a certain variation in the eccentricity of the longitudinal force. We are looking for the supremum of the force. Numerical analyses were performed for two functions, $n_i(\kappa) i = 1{,}2$.

In the case of $i = 1$, we analyse the loading path inherent in the full mechanism of depletion of the section's load carrying capacity – the case of a large eccentricity, Figure 4. On the path, we find a condition corresponding to the maximum value of the bending moment (max $M$), at which the section deforms stably, despite the plastic flow of both edge reinforcements. A further increase in the longitudinal

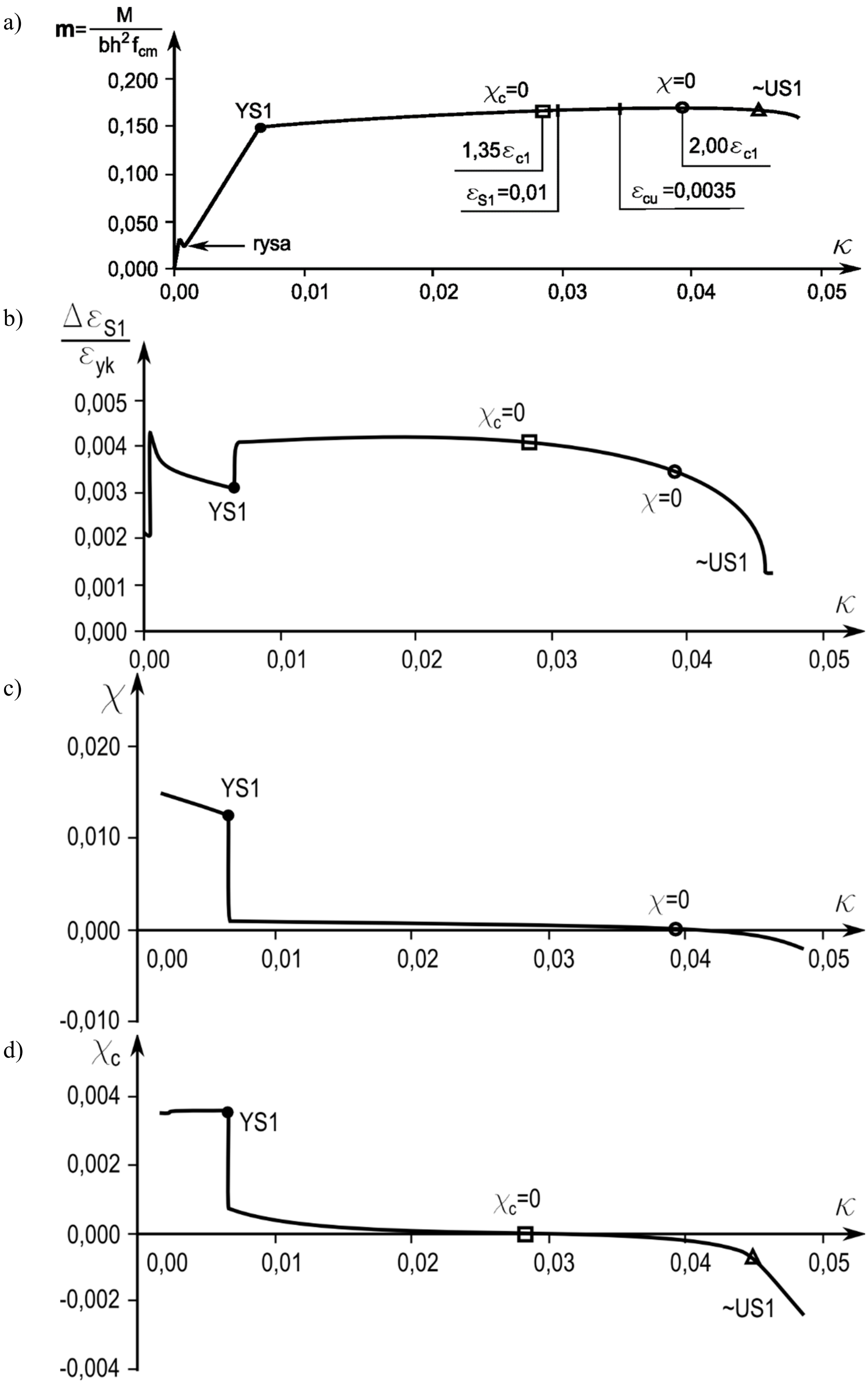


Fig. 2. Results of the test of stability of a bent reinforced concrete section in the process of forcing curvature; a) dependence $m-\kappa$, b) changes in strain $\Delta\varepsilon_{S1}$ in tensile reinforcement, c) changes in the measure of stability $\chi-\kappa$, d) changes in the measure of stability of the active zone of concrete $\chi_c-\kappa$

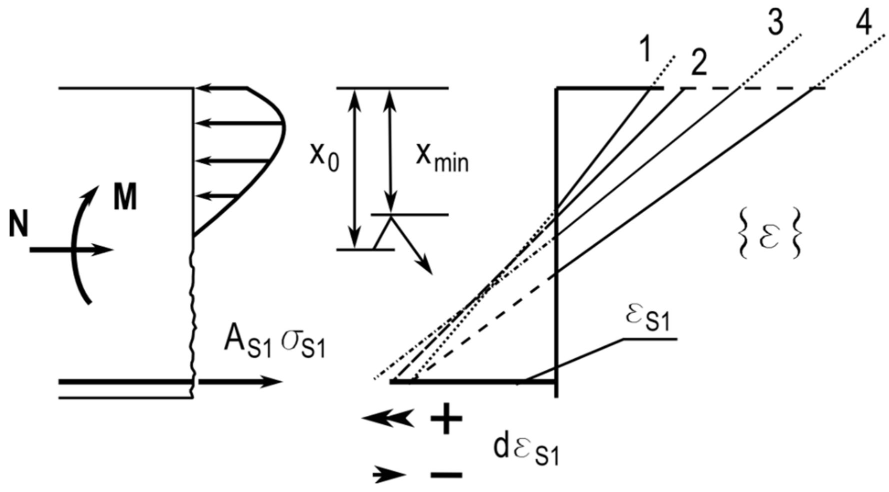


Fig. 3. Illustration of the elastic relief mechanism of plasticised tensile reinforcement $A_{S1}$

