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Analysis of Coverage and Area Spectral Efficiency under Various Design Parameters of Heterogeneous Cellular Network

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***Abstract*—As day by day the population is increasing, the use of mobile phones and different applications is increasing which requires high data rate for transmission. Homogeneous cellular network cannot fulfill the demand of mobile users, so creating a heterogeneous cellular network (HCN) is a better choice for higher coverage and capacity to fulfil the increasing demand of upcoming 5G and ultra-dense cellular networks. In this research, the impact of antenna heights and gains under varying pico to macro base stations density ratio from 2G to 5G and beyond on two-tier heterogeneous cellular network has been analyzed for obtaining optimum results of coverage and area spectral efficiency. Furthermore, how the association of UEs affects the coverage and ASE while changing the BSs antenna heights and gains has been explored for the two-tier HCN network model. The simulation results show that by considering the maximum macro BS antenna height, pico BS antenna height equal to user equipment (UE) antenna height and unity gains for both macro and pico tiers, the optimum coverage and area spectral efficiency (ASE) for a two-tier fully loaded heterogeneous cellular network can be obtained.**

***Keywords*—Heterogeneous Network, Coverage, Area Spectral Efficiency, 5G, Ultra-dense network**

I. INTRODUCTION

FUTURE wireless cellular networks require high data transmission due to high data traffic so as to fulfil the demand of 5G network which will be implemented over 4G network [1]. There are different ways to increase the capacity of wireless networks. Capacity can be enhanced by addition of spectrum that is using the higher spectrum where large bandwidth would be available, or by using multiple-input and multiple-output (MIMO) schemes, or it can also be improved by increasing the signal to interference plus noise ratio (SINR). However, the easiest way among all is adding more spectrum, but this method is not cost effective because adding more bandwidth makes the system more expensive. Thus, the only way to achieve high data transmission is to densify the network by creating heterogeneous network with multiple class of base stations such as large-scale BSs (Macro) and small-scale BSs (Pico) or even femto base stations [2-5] to boost the coverage and capacity. Thus, the important parameter of mobile wireless network is the SINR through which capacity can be increased as per Shannon law.

When the cellular networks are densely deployed with multiple class of base stations like macro, pico, femto, or some kind of relays and remote radio heads (RRHs), or mixture of these then the cellular networks become complex enough to be treated for analysis due to heterogeneity with conventional methods of wireless communication. Hence, other methods are being used to model such complex networks for analysis purpose where the wireless nodes are distributed as per specific probability distribution [6,7]. One of them is the stochastic geometry which models the situation where the objects are randomly distributed [8]. Hence, Poisson point process (PPP) is being utilized to analyze the 4G and 5G networks because of its absolute spatial randomness and tractability.

A lot of work has been done in the area of heterogeneous cellular networks and different researchers have worked on different aspects of it like coverage and rate analysis in [9,10], wireless power transfer in downlink (DL) and information transmission for uplink (UL) in [11], load balancing in [12,13] and about energy efficiency in [14,15]. The Authors have shown in [16] that by decreasing vertical beam width and applying antenna downtilt, in heterogeneous networks (HetNets), the area spectral efficiency (ASE) and average throughput can be increased along with improvement in network efficiency. Regarding the densification of HetNets, the authors in [17] have maximized the energy efficiency (EE) under the constraint of ASE by optimizing the BS densities for an ultra-dense HetNets. Some of the researchers have worked on coverage and area spectral efficiency which are the basic performance metrics of the HCNs while considering the three dimensional distance between the user and BS and also the antenna pattern and downtilt under realistic path loss model in [18,19] where they have shown that when the absolute difference of height between user and BS is greater than zero, then by increasing BS density of small cell network to ultra-dense, both the coverage and ASE decrease constantly towards zero. Furthermore, a method was adopted to solve this problem by reducing the BS antenna height equal to user antenna height. However, this all has been

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considered for homogeneous small cell ultra-dense network but not for the heterogeneous network.

In this paper, a two-tier heterogeneous cellular network is considered to analyze the impact of different antenna heights and gains of both macro and pico tiers on coverage and area spectral efficiency under three dimensional (3D) distance which is equal to the square root of the sum of square of two dimensional distance and antenna height while using single slope path loss model when the network is densified from sparse to fully loaded network. Furthermore, how the association of UEs affect the coverage and ASE when changing the BSs antenna heights and gains has been explored for the considered network model. The results obtained in this work are different from those acquired in [18,19] due to heterogeneity.

