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***Brief note***

# HYDROMAGNETIC FLOW PAST AN EXPONENTIALLY ACCELERATED ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION IN THE PRESENCE OF CHEMICAL REACTION OF FIRST ORDER

R. MUTHUCUMARASWAMY*
Department of Applied Mathematics
Sri Venkateswara College of Engineering Sriperumbudur, INDIA
E-mail: msamy@svce.ac.in

V. VALLIAMMAL
Research Scholar of SCSVMV University
KANCHEEPURAM, INDIA

An exact solution of an unsteady flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion in the presence of a transverse magnetic field has been studied. The plate temperature is raised to $T_w$ and the species concentration level near the plate is also made to rise $C'_w$. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration profiles are studied for different physical parameters such as the magnetic field parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number, time and a. It is observed that the velocity decreases with increasing the magnetic field parameter.

**Key words:** accelerated, isothermal, vertical plate, exponential, heat and mass transfer, chemical reaction, magnetic field.

## 1. Introduction

Magnetoconvection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in motion of the earth core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of a transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar *et al*. (1979). MHD effects on an impulsively started vertical infinite plate with variable temperature in the presence of a transverse magnetic field were studied by Soundalgekar *et al*. (1981). MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh (1983). The dimensionless governing equations were solved using the Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar (1984). The skin friction for an accelerated vertical plate was studied analytically by Hossain and Shayo (1986). Gupta (1960) studied the flow of an electrically conducting fluid near an uniformly

* To whom correspondence should be addressed

accelerated vertical plate in the presence of a uniform magnetic field. Raptis *et al*. (1981) discussed MHD flow past an accelerated vertical plate with variable suction and heat flux. Singh (1984) analyzed MHD flow past an exponentially accelerated vertical plate with uniform temperature.

The effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in the solution. Chambre and Young (1958) analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al*. (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on a moving isothermal vertical plate in the presence of chemical reaction were studied by Das *et al*. (1999). The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Hence it is proposed to study these effects on unsteady MHD flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion, in the presence of a homogeneous chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study was found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

## 2. Mathematical formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform mass diffusion in the presence ofa magnetic field has been considered. The $x'$-axis is taken along the plate in the vertically upward direction and the $y'$-axis taken normal to the plate. A transverse magnetic field of uniform strength $B_0$ is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation are assumed to be negligible. At time $t' \leq 0$ the plate and fluid are at the same temperature $T_\infty$. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to $T_w$ and the mass is diffused from the plate to the fluid uniformly. A transverse magnetic field of uniform strength $B_0$ is assumed to be applied normal to the plate. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta\left(T - T_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \tag{2.1}$$

$$\rho C_P \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = \mathrm{D}\frac{\partial^2 C'}{\partial y^2} - k_1 C'. \tag{2.3}$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order $n$, if the reaction rate if proportional to the $n^{th}$ power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

The initial and boundary conditions are as follows

$$u = 0, \qquad T = T_\infty, \qquad C' = C'_\infty \quad \text{for all} \quad y, t' \le 0,$$

$$t' > 0 : u = u_0 \exp(a't'), \quad T = T_w, \qquad C' = C'_w \quad \text{at} \quad y = 0, \tag{2.4}$$

$$u \to 0, \qquad T \to T_\infty, \qquad C' \to C'_\infty \quad \text{as} \quad y \to \infty.$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \qquad t = \frac{t' u_0^2}{\nu}, \qquad Y = \frac{y u_0}{\nu}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\mathrm{Gr} = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \qquad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \qquad \mathrm{Gc} = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \tag{2.5}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}, \qquad a = \frac{a' \nu}{u_0^2}, \qquad \mathrm{Sc} = \frac{\nu}{D}, \qquad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \qquad K = \frac{\mathrm{V} k_1}{\mathrm{v}_0^3},$$

in Eqs (2.1) to (2.4), leads to

$$\frac{\partial U}{\partial t} = \mathrm{Gr}\theta + \mathrm{Gc}C + \frac{\partial^2 U}{\partial Y^2} - MU, \tag{2.6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta}{\partial Y^2}, \tag{2.7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2} - KC. \tag{2.8}$$

The negative sign of $K$ in the last term of Eq.(2.8) indicates that the chemical reaction takes place from higher level of concentration to lower level of concentration.

