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APPLIED MULTI-CRITERIA MODEL OF GAME THEORY ON SPATIAL ALLOCATION PROBLEM WITH THE INFLUENCE OF THE REGULATOR

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Abstract: The consideration of space in economic analysis and economic theory has continuously been gaining more attention. Location is essential from the point of view of companies in terms of their operations in the market where companies try to operate as efficiently as possible. Therefore, companies have to make strategic decisions about the location of their operations. Faced with the need for financial support, companies often turn to state subsidies. However, subsidies are not just important for companies, but are also a key tool for providers of subsidies so that they can pursue their own interests. This paper focuses on a specific game theory problem situation in which companies decide on the location of their operations, a decision, however, which is also influenced by the pertinent regulatory authority which, in its turn, is striving to implement its own goals. The regulator selects effective tools in order to achieve its goals. In the paper, two tools are considered, namely, subsidizing the product price for the regulator's preferred area and influencing consumers' behavior through active advertising campaigns. This paper sets forth an original model of spatial competition using game theory, its effectiveness residing in the competitive character of the situation outlined. The model's results serve as information for decision-making about the final allocation decisions of companies' operations, the cost of financial subsidies in cases where the first tool is utilized, or the costs of advertising campaigns in cases where the second tool is utilized. The mining regions' support analysis is presented as a concrete case study. It is a current heated topic because of the current situation in the Slovak Republic, where coal mines are scheduled to be closed. The proposed spatial competition game theory model is unique, and its novelty can be considered as a contribution to the current state of the art of game theory knowledge.

Keywords: location, spatial competition, regulator, subsidy, multi-criteria game.

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Introduction

The foundation of an open market economy is free competition, and competition represents a clash of interests of subjects operating in the market. For a company to operate in the market and maintain or improve its position, it must be competitive. The company must proceed strategically to secure its position and maintain or increase market share. The concept of strategy can be encountered in various fields, including game theory. It represents a tool for analyzing players' optimal and strategic behavior, and a player can represent any subject in a decision-making situation. Such situations result from the conflict of interests of decision-making entities. Of all the areas in which game theory is applicable, it is precisely economics in which it has registered striking success. In the context of the national economy, it is possible to mention various industries in which it has a key role to play as an analytical tool, for example where researchers have focused on the fuel market (Cardoso et al., 2022), or where they have drawn attention to its use in the increasingly pressing topic of pollution and climate change (Halat and Hafezalkotob, 2019).

In the products and services market, companies find themselves in conflict with competing companies offering the same or similar products and services. Each of these companies aims to acquire as many customers as possible, increase their market share and maximize their profits, two primary goals where companies focus their actions. When analyzing the competition, it is essential to define the parameters which are targeted. This is an increasingly familiar phenomenon nowadays when the competition and the market have already been transferred to the online space. For example, one group of researchers (Nana et al., 2022) have used game theory to analyze the competitive space in electronic commerce. An analysis of consumer behavior in the case of online vs. merchants' offices has been the focus of a group of academics (Shi et al., 2019), who state that the type of goods offered online is essential, and that a hybrid operating model may not always represent an advantage for the merchant.

Several factors affect the implementation of business plans. From the point of view of strategies, choosing the location where the company wants to operate is essential. Several factors play a role in location decisions, such as the economic potential of the region chosen, the availability of resources, foreign trade opportunities, competitive conditions, consumers' purchasing power, and their interest in the product or services offered. The organization of economic activities or, overall human activity in this space is often discussed, but it still lacks a significant place in economic theory. The importance of distance was outlined by Krugman (1995), who contributed to creating the concept of the New Economic Theory which, unlike more classical economic theories, takes space into account. The influence of distance, the importance of transport, and associated transportation costs have been introduced into models by several other economists. Space and the dynamics of relations between subjects has led to spatial competition. Companies compete primarily on prices and locations in those models which focus on these factors. Spatial games

focused on analyzing imperfect competition from a spatial point of view are a specific area of game theory. In this instance, it deals with market entities operating in the role of competitors to attract customers and find the best location for their branch. Many authors address the application of spatial economics and spatial competition. At the same time, their approaches to the problem differ and are applied in various areas of life - in the economic sphere, logistics, planning, politics, and ecology. It is possible to consider the connection of spatial competition with game theory to be a specific and fruitful topic for future research.
One critical factor in implementing business plans is to have sufficient financial resources. However, a company's financial situation also depends on external, environmental determinants, such as the development of the economy and its cycles or events in the world, the adverse development of which may threaten the further operation of these enterprises. In such cases, one possible approach is to use state aid, which has been a tool in the arsenal of individual state governments since time immemorial. Within its competencies, the government can define strategic goals in various areas of assistance. Support for the least developed regions is often a key focus of regional development policies. These regions' problems result from long-standing problems such as lack of investment and opportunities, low education levels, weak infrastructure, and the constantly deepening differences between them. Eliminating these differences can ensure an inflow of investments resulting in the creation of job opportunities and thus raising the standard of living in the least developed regions.
This paper focuses precisely on the issue of state aid from the point of view of supporting the least developed regions. It presents the case of the decision-making situation of two entities (companies) in the upper Nitra region of the Slovak Republic, where coal mines are expected to be closed, resulting in an increase in unemployment. In the analysis, companies entering the market choose from a range of possible locations, whereas the regulator formulates their decisions based on their status as an authority following its own specific interests.
Such a situation is presented through a model in which two companies represent players offering homogeneous products at known prices and unlimited quantities. The basic idea of the game is based on paper (Čičková and Lopez, 2018). The problem analyzed in this article is defined on a graph, the nodes representing potential company facility locations. The graph nodes also represent customer locations where the demand originates. Different demand quantities for each node are assumed, as the player's general interest in a given node can be described by its size. Based on this idea, some nodes are naturally more attractive to the player than others. Demand size can be linked to the number of inhabitants or admissible customers in a considered location. At the same time, each of these customers must be served either by one or the other player. Potential customers are indifferent to the players, so they do not prefer one or the other. When making a purchase, customers decide based only on total costs criteria, which consist of the price of the purchased product and the transportation cost if customers travel to make the purchase. A