Further, the paper is organized as follows. Section II discusses the heterogeneous network model and user association algorithm. Section III discusses the performance metrics. Section IV presents the simulation results and their discussion. Finally, the conclusions drawn from this paper are discussed.

II. Network Model and User Association Algorithm

A two tier heterogeneous network model with macro and pico base stations is shown in Fig. 1. The base stations are uniformly distributed among the two tiers and their locations are modeled by independent homogeneous PPPs Φ_i having the densities λ_i and powers P_i where i =1 for macro tier and i = 2 for pico tier. Both independent homogeneous PPPs combine to form a homogeneous PPP Φ_t with total density $\lambda_t = \lambda_1 + \lambda_2$. It should be noted that P_2 is less than P_1 and both are fixed. Also λ_1 is less than λ_2 and the ratio of λ_2 to λ_1 is denoted by β which is not kept fixed as it is varied as per cellular generations from 2G to 5G and beyond to analyze its impact on network performance. The single slope path loss model as per [9] has been considered but the distance considered between the user and its associated BS is the three dimensional distance as shown in Fig. 1 where the user is considered to be at the origin of the network.

Figure 1 shows the network deployment in the form of macro and pico BSs and user equipment (UE) is considered to be at the origin of the network. The UE can be associated to any BS of the tiers as per different applicable camping criteria such as physically nearest BS, averaged channel state information (CSI) or instantaneous CSI. A lot of previous work associate users using nearest association [20,21], maximum average receive power in the downlink [20,21] or maximum instantaneous receive power [22] and this criterion has been chosen in this work. Thus, the UE is associated to the BS of any tier as per following algorithm:

1) The maximum instantaneous power received by UE at the origin by all macro BSs belonging to tier Φ_1 is calculated and the macro BS (MBS) which provides the maximum instantaneous power at UE among all macro BSs of tier-1 is selected.

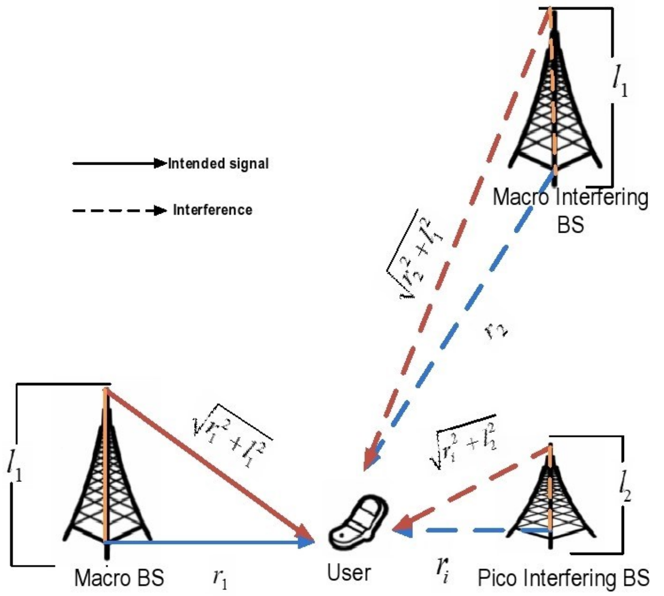

Fig. 1. Heterogeneous cellular network model showing interference and intended signals.

2) Similarly, the maximum instantaneous power received by UE at the origin by all pico BSs belonging to tier Φ2 is calculated and the pico BS (PBS) which provides the maximum instantaneous power at UE among all pico BSs of tier-2 is selected.
3) If the instantaneous power of selected MBS from tier-1 is greater than the instantaneous power of selected PBS from tier-2 at UE, associate the UE to MBS; otherwise, associate UE to PBS.

The network model in Fig. 1 shows that the UE is receiving its intended signal from macro base station and interference signals from all other macro and pico BSs of both tiers. The intended and interference signals are represented by straight solid line and dashed lines respectively. The intended BS for the UE is considered as per association algorithm discussed above.