The initial and boundary conditions in non-dimensional quantities are

$$U = 0, \qquad \theta = 0, \qquad C = 0 \quad \text{for all} \quad Y, \quad t \le 0,$$

$$t > 0: \quad U = \exp(at), \quad \theta = 1, \qquad C = 1 \quad \text{at} \quad Y = 0,$$

$$U \to 0, \qquad \theta \to 0, \qquad C \to 0 \quad \text{as} \quad Y \to \infty.$$

## 3. Solution procedure

The dimensionless governing Eqs (2.6) and (2.8), subject to the boundary conditions (2.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = \operatorname{erfc}(\eta\sqrt{\Pr}), \tag{3.1}$$

$$C = \frac{1}{2}\left[\exp\left(2\eta\sqrt{Kt\mathrm{Sc}}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Kt\mathrm{Sc}}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}} - \sqrt{Kt}\right)\right], \tag{3.2}$$

$$\begin{aligned}
U = {} & \frac{\exp(at)}{2}\Big[\exp\left(2\eta\sqrt{(M+a)t}\cdot \operatorname{erfc}(\eta + \sqrt{(M+a)t}\right) + \\
& \quad +\exp(-2\eta\sqrt{(M+a)t}\cdot \operatorname{erfc}(\eta - \sqrt{(M+a)t})\Big] + \\
& +(d+e)\left[\exp\left(2\eta\sqrt{Mt}\right)\cdot \operatorname{erfc}\left(\eta + \sqrt{Mt}\right) + \exp\left(-2\eta\sqrt{Mt}\right)\cdot \operatorname{erfc}\left(\eta - \sqrt{Mt}\right)\right] + \\
& -d\exp(bt)\Big[\exp\left(2\eta\sqrt{(M+b)t}\right)\cdot \operatorname{erfc}\left(\eta + \sqrt{(M+b)t}\right) + \\
& +\exp\left(-2\eta\sqrt{(M+b)t}\right)\cdot \operatorname{erfc}\left(\eta - \sqrt{(M+b)t}\right)\Big] + \\
& -e\cdot\exp(ct)\Big[\exp\left(2\eta\sqrt{(M+c)t}\right)\cdot \operatorname{erfc}\left(\eta + \sqrt{(M+c)t}\right) + \\
& \quad +\exp\left(-2\eta\sqrt{(M+c)t}\right)\cdot \operatorname{erfc}\left(\eta - \sqrt{(M+c)t}\right)\Big] + \\
& -2d\operatorname{erfc}\left(\eta\sqrt{\Pr}\right) + d\exp(bt)\Big[\exp\left(2\eta\sqrt{\Pr bt}\right)\cdot \operatorname{erfc}\left(\eta\sqrt{\Pr} + \sqrt{bt}\right) + \\
& +\exp\left(-2\eta\sqrt{\Pr bt}\right)\cdot \operatorname{erfc}\left(\eta\sqrt{\Pr} - \sqrt{bt}\right)\Big] + \\
& -e\left[\exp\left(2\eta\sqrt{Kt\mathrm{Sc}}\right)\cdot \operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Kt\mathrm{Sc}}\right)\cdot \operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}} - \sqrt{Kt}\right)\right] + \\
& +e\exp(ct)\Big[\exp\left(2\eta\sqrt{(K+C)t\mathrm{Sc}}\right)\cdot \operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}} + \sqrt{(K+C)t}\right) + \\
& +\exp\left(-2\eta\sqrt{(K+C)tSc}\right)\cdot \operatorname{erfc}\left(-2\eta\sqrt{(K+C)t\mathrm{Sc}}\right)\Big]
\end{aligned} \tag{3.3}$$

where $b = \dfrac{M}{(\Pr - 1)}$, $C = \dfrac{M - K\mathrm{Sc}}{(\mathrm{Sc} - 1)}$, $d = \dfrac{\mathrm{Gr}}{2b(1-\Pr)}$, $e = \dfrac{\mathrm{Gc}}{2c(1-\mathrm{Sc})}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