customer is served by player one if the customer's total costs are lower than it would be if purchasing from player two. Conversely, if player one's costs are higher, the customer will be served by player two. In cases where the costs are equal, the players will divide the demand evenly.

As already mentioned, the regulator enters the game as an authority with its own goals, mainly motivating companies to build their facilities in areas preferred by the regulator. For this motivation, the regulator chooses two tools, a financial contribution in the form of a subsidy and an alternative whereby consumers are exposed to targeted advertising, which subsequently influences the game's outcome. Given that these tools represent costs for the regulator, the regulator's second goal is to minimize their financial outlay.

The paper presents the task formulated using two scenarios. In the first one, the regulator limits the costs for individual instruments it is willing to spend on implementing its plans. Conversely, in the second scenario there are no limits on the costs of implementing these instruments. Comparing these two cases will present the overall impact of using the proposed tools on the outcome of the games.

The GAMS (General Algebraic Modeling System) software is used to solve the tasks outlined above. This software is suitable for solving more complex optimization tasks and focuses primarily on modeling linear, non-linear, and mixed integer problems. After the initial experiments, the Couenne solver algorithm (version 25.1.3) was used to obtain the solution because its use proved to be the most advantageous in terms of time and the results' quality. Experiments were performed on a PC with an Intel® Core™ i7-8650U CPU @ 1.9 GHz 2.10 GHz with 16 GB RAM.

Literature Review

One of the first to address the issue of spatial competition was the mathematician and economist Hotelling (1929), who presented a model based on the presence of two firms seeking the most advantageous position in a linear market. The proposed model serves as the base for many theories of product differentiation and location. Despite its usefulness, it has undergone many criticisms. For example, d'Aspremont, Gabszewicz, and Thisse (1979) point out its flaws and prove that equilibrium cannot exist when firms are in close proximity to one another in the market. The result of their modification is a model whose solution ensures the existence of equilibrium in any place on the market. Other publications, such as (Greenhut et al., 1987), (Kats, 1995) or (Graubner et al., 2021), also show that Hotelling's model laid the foundations for several subsequent works devoted to this issue.

Weber's work (Weber and Friedrich, 1929) ranks in primary position among the fundamental theories dealing with location issues. Weber looked for a location for a selected industry, taking into consideration transport costs and labor costs as the primary factors influencing the choice of this location. Several authors have contributed to the Weber location problem development. From many publications, a recent academic paper (Labbé et al., 2019) is highly germane: the authors propose a new two-level location model with one leader choosing a location and one or more followers trying to choose their location as close to the leader as possible.

The available spatial and demographic information affects companies' location decisions and strategic spatial planning decisions on local, regional, and national levels. Maleki et al., (2020) model a game in which the land owners represent the players with the aim of maximizing its most efficient use. Such available information is also used in (Olszewski et al., 2021). Here, the authors' analyses focus on investigating different types of public policy to find the most profitable one for the city, its municipality, and its businesses and citizens.

Fetter (1924) is among the first authors who laid the foundations for analyzing the relationships and interdependence between firms. Unlike Hotelling, Fetter focused on modeling demand behavior, not optimal decisions (Biscaia and Mota, 2013). Further expansion and development of Fetter's work can be found in publications such as (Hamoudi and Risueño, 2012). Bárcena-Ruiz et al., (2016) investigated the zonal mechanism used to find the location of two firms in linear space. The regulator enters their model in three ways. In the first case, it prevents the companies from locating their branches outside the city limits. In the second case, the regulator is interested in the companies and permits the location of branches outside the city limits within an extended zone. In the third case, the regulator is biased towards the companies and allows them to be located only outside the city limits up to a certain distance. The regulation of asymmetric zoning in the duopoly location model is the focus of one particular study (Lai and Tsai, 2004) where the authors delineate this approach’s method of prohibiting firms from locating within a specific interval in a small open linear city, while it is a policy instrument which ensures the limitation of their excess profits. Other researchers have studied the aspect of social welfare, as in (Chen and Lai, 2008), where the authors focus on symmetric zoning in a spatial model under the conditions of Cournot competition (Allaz and Vila, 1993) and its effect on location choice.

The extent to which, and also how, regulatory authorities enter various market situations depends on various factors. However, the intention of benefiting society should be the underlying intention behind each intervention. State oversight and regulation have been the focus of a recent paper (Li et al., 2022) where the authors discussed safe coal mine production and regulatory oversight, contingent on government support. In another paper (Sheng et al., 2020), specific attention was paid to national government analyses of the strategies suitable for local governments and enterprises to examine environmental regulation policies. Among the authors' conclusions is the claim that government oversight and intervention are important for achieving the goals of these policies. The regulator’s deployment of the two-phase model of spatial competition has also been the focus of academic research (Anderson and Engers, 1994). In the first phase, firms choose their locations and then focus on the pricing strategy, which is not only within their competence, but is overseen by the regulatory body. Researchers (Skalicka et al., 2022) have also contended that can justify the regulator’s participation as a motivating factor in the investment decision-making processes of an entrepreneur. The entrepreneur's trustworthiness is an essential element affecting an investor's final decision.