III. Performance Metrics

In this section, the performance metrics of the heterogeneous cellular network are discussed. The basic performance indicator of a cellular HetNet is the coverage probability. The coverage probability of the user equipment located at the origin is its signal-to-Interference plus Noise Ratio (SINR) above the specified threshold χ .

$$\Pr[SINR > \chi], \tag{1}$$

and SINR is given by

$$SINR = \frac{S}{I_T + N_0}, \tag{2}$$

where S is the instantaneous power received by UE from its tagged BS and IT is the total interference received by UE. The thermal noise N0 = KTB = -104 dBm where K is a Boltzmann constant, T is a room temperature and B is the bandwidth of 10 MHz. As N0 is negligible thus, SINR is actually SIR and is given by

$$SIR = \frac{P_k G_k H_k D_k^{-\alpha}}{\sum_{i:bi\in\Phi t\backslash bo} P_k g_{k,i} h_{k,i} d_{k,i}^{-\alpha}}, \quad (3)$$

The capital P_k, G_k, H_k, and D_k in the numerator of (3) are the power, gain, Rayleigh distributed small-scale fading ($H_k \sim exp(1)$) and the three dimensional distance respectively between the associated user and its serving BS b_0 of kth tier. Path loss exponent is denoted by α having value equal to 4 for both the tiers. Similarly, in the denominator of (3), P_k is also the power of interfering BS of kth tier and $g_{k,i}$, $h_{k,i}$, and $d_{k,i}$ are gain, Rayleigh distributed small-scale fading ($H_k \sim exp(1)$) and the three dimensional distance respectively between UE and ith interfering BS of the kth tier. The three dimensional distance between the user and the BS is given by

$$D_k / d_{k,i} = \sqrt{r_{k,i}^2 + l_k}, \quad (4)$$

$r_{k,i}$ is a two dimensional distance between the user and the ith BS from kth tier and l_k is the height of the BS from kth tier.

Another performance metric of a heterogeneous cellular network is an area spectral efficiency (ASE) which can be defined as the maximum data rate that can be achieved per unit bandwidth and unit area for a specified randomly located user in bps/Hz/m^2. In this work, ASE has been calculated in bps/Hz/Km^2. It has also been considered that one UE per BS is scheduled because of sufficiently large UEs density which is known as full buffer model. Thus, the ASE of a considered network model can be written as

$$ASE = \sum_{k=1}^{2} \lambda_k SE_k, \quad (5)$$

where SE_k is the spectral efficiency of kth tier and is given by

$$SE_k = log_2(1 + SIR_k), \quad (6)$$

IV. Simulation Results and Discussion

In this section, the simulation results are being presented for the heterogeneous cellular network model as shown in Fig. 1. The simulation parameters with their values and units of the heterogeneous cellular network model are shown in Table I. The Coverage and Area Spectral Efficiency (ASE) results under the impact of macro and pico antenna heights and gains will be discussed in this section.

A. Coverage

The results of coverage have been obtained for the network model shown in Fig. 1 under the simulation parameters shown in Table 1. The coverage results have been obtained at χ equals to 0 dB and 10 dB SIR thresholds respectively for different macro and pico BSs antenna heights as shown in Fig. 2 and Fig. 4 respectively with the increase of β from 10 to 2500 which is pico to macro BS density as shown in Table I.

TABLE I
SIMULATION PARAMETERS

Parameters	Values with units
Pico to Macro BS density ratio $\beta = \lambda_2/\lambda_1$	{10 15 30 50 80 333 500 1000 1500 2500}
Pico BS density, λ2	{0.1 1 10 50 100 500 1000 2500 5000 10000} (BSs/km^2)
Tier BSs Power (P_1, P_2)	{46, 24} dBm [24]
Min and Max Macro Antenna Gains (GM_{min}, GM_{max})	{0, 15} dB [24]
Min and Max Pico Antenna Gains (GP_{min}, GP_{max})	{0, 5} dB [24]
UE Antenna Gain	0 dB [24]
Absolute Min and Max Macro Antenna Heights ($l\,M_{min}$, $l\,M_{max}$)	{0, 30.5} m [25]
Absolute Min and Max Pico Antenna Heights ($l\,P_{min}$, $l\,P_{max}$)	[0, 8.5] m [25]
Path loss Slopes ($\alpha_1 = \alpha_2 = \alpha$)	4
Carrier Frequency (fc)	2 GHz
Bandwidth (B)	10MHz
Thermal Noise (N0)	-104 dBm

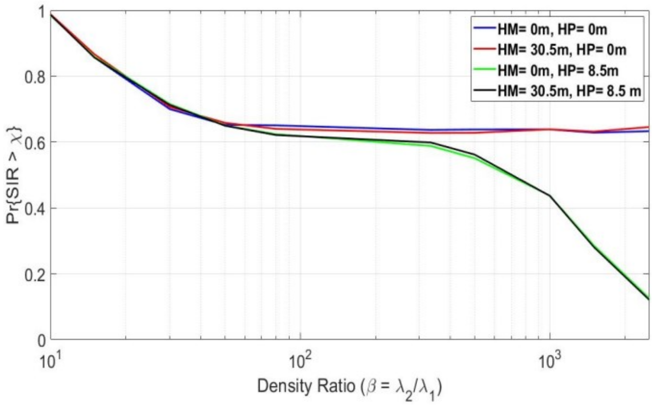

Fig. 2. Coverage at χ = 0 dB under different two-tier BSs antenna heights Vs BSs density ratio, β.

