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SCATTERING OF OBLIQUE WATER WAVES BY AN INFINITE STEP

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The present paper is concerned with the problem of scattering of obliquely incident surface water wave train passing over a step bottom between the regions of finite and infinite depth. Havelock expansions of water wave potentials are used in the mathematical analysis to obtain the physical parameters reflection and transmission coefficients in terms of integrals. Appropriate multi-term Galerkin approximations involving ultra spherical Gegenbauer polynomials are utilized to obtain very accurate numerical estimates for reflection and transmission coefficients. The numerical results are illustrated in tables.

Key words: infinite step, Havelock expansion, Galerkin approximation, Gegenbauer polynomial, reflection and transmission coefficients.

1. Introduction

Scattering problems involving fixed vertical thin barriers of various configurations were investigated long back using a variety of mathematical methods (cf. Dean [1], Ursell [2], Evans [3], Porter [4], Mandal and Dolai [5], etc.). The problems of water wave scattering by an irregular bottom have some considerable interest in the literature on linearised theory of water waves due to their importance in finding the effects of naturally occurring bottom obstacles such as sand ripples on the wave motion (cf. Roseau [6], Kreisel [7], Fitz Gerald [8], Hamilton [9], Newman [10], Miles [11], Mandal and Gayen [12], Dolai and Dolai [13]).

Problems involving the propagation of water waves in a fluid of variable depth can be divided into three categories: 'beach' problems, where the depth tends to zero, 'obstacle' problems, where the depth is a constant except for variations extending over a finite interval in space, and 'changing-depth' problems, where the depth changes from one limiting value to another limiting value. There have been many investigations of the beach and obstacle problems (cf. Stoker [14], Wehausen and Laitone [15]), but comparatively few studies have been made of the 'changing depth' case (cf. Bartholomeusz [16], Evans and McIver [17], Newman [10], Dolai [18]). The importance of wave propagation in the case of changing depth is obvious in many coastal situations such as the passage of waves over a continental shelf. As an idealization of such a problem, we consider here the case of wave propagation over a step bottom between the regions of finite and infinite depth.

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2. Formulation of the problem

We consider the motion in an inviscid, homogeneous, incompressible liquid which is supposed confined between the regions of finite and infinite depth. Cartesian axes are chosen on the mean free surface while the (x,z) plane has its origin directly above the step, and the axis of y is directed down wards into the liquid. The shallower water is of finite depth h, the deeper water is of infinite depth. A simple sketch of the problem is given in Fig.1.

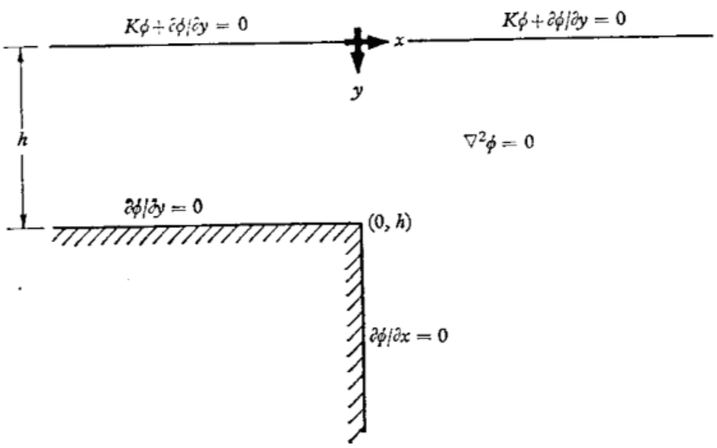

Fig.1. Geometry of the problem.

A simple harmonic progressive oblique wave train originating at $x \to +\infty$ is incident on the step, and is partially reflected and partially transmitted. Assuming linear theory, the time harmonic progressive waves from positive infinity can be represented by the velocity potentials $\operatorname{Re}\left\{\varphi_+^{\text{inc}}(x,y)\exp(i\vartheta z - i\sigma t)\right\}$ where

$$\varphi_+^{\text{inc}}(x,y) = \exp(-Ky - imx), \tag{2.1}$$

with $K = \sigma^2/g$, $\vartheta = K\sin\alpha$, $m = K\cos\alpha$, α being the angle of incidence, σ being the frequency of the incoming waves and g being the gravity. Due to the presence of the step, the oblique incident wave train is partially reflected by the step and partially transmitted through the gap. If the resulting motion is described by the velocity potential $\operatorname{Re}\left\{\varphi(x,y)\exp(i\vartheta z - i\sigma t)\right\}$, then φ satisfies