In the literature, there are more studies dealing with regulatory authorities' tools supporting the achievement of sustainable development goals. A particularly germane line of research (Vorontsova et al., 2020) deals with the regulation of education and its contribution to economic growth and the acquisition of intellectual capital of the regions. Plevný (2017) deals with regulator price subsidies of transportation fares to support urban public transport.

Spatial allocation model with the influence of the regulator formulation

The paper proposes to look at a model situation in which two companies, players, decide on the location of their operations, the prices of their products are disclosed and examine where the regulator engages in the decision-making process based on its own specific intentions and criteria. The aim is to motivate companies to build facilities in authority-preferred areas. The regulator uses two selected tools to motivate - a financial contribution as a subsidy and influencing consumers with targeted advertising to create demand at the selected location. Given that these tools represent costs incurred by the regulator, the regulator's primary goal is to minimize these costs. The proposed model solution will take place in two successive phases. In the first phase, the regulator aims to establish branches of companies in preferred areas by using the tools mentioned earlier (first goal). The first goal is at a higher priority level. Subsequently, in the second phase, the regulator aims to minimize the costs associated with the stimulation tools, i.e., the number of affected, aware customers following regulator recommendations, which are associated with the costs of the advertising campaign and the amount of the subsidy provided with regard to the number of customers purchasing from companies. The problem is modeled as a goal programming problem. Lexicographic goal programming is used while minimizing the percentage deviation at the second priority level. Next, the following parameters notations are used:

- $n \in Z^+$ – number of nodes,
- $V = \{1,2, \ldots n\}$ – the set of all nodes,
- $V^{(p)} \subseteq V$ – a set of preferred nodes
- $V^{(u)} \subseteq V$, $V = V^{(p)} \cup V^{(u)}$ – a set of non-preferred nodes,
- $d_{ij} \geq 0, i, j \in V$ – the shortest distance between nodes i and j,
- t 0 – shipping unit costs,
- $p_1 > 0$ – product price of player 1,
- $p_2 > 0$ – product price of an opponent, player 2,
- g_i, $i \in V$ – demand in i-th node,
- λ^H – the maximum percentage of customers that the regulator can influence,
- c^H – the upper limit of the financial subsidy provided by the regulator,
- M – a large positive number,
- ε – a small positive number.

and the following variables notations:

- $w_1 \in \langle 0, n \rangle$ – the number of served nodes by player 1,
- $x_i \in \langle 0,1 \rangle, i \in V$ – i-th mixed strategy of player 1,
- $y_j \in \langle 0,1 \rangle, j \in V$ – j-th mixed strategy of player 2,
- $a_{ij} \in \langle 0, n \rangle, i,j \in V$ – player 1 payoff matrix,
- $b_{kij}^{(1)} \in \{0,1\}; k,i,j \in V$ – binary variable,
- $b_{kij}^{(2)} \in \{0,1\}; k,i,j \in V$ – binary variable,
- $b_{kij} \in \langle -1,1 \rangle; k,i,j \in V$ – continuous variable,
- $z_{ij} \in \{0,1\}; i,j \in V$ – binary variable,
- $\lambda \in \langle 0,1 \rangle$ – the percentage of aware consumers,
- c – a subsidy of the regulator,
- d_1^- – deviation from the first goal,
- d_2^+ – deviation from the second goal,
- d_3^+ – deviation from the third goal,
- l_{kij} – the difference in costs compared to player 2,
- s_{kij} – auxiliary variable for determination of a_{ij},
- c_a – advertising costs per percent of affected customers.

The goals can be defined as follows:

Goal 1: $\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j \geq 2$, at the first strategic level

Goal 2: $minimize\ \lambda \in \langle 0; \lambda^H \rangle$, at the second strategic level

Goal 3: $minimize\ c \in \langle 0; c^H \rangle$, at the second strategic level

The problem is formulated as a lexicographic goal programming model as follows:

$$lex\ \min\{d_1^-; (d_2^+ + d_3^+)\} \quad (1)$$

$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^- \geq 2 \quad (2)$$

$$\lambda * c_a - d_2^+ \leq 0$$

$$c * \left(\sum_{i \in V^{(p)}} \sum_{j \in V} a_{ij} x_i y_j + \sum_{j \in V^{(p)}} \sum_{i \in V} b_{ji} y_j x_i \right) - d_3^+ \leq 0 \quad (3)$$

$$d_1^-, d_2^+, d_3^+ \geq 0$$

Where the subsidy amount depends on the number of nodes served in cases where the facility location is in a preferred location.

The solution will take place in two consecutive stages:

1. *stage*

$$\min d_1^- \quad (4)$$

$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^- \geq 2 \quad (5)$$

$$d_1^- \geq 0$$

the optimal solution of the decision variable is denoted as d_1^{-*}.