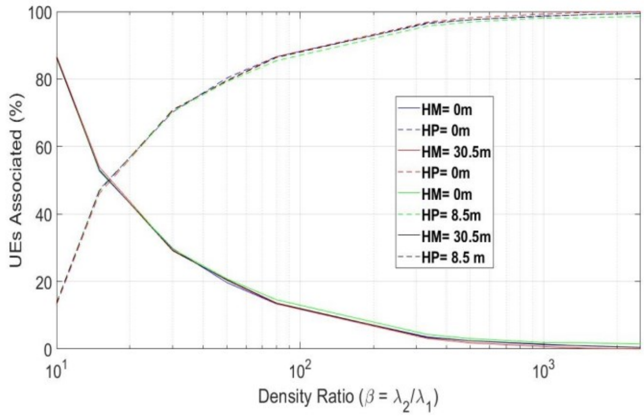

Fig. 3. UEs Association to macro and pico BSs under different two-tier BSs antenna heights Vs BSs density ratio, β

The ratio β has been proposed as per [23] which is shown in Table 2 as per required density for different cellular generations from 2G to 5G and beyond i.e., ultra-dense network (UDN).

TABLE II
BSs DENSITY RATIO (β) UNDER DIFFERENT GENERATIONS [23]

Number of Pico BSs per Macro BS (β)	Cellular Generations
≤ 10	2G ~ 3G
10 < β < 60	4G
60 < β < 2500	5G
B ≥ 2500	Ultra-dense Network (UDN) (Beyond 5G)

1) Impact of Antenna Heights on Coverage

The results in Fig. 2 have been acquired for different macro and pico BSs antenna heights from minimum to maximum or mix of two at χ = 0 dB which is the minimum threshold of SIR for practical receivers. It should be noted that the antenna heights of macro and pico BSs are considered to be 32 meters and 10 meters respectively whereas the height of user equipment is considered to be 1.5 meters [25]. However, the heights of macro and pico cells have been taken as the absolute difference of macro or pico BS and UE heights. Thus, the height of macro BS is 32 m – 1.5 m which equals to be 30.5 m. Similarly, the height of pico cell is 10 m – 1.5 m and is equal to 8.5 m.

From Fig. 2, it is clear that as the β increases up to approximately 50, the coverage reduces from approximately 100% to 65% for all considered antenna heights and there is no impact of antenna heights on coverage which is evident in Fig. 3 that all results of UEs association for different antenna heights are overlapping each other. The coverage only reduces because of offloading of users from macro BSs to pico BSs as shown in Fig. 3. It should be noted that when β = 10 that is there are only 10 pico cells per single macro cell, 85% of UEs are associated to macro BSs whereas only 15% are associated to pico cells (Fig. 3) and coverage is highest at 99% (Fig. 2) due to higher received instantaneous power from macro cells as their transmit power in the downlink is higher than the pico cells. Another reason for highest coverage is that when network is sparse (β = 10), the overall network interference is low due to large two dimensional (2D) distance between UEs and their associated BSs. Furthermore, with the offloading of UEs from macro to pico tier as the number of pico BSs per macro BS reaches 50, there is no impact of antenna heights because 2D distance is quite high enough and at this stage, 80% of UEs are associated to small cells and only 20% to large cells where coverage reaches to 65%. However, as β has been increased higher and higher beyond 50, the impact of antenna heights can be seen in Fig. 2. With the increase of β beyond 50 in the range of 5G and UDN, more than 80% users are associated to pico cells and in the UDN range, almost all UEs are associated to small cells. Similarly, 2D distance tends to zero in the 5G and UDN range but still there is minimum distance in the form antenna height is linked to signal power [18] [19]. Thus, the higher density of interference paths due to huge density of BSs and the cap on signal power tied in the form of antenna height leads coverage towards zero when the absolute difference of height of small cells and UE is greater than zero whereas the height of macro BS does not matter. On the other hand, when the absolute difference of height of small cells and UE is equal to zero, no matter whatever is the height of macro BS, the coverage approaches to a constant of 63% in the 5G and UDN range. The reason behind the constant coverage in the range of 5G and UDN where the BS density is quite high is that there is no antenna height cap and the interference of the dense network is compensated by the signal power as the intended BS and UE are very close to each other with the densification of the network. It should be noted that the height of macro BS does not impact the coverage as most of the macro BSs are in outage in the 5G and UDN regime.