$$\nabla^2\varphi - \vartheta^2\varphi = 0 \quad \text{in the fluid region}, \tag{2.2}$$

the free surface condition

$$K\varphi + \frac{\partial\varphi}{\partial y} = 0 \quad \text{on} \quad y = 0, \tag{2.3}$$

the bottom conditions

$$\frac{\partial\varphi}{\partial y} = 0 \quad \text{on} \quad y = h, \quad -\infty < x < 0, \tag{2.4}$$

$$\nabla\varphi \to 0 \quad \text{as} \quad y \to \infty, \quad 0 < x < \infty, \tag{2.5}$$

the condition on the step

$$\frac{\partial\varphi}{\partial x} = 0 \quad \text{on} \quad x = 0, \quad h < y < \infty, \tag{2.6}$$

the edge condition

$$r^{1/3}\nabla\varphi \quad \text{is bounded as} \quad r \to 0, \tag{2.7}$$

r is the distance from the edge $(0, h)$, and the infinity condition

$$\varphi(x,y) \to \begin{cases} T_1 \dfrac{\cosh k_0(h-y)}{\cosh k_0 h} \exp(-i\mu x) & \text{as} \quad x \to -\infty \\ \exp(-Ky - imx) + R_1 \exp(-Ky + imx)\} & \text{as} \quad x \to \infty \end{cases} \tag{2.8}$$

where k_0 satisfies $k_0 \tanh k_0 h = K$, $\mu^2 = k_0^2 - \vartheta^2$ and T_1, R_1 are the unknown transmission and reflection coefficients to be determined.

3. Method of solution

Since $\varphi_x(x,y)$ and $\varphi(x,y)$ are continuous across $(0, 0)$ to $(0, h)$, we can write

$$\left(\frac{\partial\varphi}{\partial x}\right)_{x=0+} = \left(\frac{\partial\varphi}{\partial x}\right)_{x=0-} = f(y), \quad \text{say,} \quad \text{for} \quad 0 < y < h, \tag{3.1}$$

$$(\varphi)_{x=0+} = (\varphi)_{x=0-} \quad \text{for} \quad 0 < y < h. \tag{3.2}$$

A solution for $\varphi(x,y)$ satisfying Eqs (2.2), (2.3), (2.4), (2.5) and (2.8) can be represented as

$$\varphi(x,y) \to \begin{cases} T_1 \dfrac{\cosh k_0(h-y)}{\cosh k_0 h} \exp(-i\mu x) + \displaystyle\sum_1^{\infty} B_n \cos k_n(h-y) \exp(s_n x), & x \le 0, \\ \exp(-Ky)\{\exp(-imx) + R_1 \exp(imx)\} + \displaystyle\int_0^{\infty} A(k)(k \cos ky - K \sin ky) \exp(-\xi x)\, dk, & x \ge 0 \end{cases} \tag{3.3}$$

where $s_n^2 = k_n^2 + \vartheta^2, \xi^2 = k^2 + \vartheta^2$, k_n satisfy $k_n \tan k_n h + K = 0$.

Using Eqs (3.3) in Eqs (3.1) and (3.2), we find

$$f(y) = -i\mu T_1 \frac{\cosh k_0(h-y)}{\cosh k_0 h} + \sum_1^\infty s_n B_n \cos k_n (h-y), \ 0 < y < h,$$

$$= im(R_1 - 1)\exp(-Ky) - \int_0^\infty \xi A(k)(k \cos ky - K \sin ky)dk, \quad 0<y<\infty, \tag{3.4}$$

and

$$T_1 \frac{\cosh k_0(h-y)}{\cosh k_0 h} + \sum_1^\infty B_n \cos k_n (h-y) =$$
$$= (1+R_1)\exp(-Ky) + \int_0^\infty A(k)(k \cos ky - K \sin ky)dk, \quad 0<y<h \tag{3.5}$$

Use of Havelock's [19] inversion theorem in Eq.(3.4) produces

$$-i\mu T_1 = \frac{4k_0 \cosh k_0 h}{2k_0 h + \sinh 2k_0 h}\int_0^h f(y)\cosh k_0(h-y)dy, \tag{3.6}$$

$$s_n B_n = \frac{4k_n}{2k_n h + \sin 2k_n h}\int_0^h f(y)\cos k_n(h-y)dy, \tag{3.7}$$

$$m(R_1 - 1) = -2iK\int_0^h f(y)\exp(-Ky)dy, \tag{3.8}$$

$$A(k) = -\frac{2}{\pi\xi(k^2+K^2)}\int_0^h f(y)(k \cos ky - K \sin ky)dy. \tag{3.9}$$