2. *stage*

$$\min(d_2^+ + d_3^+) \quad (6)$$

$$\lambda * c_a - d_2^+ \leq 0$$
$$c * \left(\sum_{i \in V^{(p)}} \sum_{j \in V} a_{ij} x_i y_j + \sum_{j \in V^{(p)}} \sum_{i \in V} b_{ji} y_j x_i \right) - d_3^+ \leq 0 \tag{7}$$
$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^{-*} \geq 2$$
$$d_2^+, d_3^+ \geq 0$$

The application of the regulator's tools and associated costs depend on the final company's choice, i.e., whether companies decide to open their facilities in the preferred areas. Three possible scenarios can occur:

1. both companies decide to open their facilities in preferred nodes, and therefore the regulator will provide a price subsidy to both companies,
2. only one of the companies decides to open its facility in the preferred nodes and receives the subsidy
3. neither company decides to open their facilities in the preferred nodes; therefore, the regulator will not provide a subsidy to either of the companies.

The final prices at which consumers purchase the offered products depend on the final location decision. The given scenarios, where player 1 chooses node *i*, player 2 chooses node *j*, and subject to preferred locations condition, can be written as follows:

$$t * d_{kj} + p_2 - c - (t * d_{ki} + p_1 - c) = l_{kij}; k, i \in V^{(p)}, j \in V^{(p)} \tag{8}$$
$$t * d_{kj} + p_2 - (t * d_{ki} + p_1 - c) = l_{kij}; k, i \in V^{(p)}, j \in V^{(u)} \tag{9}$$
$$t * d_{kj} + p_2 - c - (t * d_{ki} + p_1) = l_{kij}; k, i \in V^{(u)}, j \in V^{(p)} \tag{10}$$
$$t * d_{kj} + p_2 - (t * d_{ki} + p_1) = l_{kij}; k, i \in V^{(u)}, j \in V^{(u)} \tag{11}$$

where l_{kij} is the comparison of consumer costs from the *k*-th node when choosing a player from whom to buy (player 1 selects node *i*, player 2 selects node *j*).

The relationship (8) describes the scenario if both players decide to build their facilities in preferred nodes,
Relationship (9) describes the scenario if only player 1 decides to build their facility in the preferred node,
Relationship (10) describes the scenario that occurs if only player 2 decides to build their facility in the preferred node,
Relationship (11) describes the scenario if neither player decides to build their facilities in the preferred node.

The allocation of a customer to a player can be carried out using the Signum function, and its discontinuity problem can be solved by introducing the binary variables $b_{kij}^{(1)} \in \{0,1\}; k, i, j \in V$, $b_{kij}^{(2)} \in \{0,1\}; k, i, j \in V$, the continuous variable $b_{kij} \in \langle -1,1 \rangle; k, i, j \in V$ and the following constraints:

$$l_{kij} \leq M * b_{kij}^{(1)}; k,i,j \in V \tag{12}$$

$$l_{kij} \geq -M * b_{kij}^{(2)}; k,i,j \in V \tag{13}$$

The constraint (12) ensures that if the difference in costs is $l_{kij} > 0$, $b_{kij}^{(1)} = 1$. The constraint (13), on the other hand, ensures that if $l_{kij} < 0$, $b_{kij}^{(2)} = 1$. The following constraint (14) models the situation when these two cases could occur at the same time:

$$b_{kij}^{(1)} + b_{kij}^{(2)} \leq 1; k,i,j \in V \tag{14}$$

Let the demand in nodes be characterized by the vector $g = (g_i), i \in V$, then variables $b_{kij} \in \langle -1,1 \rangle; k,i,j \in V$ together with information on the percentage of aware customers influenced by the regulator $\lambda \in \langle 0, \lambda^H \rangle$ (the upper limit is also set by the regulator) is used to calculate the elements a_{ij}:

$$b_{kij} = b_{kij}^{(1)} - b_{kij}^{(2)}; k,i,j \in V \tag{15}$$

$$b_{kij}^{(1)} * l_{kij} \geq \varepsilon * b_{kij}^{(1)}; k,i,j \in V \tag{16}$$

$$b_{kij}^{(2)} * l_{kij} \leq -\varepsilon * b_{kij}^{(2)}; k,i,j \in V \tag{17}$$

$$s_{kij} = \frac{b_{kij} + 1}{2}; k,i \in V^{(p)}, j \in V^{(p)} \tag{18}$$

$$s_{kij} = \frac{b_{kij} + 1}{2} * (1 - \lambda) + \lambda; k,i \in V^{(p)}, j \in V^{(u)} \tag{19}$$

$$s_{kij} = \frac{b_{kij} + 1}{2} * (1 - \lambda); k,i \in V^{(u)}, j \in V^{(p)} \tag{20}$$

$$s_{kij} = \frac{b_{kij} + 1}{2}; k,i \in V^{(n)}, j \in V^{(n)} \tag{21}$$

$$a_{ij} = \sum_{k \in V} s_{kij} * g_k\, ; i,j \in V \tag{22}$$

Constraints (15) and (17) meet the situation when $l_{kij} = 0$. The elements a_{ij} simultaneously depend on the number of customers affected by the regulator, which is modeled by constraints (18) - (22).