In Fig. 4, the results are shown for coverage at χ = 10 dB. As the threshold has been increased, surely, the coverage is lower at χ = 10 dB than the results produced at χ = 0 dB in

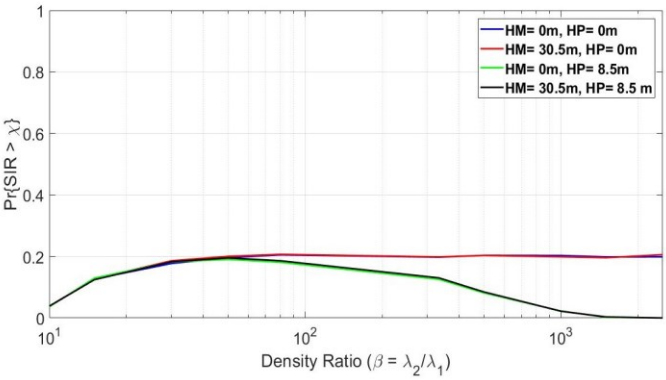

Fig. 4. Coverage at χ = 10 dB under different two-tier BSs antenna heights Vs BSs density ratio, β

Fig. 2. It is evident from the results that when network is not heavily populated with the BSs i.e. β = 10, the coverage is about 4% which is very low. The reason for very low coverage is the higher threshold which cannot be achieved with low density. However, as the β has been escalated to 50 and above, coverage is improving with the BSs density and approaches to a constant value of 20% when the pico cell antenna height is equal to UE antenna height. Similarly, when the pico cell antenna height is greater than zero i.e. 8.5 m for whatever the macro cell antenna height is, the coverage touches the maximum 20% value and then approaches towards zero. The reasons for constant coverage when pico cell antenna height is zero and approaches towards zero when pico cell antenna height is greater than zero are same as discussed above for the coverage results achieved at χ = 0 dB.

2) Impact of Antenna Gains on Coverage

While observing the impact of antenna heights on coverage, the gains of antenna were considered to be unity (0dB). From those results, it has been noticed that the coverage is highest when the pico cell antenna height is zero meters (equal to UE antenna height) whereas the macro cell antenna height does not matter. However, making the macro cell antenna height equal to zero is not practical, therefore, the results of highest coverage when pico cell antenna height equals to zero and macro cell absolute antenna height is maximum (30.5 m) are considered to further analyze the impact of antenna gains of large and small cells on coverage.

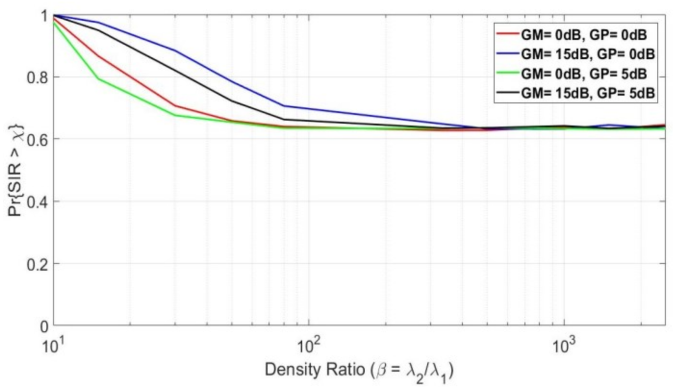

Fig. 5. Coverage at χ = 0 dB under different two-tier BSs antenna gains Vs BSs density ratio, β

The impact of antenna gains on coverage at χ = 0 dB and χ = 10 dB have been shown in Fig. 5 and Fig. 7 respectively while considering the maximum macro antenna height and zero meters pico antenna height.

From results shown in Fig. 5 at χ = 0 dB, it can be easily noticed that by taking the gain of macro BS to its maximum value and keeping the pico antenna gain to its minimum value, the coverage is highest and it approaches to constant for 5G and UDN range. The reason is that macro BS gain is highest which has increased the number of UEs association to macro BS as shown in Fig. 6 specially when network is sparse and as a result, the coverage is highest.