## 4. Discussion of results

For physical understanding of the problem numerical computations are carried out for different physical parameters $M$, $a$, Gr, Gc, $K$, Sc and $t$ upon the nature of the flow and transport. The values of the Prandtl number Pr are chosen such that they represent water (Pr=*7.0*). The numerical values of the velocity are computed for different physical parameters such as the magnetic field parameter, chemical reaction parameter, $a$, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time. The velocity profile for different values of time ($t$=*0.2*, *0.4*, *0.6*, *0.8*),Sc=*0.6*, Gr = Gc = *2*, $M$ = *0.5*, $K$ = *2* and $a$ = *0.5* are shown in Fig.1. It is observed that velocity increases with increasing the values of time.

Figure 2 demonstrates effects of the magnetic field parameter on velocity when $M$=*0.2*, *2*, *5*, *8*, Sc=*0.6*, Gr=Gc=*5*, $t$ =*0.2*, $K$=*8* and $a$=*0.5*. It is observed that velocity increases with decreasing the magnetic field parameter.

The velocity profile for different values $a$ = *0.2*, *0.5*, *1*, Sc = *0.6*, Gr = Gc = *5*, $M$ = *0.5*, $t$ = *0.2* and $K$ = *8* are studied and presented in Fig.3. It is observed that velocity increases with increasing values of $a$.

Figure 4 represents the effect of concentration profiles for different values of the Schmidt number (Sc = *0.16*, *0.3*, *0.6*, *2.01*), $K$ = *0.2* and $t$ = *1*. The effect of concentration is important in the concentration field. The profile has the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

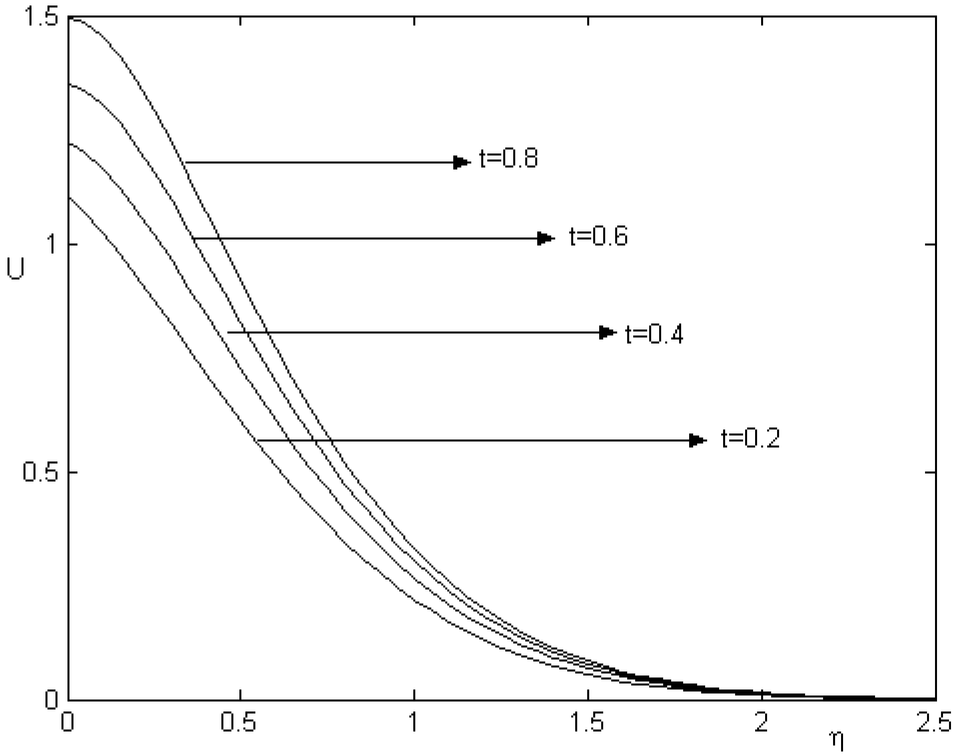


Fig.1. Velocity profiles for different values of *t*.