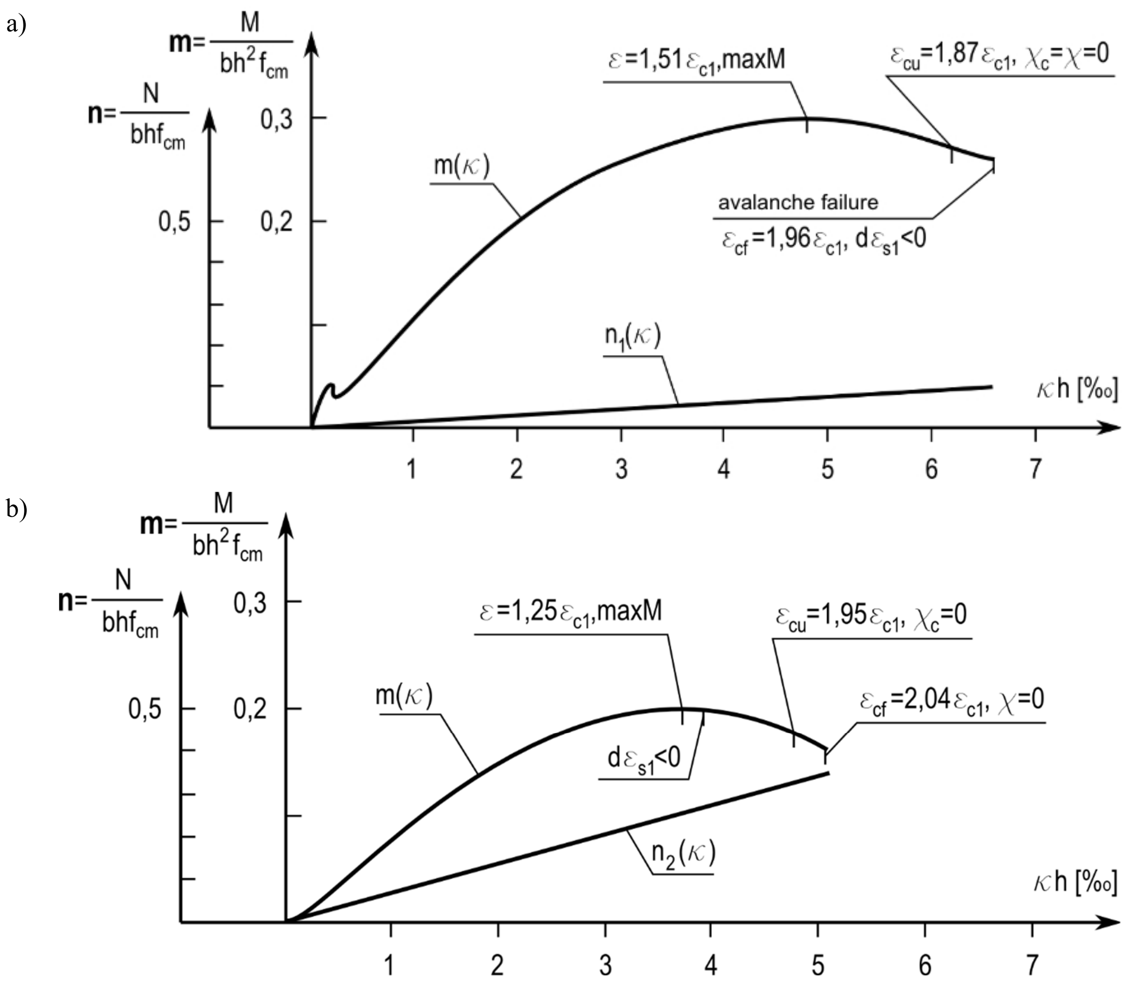


Fig. 4. Path of eccentric loading of a doubly reinforced section; a) case of large eccentricity $-n_1(\kappa)$, b) case of small eccentricity $-n_2(\kappa)$

force is accompanied by a slight decrease in the value of the moment due to the redistribution of stresses in the active zone of the concrete, with the section near the edge showing weakening. A state of simultaneous loss of stability of the active zone of concrete and at the same time of the entire section is reached, $\chi_c = \chi = 0$. We will consider the state as the state of exhaustion of the section's load-bearing capacity. Then the process of weak instability begins, controlled by further weakening of the concrete. After the plasticised tensile reinforcement is relieved, it turns into a process of avalanche failure. The beginning of the process is considered the failure of the section.

In the process of force loading $n_2(\kappa)$, the mechanism of partial exhaustion of the load carrying capacity, which is inherent to a small eccentric, is realised. The loading path is shown in Figure 4. In the process of loading the section, the tensile reinforcement deforms elastically. Due to the development of weakening, the concrete experiences stress relief. The behaviour of the active zone of the concrete determines the stability of the process. The state in which it is reached, $\chi_c = 0$ should be considered the exhaustion of load-bearing capacity. The short-lived process of weak instability ends with the destruction of the section, for which the measure of the stability of the entire section reaches $\chi = 0$.

The failure of critical sections of reinforced concrete elements is constituted by the material weakening property of concrete. The paper shows that taking into account realistic $\sigma$-$\varepsilon$ diagrams for concrete, which should be considered curves with descending branches, it is possible to determine the state of exhaustion of the load-bearing capacity of the section without the need to introduce the limiting strain $\varepsilon_{cu}$ assigned only to the concrete class.

The state should be considered the state of loss of stability of the deformation process of the section, which is characterised by the stability measure $\chi = 0$. It is also possible to determine the state of onset of avalanche destruction of the section defined by the strain value in the outermost layer of the concrete compression zone. In the paper, the value is defined as the failure strain $\varepsilon_{cf}$. The method of determining the value of the ultimate strain and the destructive strain based on the analysis of the load path of a reinforced concrete section is presented.

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