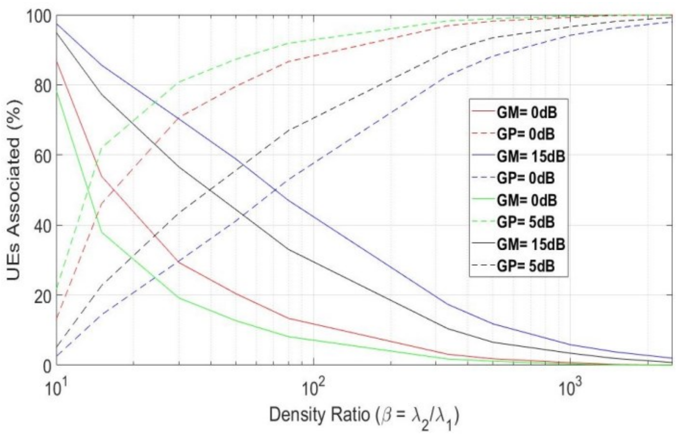

Fig. 6. UEs Association to macro and pico BSs under different two-tier BSs antenna gains Vs BSs density ratio, β.

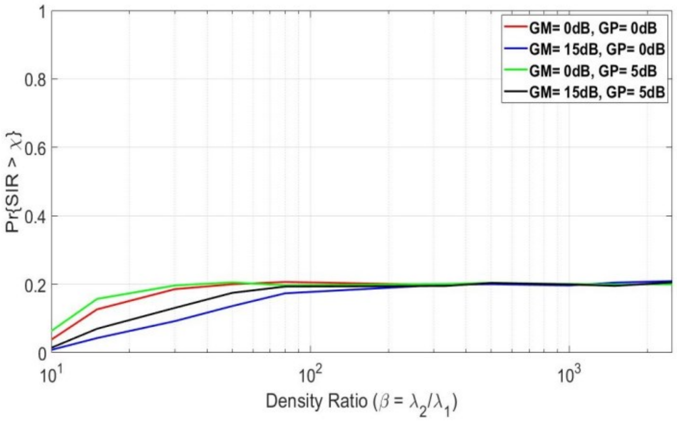

Fig. 7. Coverage at χ = 10 dB under different two-tier BSs antenna gains Vs BSs density ratio, β.

On the other hand, for χ = 10 dB, the coverage results are opposite to the results obtained at χ = 0 dB. The reason is simple as discussed already that when SIR threshold is increased from 0dB to 10 dB, the macro BSs are less populated and are in outage and UEs associate to pico BSs. Thus, when pico BS gain is highest and macro BS gain is lowest, the coverage is highest. Thus, trade-off values for the gains may be considered which are GM_{max} and GP_{max} but at these gains, ASE decreases which will be discussed in the next section.

B. *Area Spectral Efficiency*

Another important performance indicator for analyzing the heterogeneous cellular networks is an area spectral efficiency (ASE) which is the maximum data rate achieved per unit bandwidth and unit area for a randomly located user in bps/Hz/m^2. In this work, ASE has been calculated in bps/Hz/Km^2. The results of ASE have been obtained for the heterogeneous cellular network model shown in Fig. 1 under the simulation parameters shown in Table I.

1) *Impact of Antenna Heights on Area Spectral Efficiency*

The results regarding the impact of macro and pico BS antenna heights on ASE are shown in Fig. 8. It can be observed from the results shown in Fig. 8 that when macro antenna height is maximum (lM_{max}) and pico antenna height is minimum (lP_{min}), area spectral efficiency is highest. The reason is same as discussed in section A(1) that when the density ratio β is in the 5G and UDN range, mostly UEs are associated to small cells and because of no antenna height cap and the interference of the dense network is compensated by the signal power as the intended BS and UE are very close to each other with the densification of the network, an Area spectral efficiency is highest but it degrades to minimum value when small cell antenna height is considered. However, it should be noted that the height of macro BS does not impact the coverage as most of the macro BSs are in outage in the 5G and UDN regime but it has the impact on ASE and macro BS antenna height improves the ASE in 5G and UDN range. Thus, ASE is highest when macro antenna height is maximum and pico antenna height is minimum (equal to UE antenna height).