Allow recapitulate the proposed model in which the regulator pursues its goals:

$$d_1^- \rightarrow \min \tag{23}$$

$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^- \geq 2 \tag{24}$$

$$t * d_{kj} + p_2 - c - (t * d_{ki} + p_1 - c) = l_{kij}; k,i \in V^{(p)}, j \in V^{(p)} \tag{25}$$

$$t * d_{kj} + p_2 - (t * d_{ki} + p_1 - c) = l_{kij}; k,i \in V^{(p)}, j \in V^{(u)} \tag{26}$$

$$t * d_{kj} + p_2 - c - (t * d_{ki} + p_1) = l_{kij}; k,i \in V^{(u)}, j \in V^{(p)} \tag{27}$$

$$t * d_{kj} + p_2 - (t * d_{ki} + p_1) = l_{kij}; k,i \in V^{(u)}, j \in V^{(u)} \tag{28}$$

$$l_{kij} \leq M * b_{kij}^{(1)}; k,i,j \in V \tag{29}$$

$$l_{kij} \geq -M * b_{kij}^{(2)}; k,i,j \in V \tag{30}$$

$$b_{kij}^{(1)} + b_{kij}^{(2)} \leq 1; k, i, j \in V \quad (31)$$

$$b_{kij} = b_{kij}^{(1)} - b_{kij}^{(2)}; k, i, j \in V \quad (32)$$

$$b_{kij}^{(1)} * l_{kij} \geq \varepsilon * b_{kij}^{(1)}; k, i, j \in V \quad (33)$$

$$b_{kij}^{(2)} * l_{kij} \leq -\varepsilon * b_{kij}^{(2)}; k, i, j \in V \quad (34)$$

$$s_{kij} = \frac{b_{kij} + 1}{2}; k, i \in V^{(p)}, j \in V^{(p)} \quad (35)$$

$$s_{kij} = \frac{b_{kij} + 1}{2} * (1 - \lambda) + \lambda; k, i \in V^{(p)}, j \in V^{(u)} \quad (36)$$

$$s_{kij} = \frac{b_{kij} + 1}{2} * (1 - \lambda); k, i \in V^{(u)}, j \in V^{(p)} \quad (37)$$

$$s_{kij} = \frac{b_{kij} + 1}{2}; k, i \in V^{(n)}, j \in V^{(n)} \quad (38)$$

$$a_{ij} = \sum_{k \in V} s_{kij} * g_k ; i, j \in V \quad (39)$$

$$w \leq \sum_{i \in V} a_{ij} * x_i ; j \in V \quad (40)$$

$$w \geq \sum_{j \in V} a_{ij} * y_j ; i \in V \quad (41)$$

$$\sum_{i \in V} x_i = 1 \quad (42)$$

$$\sum_{j \in V} y_j = 1 \quad (43)$$

The value of the objective function (23) represents the deviation from the goal of building players' facilities in preferred areas (24). Constraints (25) to (34) are used to calculate the payoff matrix of player 1. Constraints (35) to (39) determine the elements of matrix **A** depending on the percentage of aware customers according to the regulator's preference. Conditions (40) to (43) allow the determining of the equilibrium strategies of players 1 and 2.

Solving the first stage of the problem involves determining the feasible solutions for opening facilities in the specified preferred nodes. The goal of securing the allocation in preferred areas is at a higher priority level. Objectives 2 and 3, i.e., the objective to minimize the campaign cost and the subsidy amount, are at the same lower priority level. In the second stage, the objective function (23) and constraint (24) are replaced by the following formulations:

$$\min(d_2^+ + d_3^+) \quad (44)$$

$$\lambda * c_a - d_2^+ \leq 0$$

$$c * \left(\sum_{i \in V^{(p)}} \sum_{j \in V} a_{ij} x_i y_j + \sum_{j \in V^{(p)}} \sum_{i \in V} b_{ji} y_j x_i \right) - d_3^+ \leq 0 \quad (45)$$

$$d_2^+, d_3^+ \geq 0$$

Whereas in the second phase, the first phase solution is reflected by adding the following constraint:

$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^{-*} \geq 2 \qquad (46)$$

The result of solving the second stage of the problem is information on the regulator's total costs associated with selected tools.

Research Results

The presented paper focuses precisely on the issue of state aid from the point of view of supporting the least developed regions. Considering the low competitiveness of the coal mining industry compared to other sectors and following the environmental objectives and challenges of the transition to clean energy, the European Commission adopted a decision on a platform supporting coal regions in the process of undergoing transformation. Transitioning to a low-carbon economy is a demanding process in which member states and individual regions need help in the economic, socio-social, and environmental fields. The aid aims to support business activities, job creation, and the comprehensive attractiveness of the region to new investors. Also, for this reason, the regulator's efforts revolve around providing public support to facilitate and mitigate the consequences of closing mines.

The decision-making situation of two entities (companies) entering the market is presented in the upper Nitra region of the Slovak Republic, where coal mines are scheduled to be closed, resulting in an increase in unemployment. Companies entering the market choose from possible locations while the regulator engages in their decisions as an authority following its interests. Taking into account the aims of the regulator to help less developed regions, in the example, the first area preferred by the regulator will be the district of Prievidza (representing the region of Upper Nitra). The second preferred region will be the district of Lučenec in the Banská Bystrica region, which ranks among the least developed regions of the Slovak Republic.

In the paper is considered a case with ten nodes (regions) representing potential locations for building branches of two companies entering the market. These are nodes 1 - Martin, 2 - Prievidza, 3 - Poprad, 4 - Lučenec, 5 - Prešov, 6 - Brezno, 7 - Zvolen, 8 - Žilina, 9 - Bardejov, 10 - Dolný Kubín; among these are larger cities, but also districts requiring support from the state. The nodes 2 – Prievidza and 4 – Lučenec will be the preferred nodes of the regulator. The region's demand is determined by the number of potential customers reflecting the number of residents over the age of 18. Residents also represent potential employees for companies coming into the region. The following vector represents the demand:

$$\boldsymbol{g} = (80; 112; 84; 60; 139; 50; 57; 129; 62; 32)^T$$

The values are given in thousands and are rounded.