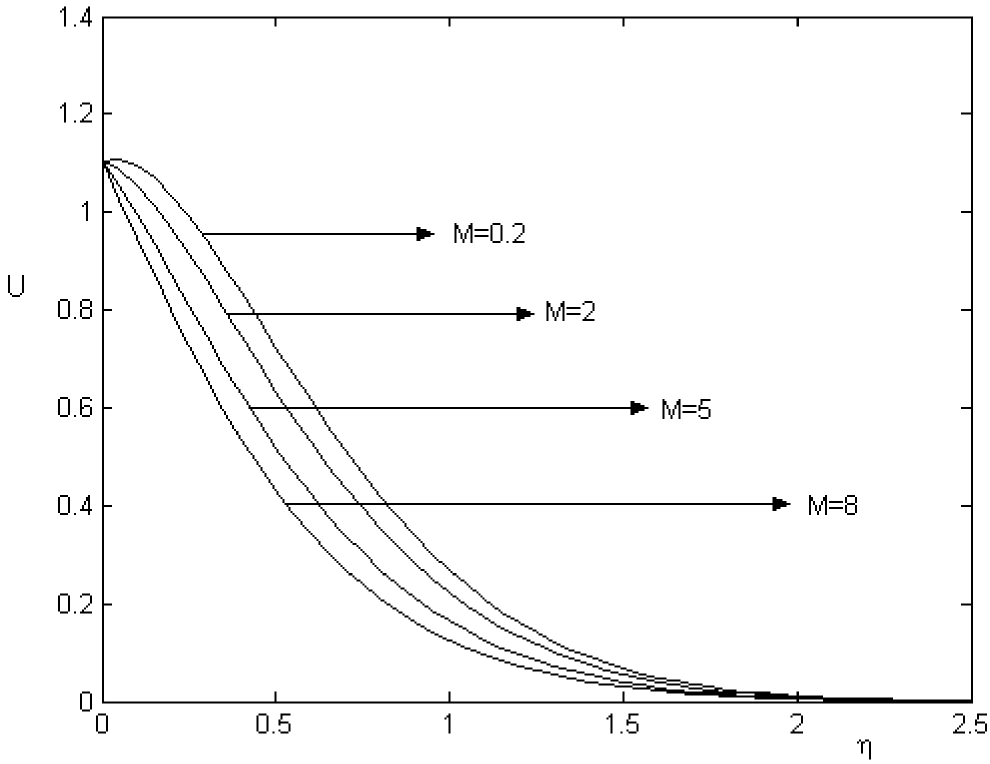


Fig.2. Velocity profiles for different values of *M*.