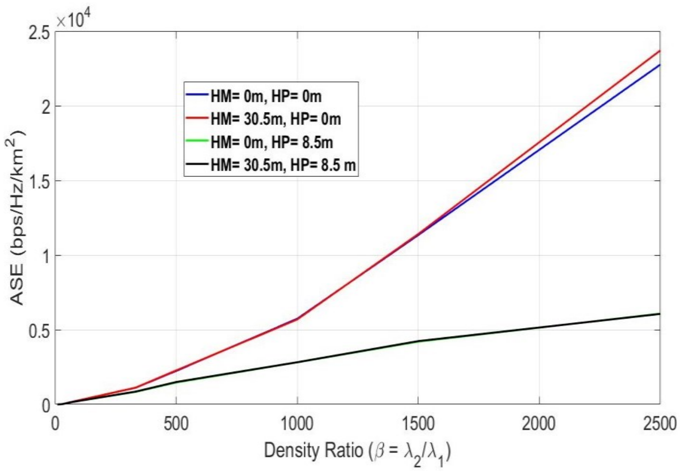

Fig. 8. ASE under different two-tier BSs antenna heights Vs BSs density ratio, β.

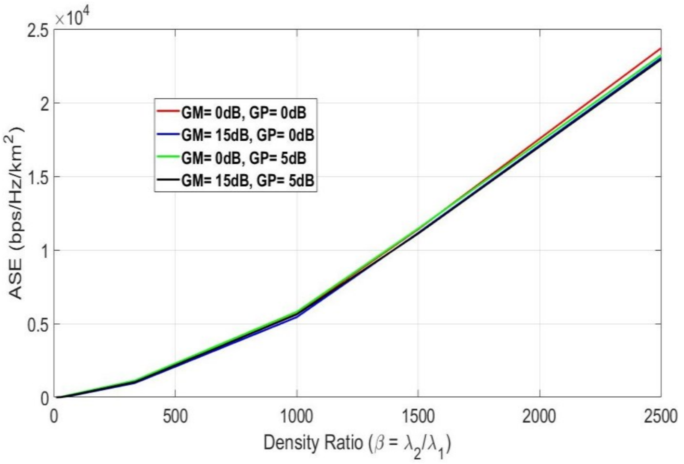

Fig. 9. ASE under different two-tier BSs antenna gains Vs BSs density ratio, β.

2) *Impact of Antenna Gains on Area Spectral Efficiency*

The impact of antenna heights on ASE were acquired where the gains of both macro and pico antennas were considered unity (0dB). From those results, it can be seen that an ASE is highest when the pico cell antenna height is zero meters (equal to UE antenna height) whereas the macro cell antenna height is maximum. Thus, these results where ASE is highest under the maximum macro antenna height and minimum pico antenna height are considered to further examine the impact of antenna gains of large and small cells on ASE.

The results with the impact of antenna gains on ASE are shown in Fig. 9. It should be noted by increasing the gain of any tier reduces the ASE in the 5G and UDN range because by increasing the gains, total number of BSs to be associated by UEs reduces. If both of the tiers' gains are unity then the ASE is highest, however, if the gain of macro tier is considered, the ASE reduces. In addition, if the gain of pico is also considered, the ASE reduces further. Therefore, the effect of ASE reduction enhances if larger gains are to be considered for the reason as discussed above. Thus, gains of both the macro and pico tiers must be unity (0dB) for highest ASE.

Conclusion

In this work, a two-tier heterogeneous cellular network (HCN) was considered to analyze the impact of antenna heights and gains under varying pico to macro base stations density ratio from 2G to 5G and beyond (Ultra-dense network). It was observed that the association of UEs affects the coverage and ASE while changing the BSs antenna heights and gains. It was examined from the simulation results that by considering the maximum macro BS antenna height, pico BS antenna height equal to user equipment (UE) antenna height and unity gains for both macro and pico tiers, the optimum coverage and area spectral efficiency (ASE) for a two-tier fully loaded heterogeneous cellular network can be obtained.