Ten nodes (regions) represent potential locations for building branches of two companies entering the market. These are nodes 1 - Martin, 2 - Prievidza, 3 - Poprad, 4 - Lučenec, 5 - Prešov, 6 - Brezno, 7 - Zvolen, 8. Furthermore, there are considered two substantial firms offering homogeneous products at available prices set at $p_1 =$

$p_2 = 150$. In addition to the product's price, which customers must pay when purchasing, customers also bear the transportation costs for the journey customers have to make if they decide to buy from the given company (if located in another region). The transportation costs within one area are not considered. Unit costs are represented by the parameter $t = 0{,}5$. The shortest distance matrix $\boldsymbol{D} = d(i,j), i,j \in V$ representing the shortest distances between individual regions are known:

$$\mathbf{D}=\begin{bmatrix} 0 & 51 & 125 & 136 & 197 & 101 & 80 & 33 & 238 & 42 \\ 51 & 0 & 176 & 121 & 248 & 123 & 65 & 63 & 289 & 93 \\ 125 & 176 & 0 & 125 & 77 & 85 & 153 & 145 & 118 & 100 \\ 136 & 121 & 125 & 0 & 193 & 77 & 60 & 165 & 233 & 151 \\ 197 & 248 & 77 & 193 & 0 & 156 & 225 & 217 & 41 & 171 \\ 101 & 123 & 85 & 77 & 156 & 0 & 63 & 134 & 197 & 96 \\ 80 & 65 & 153 & 60 & 225 & 63 & 0 & 113 & 265 & 90 \\ 33 & 63 & 145 & 165 & 217 & 134 & 113 & 0 & 257 & 62 \\ 238 & 289 & 118 & 233 & 41 & 197 & 265 & 257 & 0 & 213 \\ 42 & 93 & 100 & 151 & 171 & 96 & 90 & 62 & 213 & 0 \end{bmatrix}$$

The regulator chooses its tools to influence companies' decisions. The available tools are a subsidy affecting the price of the offered products and a promotional campaign influencing customers' behavior, affecting sales in preferred areas. The parameter value $c_a = 1500$ gives the unit price of such a campaign per one percent of customers. Let's assume that the total cost of advertising must not exceed 7500. Both available tools are sensitive to the number of customers when choosing a place of operation among regions of Prievidza and/or Lučenec. It represents a cost for the regulator that needs to be minimized under the condition of achieving the main goal – choosing one (or both) preferred locations. The regulator determines the maximum costs it is willing to bear. The upper and lower limits for the variables are set as follows: $\lambda \in \langle 0; 0{,}05 * c_a \rangle$ and $c \in \langle 0; 10 \rangle$.
The problem is formulated as a lexicographic goal programming problem with two goals:

$$lex\ \min\{d_1^-; d_2^+ + d_3^+\}$$
$$\sum_{i \in V^{(p)}} x_i + \sum_{j \in V^{(p)}} y_j + d_1^- \geq 2$$
$$d_1^- \geq 0$$
$$\lambda * c_a - d_2^+ \leq 0$$
$$c * \left(\sum_{i \in V^{(p)}} \sum_{j \in V} a_{ij} x_i y_j + \sum_{j \in V^{(p)}} \sum_{i \in V} b_{ji} y_j x_i \right) - d_3^+ \leq 0$$
$$d_2^+, d_3^+ \geq 0$$

Furthermore, problem-solving will occur in two phases: the result of the first phase ensures that the opened facilities are located in the preferred nodes. In the second phase, the regulator aims to minimize the costs of implementing the selected tool.

GAMS software and the Couenne solver are used to solve the mathematical programming problems.
The following results are obtained in the first stage of the problem solution: matrix **A**, which has the form:

A=

402,500	547,200	410,000	375,250	520,000	353,000	638,000	676,000	520,000	438,000
257,800	402,500	486,750	353,000	534,250	375,600	345,200	305,300	534,250	257,800
395,000	318,250	402,500	499,700	604,000	285,000	285,000	395,000	604,000	395,000
429,750	452,000	305,300	402,500	534,250	257,800	368,000	469,650	534,250	198,900
285,000	270,750	201,000	270,750	402,500	285,000	285,000	285,000	743,000	285,000
452,000	429,400	520,000	547,200	520,000	402,500	335,000	452,000	604,000	452,000
167,000	459,800	520,000	437,000	520,000	470,000	402,500	167,000	520,000	279,000
129,000	499,700	410,000	335,350	520,000	353,000	638,000	402,500	520,000	321,000
285,000	270,750	201,000	270,750	62,000	201,000	285,000	285,000	402,500	201,000
367,000	547,200	410,000	606,100	520,000	353,000	526,000	484,000	604,000	402,500

The individual elements of the matrix **A** $(a_{ij}, i, j \in V)$ represent the total demand served by the first firm, assuming that the first firm will perform service at node *i* and the second firm will perform service at node *j*. Thus, for example, the element $a_{14} = 375.25$ means that if the first firm provides a service in node one and the second firm provides a service in node four, the first firm fulfills 375,250 demand units (out of a total amount of 805 and the second firm fulfills 429.75 (805-375.25) demand units.
The players' equilibrium strategies are $\boldsymbol{x} = (0{,}191; 0; 0; 0; 0; 0{,}668; 0{,}140; 0)$ and $\boldsymbol{y} = (0{,}191; 0; 0; 0; 0; 0{,}668; 0{,}140; 0)$. Both players serve, on average, 402.5 demand units. The maximum deviation value from goal 1 is minimized at the value $d_1^{-*} = 2$. The percentage information about aware customers is λ=0.05, so the recommendations of the regulator will be followed by a resulting 5% of customers, and the advertising costs will be 75 units. The final amount of the price subsidy provided by the regulator will be minimized to the value c = 0.
It can be concluded that with established intervals for individual instruments $\lambda \in \langle 0; 0.05 * c_a \rangle$ and $c \in \langle 0; 10 \rangle$, the regulator will not be able to achieve the goal of locating players' facilities in the regulator's preferred areas and, therefore, will not invest any funds in the instruments. Therefore, a situation with no financial limits for the regulatory body will be illustrated.
Allow situation in which the regulator does not have a limit on its costs. Thus, its instruments can move in the intervals $\lambda \in \langle 0; 1 * c_a \rangle$ and $c \in \langle 0; 150 \rangle$. In this case, the following results are obtained:
In the first stage of the problem solution, the following results are obtained: matrix **A**, which has the form:

A=

$$\begin{bmatrix} 402{,}500 & 363{,}386 & 410{,}000 & 249{,}197 & 520{,}000 & 353{,}000 & 638{,}000 & 676{,}000 & 520{,}000 & 438{,}000 \\ 441{,}614 & 402{,}500 & 593{,}656 & 353{,}000 & 625{,}200 & 519{,}843 & 499{,}655 & 473{,}158 & 625{,}200 & 441{,}614 \\ 395{,}000 & 211{,}344 & 402{,}500 & 331{,}842 & 604{,}000 & 285{,}000 & 285{,}000 & 395{,}000 & 604{,}000 & 395{,}000 \\ 555{,}803 & 452{,}000 & 473{,}158 & 402{,}500 & 625{,}200 & 441{,}614 & 514{,}796 & 582{,}300 & 625{,}200 & 402{,}500 \\ 285{,}000 & 179{,}800 & 201{,}000 & 179{,}800 & 402{,}500 & 285{,}000 & 285{,}000 & 285{,}000 & 743{,}000 & 285{,}000 \\ 452{,}000 & 285{,}157 & 520{,}000 & 363{,}386 & 520{,}000 & 402{,}500 & 335{,}000 & 452{,}000 & 604{,}000 & 452{,}000 \\ 167{,}000 & 305{,}345 & 520{,}000 & 290{,}204 & 520{,}000 & 470{,}000 & 402{,}500 & 167{,}000 & 520{,}000 & 279{,}000 \\ 129{,}000 & 331{,}842 & 410{,}000 & 222{,}700 & 520{,}000 & 353{,}000 & 638{,}000 & 402{,}500 & 520{,}500 & 321{,}000 \\ 285{,}000 & 179{,}800 & 201{,}000 & 179{,}800 & 62{,}000 & 201{,}000 & 285{,}000 & 285{,}000 & 402{,}500 & 201{,}000 \\ 367{,}000 & 363{,}386 & 410{,}000 & 402{,}500 & 520{,}000 & 353{,}000 & 526{,}000 & 484{,}000 & 604{,}000 & 402{,}500 \end{bmatrix}$$

The players' equilibrium strategies are $x_4 = y_4 = 1$. Both players serve, on average, 402.5 of demand units. The maximum deviation value from goal 1 is minimized at the value $d_1^{-*} = 0$. The percentage information about aware customers is λ=0.369, so the recommendations of the regulator will be followed by a resulting 36.9% of customers, and the advertising costs will be 55350 financial units. The final amount of the price subsidy provided by the regulator will be minimized to the value c = 0.

In the second stage of the problem solution, taking into consideration the information obtained in the first stage, the same matrix **A** is obtained, and the equilibrium strategies and values of the above variables remain unchanged. It can be concluded that in this phase, it was impossible to reduce the regulator's cost.

At the same time, the regulator's primary objective to attract companies in the preferred areas is fulfilled, as both players have decided to open their facilities at the preferred node four. The regulator’s goal will be achieved only by using one of the tools, namely targeted advertising.

Based on the results of the solution to the problem, it can be concluded that the intervals set for individual instruments, the number of affected customers, and the financial subsidy have an impact on the outcome of the game, as the decision of the companies on the choice of their location depends on the regulator's instruments settings. The importance of space and location in economic theory is growing along with globalization, the conditions of an open market, and the developing possibilities of acting in it. Therefore, expanding the basic, already-known models of spatial competition by including new conditions, a different view of the analyzed situations, and other extensions allow us to further develop this topic from a theoretical point of view and apply it to real-world cases. This paper aims to present the possibilities of location modeling of a duopoly in the case of the entry of another entity affecting the outcome of location decisions. This entity is considered to be a state regulator in a position of authority that pursues its interests. The considered approach may be interesting for further analysis of potential tools such an authority can use to influence the market situation within available legal options. There is still room to expand the analysis to different market areas and model less developed regions' issues. The proposed spatial competition game theory model is unique, and its novelty can be considered as a germane and promising contribution to the current state of the art of game theory knowledge.

Conclusion

Models of location determination and models of spatial competition belong to the contemporary field of scientific research. The combination with game theory is a developing topic and presents a promising area for future research. The research put forward in this paper focuses on developing the spatial location model, including the regulator as an authority that enters the decision-making situation of companies entering the market. These companies choose their location of operations from feasible areas, while the regulator prefers specific ones based on certain criteria or established priorities. The regulator's goal is to motivate companies to choose at least one of the preferred areas which will fulfill the regulator's intentions, such as supporting the development of a preferred region. The regulator uses specific tools for this purpose. In the presented case, a situation in which the regulator chose a financial subsidy and targeted advertising is considered, influencing consumers' behavior toward buying companies' products. The regulator needs to invest funds in both instruments, representing incurred costs. The regulator's next goal is to minimize costs so that the first main goal remains fulfilled: opening company facilities in preferred areas.