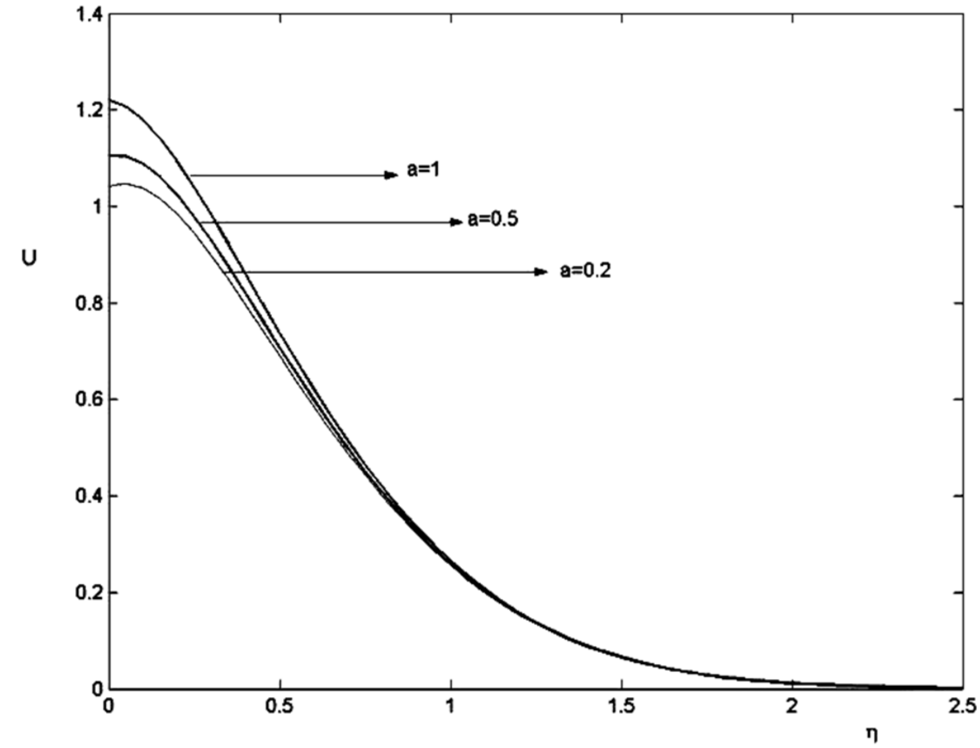


Fig.3. Velocity profiles for different values of *a.*

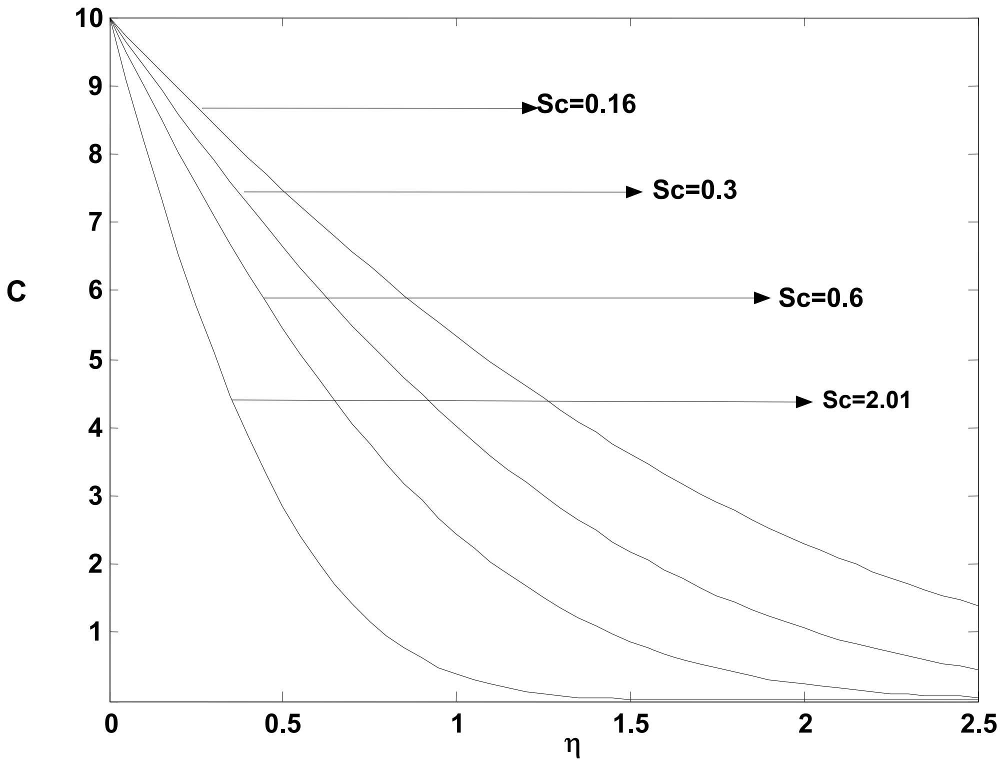


Fig.4. Concentration profiles for different values of Sc.

Figure 5 represents the effect of concentration profiles for different values of the chemical parameter ($K = 0.2, 2, 5, 10$), $t = 0.2$ and Sc = $0.6$. It is observed that the concentration increases with decreasing the chemical reaction parameter.