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References

[1] RYSAVY Research, "LTE to 5G: Cellular and Broadband Innovation," *5G Americas white paper*, 2017.
[2] J. Acharya, L. Gao, S. Gaur, "Heterogeneous Networks in LTE-Advanced," John Wiley & Sons, 2014.
[3] H. S. Dhillon, R. K. Ganti, F. Baccelli, J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 30(3), 2012, pp. 550-560.
[4] J. Chen, P. Rauber, D. Singh, C. Sundarraman, P. Tinnakornsrisuphap, M. Yavuz, "Femtocells – Architecture & Network Aspects," Qualcomm, 2010, pp. 1-6.
[5] M. Ghanbarisabagh, G. Vetharatnam, S. M. Giacoumidis, Malayer, "Capacity Improvement in 5G Networks Using Femtocell," *Wireless Personal Communications*, vol. 105, 2019, pp. 1027–1038, https://doi.org/10.1007/s11277-019-06134-2
[6] F. Baccelli, B. Btaszczyszyn, "Stochastic Geometry and Wireless Networks: Volume I: Theory," *Foundations and Trends in Networking*, Hanover, USA, 2009.
[7] M. Haenggi, "Stochastic Geometry for Wireless Networks," *Cambridge University Press*, 2012.
[8] S. N. Chiu, D. Stoyan, W. Kendall, and J. Mecke, "Stochastic Geometry and its applications," *Wiley series in Probability and Statistics*, John Wiley & Sons, 2013.
[9] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, 2011, pp. 3122–3134.
[10] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of K-tier downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, 2012, pp. 550–560.
[11] Y. Deng, L. Wang, M. Elkashlan, M. Di Renzo and J. Yuan, "Modeling and Analysis of Wireless Power Transfer in Heterogeneous Cellular Networks," *IEEE Transactions on Communications*, vol. 64, no. 12, 2016, pp. 5290-5303.
[12] Q. Ye, B. Rong, Y. Chen, M. Al-Shalash, C. Caramanis and J. G. Andrews, "User Association for Load Balancing in Heterogeneous Cellular Networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 6, 2013, pp. 2706-2716.
[13] S. Singh, and H.S. Dhillon, "Offloading in Heterogeneous Networks: Modeling, Analysis, and Design Insights," *IEEE Transactions on Wireless Communications*, vol. 12 (5), 2013, pp. 2484–2497.
[14] W. Wang and G. Shen, "Energy Efficiency of Heterogeneous Cellular Network," *IEEE 72nd Vehicular Technology Conference - Fall*, Ottawa, 2010, pp. 1-5.
[15] X. Chen, J. Wu, Y. Cai, H. Zhang and T. Chen, "Energy-Efficiency Oriented Traffic Offloading in Wireless Networks: A Brief Survey and a Learning Approach for Heterogeneous Cellular Networks," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 4, 2015, pp. 627-640.
[16] X. Li, R. W. Heath Jr., K. Linehan, and R. Butler, "Impact of metro cell antenna pattern and downtilt in heterogeneous networks," arXiv:1502.05782 [cs.IT], 2015. [Online] Available: http://arxiv.org/abs/1502.05782.
[17] L. Xiang, H. Chen, and F. Zhao, "Area Spectral Efficiency and Energy Efficiency Tradeoff in Ultradense Heterogeneous Networks," *Wireless Communications and Mobile Computing, Hindawi*, vol. 2017.
[18] M. Ding and D. Lopez Perez, "Please Lower Small Cell Antenna Heights in 5G," *IEEE Global Communications Conference (GLOBECOM)*, Washington, DC, 2016, pp. 1-6.
[19] M. Ding and D. López-Pérez, "Performance Impact of Base Station Antenna Heights in Dense Cellular Networks," *IEEE Transactions on Wireless Communications*, vol. 16, no. 12, 2017, pp. 8147-8161.
[20] M. M. Shaikh, M. C. Aguayo-Torres, "Joint Uplink/Downlink Coverage and Spectral Efficiency in Heterogeneous Cellular Network," *Springer, Wireless Personal Communications Journal*, 2016, DOI: 10.1007/s11277- 016-3889-1.
[21] M. M. Shaikh, M. C. Aguayo-Torres, "Fairness and Rate Coverage of Symmetric Transmission over Heterogeneous Cellular Networks under Diverse Coupling and Association Criteria," *Springer Wireless Personal Communications Journal*, 2017, DOI: 10.1007/s11277-017-4418-6.
[22] S. Mukherjee, "Analytical Modeling of Heterogeneous Cellular Networks: Geometry, Coverage, and Capacity," Cambridge University Press, 2014.

[23] M. Ding, D. Lopez-Perez, H. Claussen, M. A. Kaafar, "On the Fundamental Characteristics of Ultra-Dense Small Cell Networks," *IEEE Network*, vol. 32, no. 3, 2018, pp. 92-100.

[24] 3GPP, "TR 36.828 V11.0.0: 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Further enhancements to LTE Time Division Duplex (TDD) for Downlink-Uplink (DL-UL) interference management and traffic adaptation (Release 11)," 2012.

[25] 3GPP, "TR 36.814, V2.2.0: 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Further advancements for E-UTRA physical layer aspects," 2017.