For analysis it is deployed an original mathematical model of spatial competition to analyze the situation formulated. Game theory is used as an effective tool for making strategic decisions. The problem was formulated as a goal mathematical programming problem, solved in two consecutive phases. In the first phase, the goal was to ensure the facility opened in preferred areas and, in the second, to minimize total costs incurred.

The developed model was applied to two examples based on the strategy of supporting the least developed regions of the Slovak Republic. Based on analysis, it is possible to identify ten permissible areas in the Slovak Republic in which companies could open their branches and offer their products at given prices. At the same time, two of them were preferred by the regulator. In the first example, the situation in which the regulator limited the funds it was willing to invest in individual instruments was examined. In this case, the regulator failed to secure the choice of preferred districts for establishing facilities of these companies in the first phase. In the second example, the boundary assumption was dropped. Wider intervals for both instruments ensured the motivation of companies to choose their preferred areas, thereby fulfilling the main objective of the regulatory body. It was possible to conclude that the chosen intervals impact the final strategies of the players (companies entering the market). The amount of funds invested in the regulator's tools affects whether it is worthwhile for companies to open branches in designated areas or non-preferred ones.

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ZASTOSOWANIE WIELOKRYTERIALNEGO MODELU TEORII GIER W ZAGADNENIU ALOKACJI PRZESTRZENNEJ UWZGLĘDNIAJĄCEJ WPŁYWOPERATORA

Streszczenie: Uwzględnienie przestrzeni w analizie ekonomicznej i teorii ekonomii stale zyskuje większą uwagę coraz więcej uwagi. Lokalizacja jest istotna z punktu widzenia przedsiębiorstw w zakresie ich działalności na rynku, na którym firmy starają się działać jak najbardziej efektywnie. W związku z tym przedsiębiorstwa muszą podejmować strategiczne decyzje dotyczące lokalizacji swojej działalności. W obliczu potrzeby wsparcia finansowego firmy często sięgają po dotacje państwowe. Dotacje są jednak ważne nie tylko dla firm, ale są również kluczowym narzędziem dla podmiotów udzielających dotacji, dzięki którym mogą realizować własne interesy. Niniejszy artykuł skupia się na specyficznej sytuacji problemowej z zakresu teorii gier, w której przedsiębiorstwa decydują o lokalizacji swojej działalności, na którą jednak wpływ ma również odpowiedni operator, który z kolei dąży do realizacji własnych celów. Operator dobiera skuteczne narzędzia do realizacji swoich celów. W artykule rozważane są dwa narzędzia, a mianowicie subsydiowanie ceny produktu dla obszaru preferowanego przez operatora oraz wpływanie na zachowania konsumentów poprzez aktywne kampanie reklamowe. Artykuł przedstawia autorski model współzawodnictwa przestrzennego z wykorzystaniem teorii gier, którego skuteczność wynika z konkurencyjnego charakteru przedstawianej sytuacji. Wyniki modelu służą jako informacja do podejmowania decyzji o ostatecznych decyzjach alokacyjnych działalności firm, kosztach dotacji finansowych w przypadku wykorzystanie pierwszego narzędzia lub kosztach kampanii reklamowych w przypadku wykorzystania drugiego narzędzia. Analiza wsparcia regionów górniczychb jest przedstawiona jako konkretne studium przypadku. Jest to aktualnie gorący temat ze względu na obecną sytuację na Słowacji, gdzie planowane jest zamknięcie kopalń węgla.Proponowany model teorii gier konkurencji przestrzennej jest unikalny, a jego nowatorstwo można uznać za wkład w aktualny stan wiedzy z zakresu teorii gier.

Słowa kluczowe: lokalizacja, konkurencja przestrzenna, regulator, dotacja, gra wielokryterialna

具有调节器影响的空间分配问题博弈论应用多准则模型

摘要：经济分析和经济理论中对空间的考虑一直受到越来越多的关注。 **从公司的角度来看，就其在公司**试图尽可能高效地运营的市场中的运营而言，位置至关重要。**因此，公司必**须对其运营地点做出战略决策。 **面**对资金支持的需要，企业往往求助于国家补贴。 **然而，**补贴不仅对公司很重要，而且也是补贴提供者追求自身利益的重要工具。 **本文**侧重于一个特定的博弈论问题情况，在这种情况下，公司决定其运营地点，然而，这一决定也受到相关监管机构的影响，而监管机构反过来也在努力实现自己的目标。 监管机构选择有效的工具以实现其目标。 **在本文中，考**虑了两种工具，即为监管机构的首选区域提供产品价格补贴，以及通过积极的广告活动影响消费者的行为。 **本文利用博弈**论提出了空间竞争的原始模型，其有效性在于概述了竞争态势的特征。 该模型的结果可作为决策信息，用于公司运营的最终分配决策、使用第一种工具时的财政补贴成本或使用第二种工具时的广告活动成本。 矿区的支持分析作为一个具体的案例研究提出。 **由于斯洛伐克共和国目前的情况，煤**矿计划关闭，这是当前的热门话题。 **所提出的空**间竞争博弈论模型是独一无二的，其新颖性可以被视为对当前博弈论知识艺术水平的贡献

关键词：区位、空间竞争、监管机构、补贴、多标准博弈