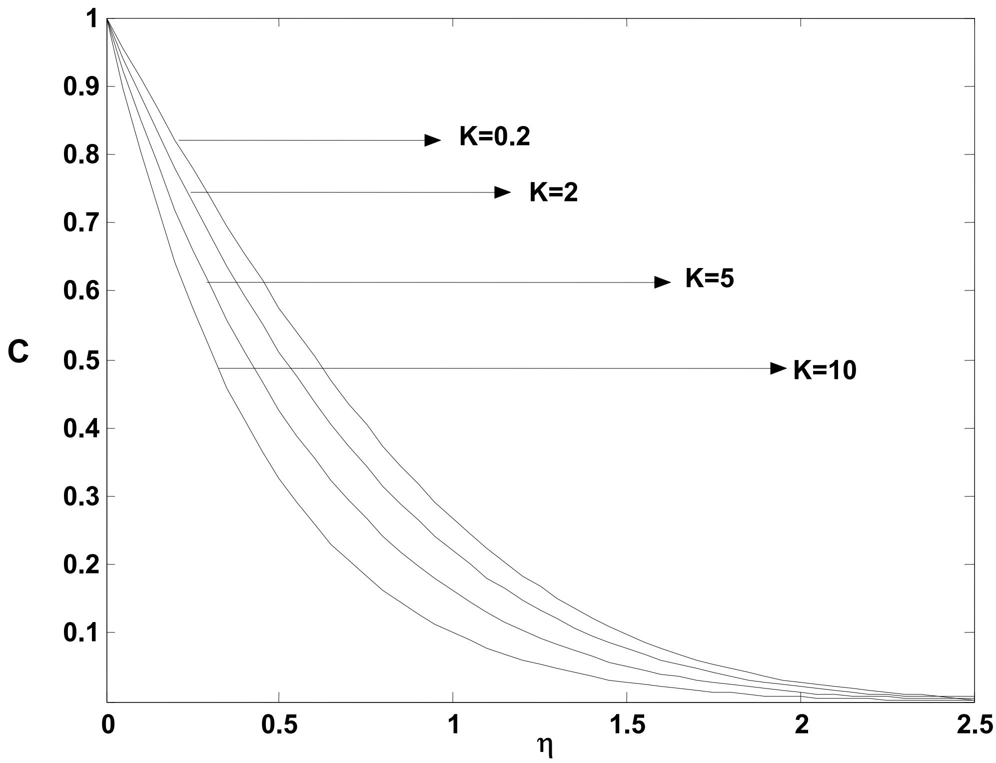


Fig.5. Concentration profiles for different values of *K*.

## 5. Conclusions

An exact analysis of a hydromagnetic flow past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion in the presence of a chemical reaction of first order has been studied. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters such as the magnetic field parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, 'a' and t are studied graphically. The conclusions of the study are as follows:

(i) Velocity decreases with increasing the magnetic field parameter *M*.
(ii) Velocity increases with increasing the values of 'a' and time t in the presence of the magnetic field parameter.
(iii) The wall concentration increases with decreasing the chemical reaction parameter or Sc.

## Nomenclature

$C$ – dimensionless concentration
$C'$ – species concentration in the fluid
$C_p$ – specific heat at constant pressure
$C_w$ – concentration of the plate

$C_\infty$ – concentration in the fluid far away from the plate
$D$ – mass diffusion coefficient
erfc – complementary error function
Gc – mass Grashof number
Gr – thermal Grashof number
$g$ – acceleration due to gravity
$k$ – thermal conductivity
Pr – Prandtl number
Sc – Schmidt number
$T$ – temperature of the fluid near the plate
$T_w$ – temperature of the plate
$T_\infty$ – temperature of the fluid far away from the plate
$t$ – dimensionless time
$t'$ – time
$U$ – dimensionless velocity
$u$ – velocity of the fluid in the $x'$-direction
$u_0$ – velocity of the plate
$Y$ – dimensionless coordinate axis normal to the plate
$y$ – coordinate axis normal to the plate
$\alpha$ – thermal diffusivity
$\beta$ – volumetric coefficient of thermal expansion
$\beta^*$ – volumetric coefficient of expansion with concentration
$\mu$ – cofficient of viscosity
$\nu$ – kinematic viscosity
$\rho$ – density of the fluid
$\tau$ – dimensionless skin-friction
$\theta$ – dimmensionless temperature
$\eta$ – similarity parameter

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