Using Eqs (3.6), (3.7), (3.9) in Eq.(3.5) and from Eq.(3.8), we find

$$\int_0^h F_1(u)M_1(y,u)du = \exp(-Ky), \quad 0 < y < h, \tag{3.10}$$

$$\int_0^h F_1(y)\exp(-Ky)dy = C_1 \tag{3.11}$$

where

$$F_1(u) = \frac{f(u)}{1+R_1}\frac{2K}{m},$$

$$M_1(y,u)=\left[\frac{2ik_0\cosh k_0(h-y)\cosh k_0(h-u)}{\mu(2k_0h+\sinh 2k_0h)}+\sum_1^\infty\frac{2k_n\cos k_n(h-y)\cos k_n(h-u)}{s_n(2k_nh+\sin 2k_nh)}+\right.$$

$$\left.+\frac{1}{\pi}\int_0^\infty\frac{(k\cos ky-K\sin ky)(k\cos ku-K\sin ku)}{\xi(k^2+K^2)}dk\cos\alpha\right.,$$

$$C_1=\frac{i(R_1-1)}{1+R_1}. \tag{3.12}$$

It may be noted that the function $F_1(y)$ and the constant C_1 are real. The integral Eq.(3.10) is to be solved by $(N+1)$ multi-term Galerkin approximations of $F_1(y)$ in terms of ultra spherical Gegenbauer polynomials $C_{2n}^{1/6}(y/h)$ by noting the behavior of $F_1(y)\sim(h-y)^{-1/3}$ as $y\to h-0$ given by (cf. Kanoria *et al.* [20])

$$F_1(y)=\sum_{n=0}^N a_nf_n(y),\quad 0<y<h \tag{3.13}$$

where

$$f_n(y)=-\frac{d}{dy}\exp(-Ky)\int_y^h\exp(Kt)\hat{f}_n(t)dt,\qquad 0<y<h,$$

with

$$\hat{f}_n(y)=\frac{2^{7/6}\Gamma(1/6)(2n)!}{\pi\Gamma(2n+1/3)h^{1/3}\left(h^2-y^2\right)^{1/3}}C_{2n}^{1/6}(y/h).$$

The unknown coefficients a_n $(n=0,1,2,\cdots,N)$ are obtained by solving the system of linear equations

$$\sum_{n=0}^N a_n\Re_{nm}=d_m,\qquad m=0,1,2,\cdots,N \tag{3.14}$$

where

$$\Re_{nm}=\cos\alpha\left[4(-1)^{n+m}\left\{\sum_{r=1}^\infty\frac{2k_r\cos^2 k_rh}{s_r(2k_rh+\sin 2k_rh)}\frac{J_{2n+1/6}(k_rh)J_{2m+1/6}(k_rh)}{(k_rh)^{1/3}}+\right.\right.$$

$$\left.\left.+\frac{1}{\pi}\int_0^\infty\frac{k^2J_{2n+1/6}(kh)J_{2m+1/6}(kh)}{\left(k^2+K^2\right)\left(k^2+\vartheta^2\right)^{1/2}(kh)^{1/3}}dk\right\}+\frac{2ik_0\cosh^2 k_0h}{\mu(2k_0h+\sinh 2k_0h)}\frac{I_{2n+1/6}(k_0h)I_{2m+1/6}(k_0h)}{(k_0h)^{1/3}}\right],$$

$$d_m = \frac{I_{2m+1/6}\left(Kh\right)}{\left(Kh\right)^{1/6}}.$$

Once a_n $(n = 0, 1, 2, \cdots, N)$ are solved, the real constant C_1 can be determined from Eq.(3.11)

$$C_1 = \sum_{n=0}^{N} a_n d_n. \tag{3.15}$$

Then R_1 can be found using Eq.(3.12) and T_1 can be found from Eq.(3.6) using Eq.(3.13) as

$$R_1 = \frac{1 - iC_1}{1 + iC_1}, \tag{3.16}$$

$$T_1 = \frac{iA\cosh k_0 h}{\left(k_0 h\right)^{1/6}} \sum_{n=0}^{N} a_n I_{2n+1/6}\left(k_0 h\right) \tag{3.17}$$

where

$$A = \frac{2k_0\left(1 + R_1\right)\cos\alpha\cosh k_0 h}{\mu\left(\left(2k_0 h + \sinh 2k_0 h\right)\right)}$$

If a simple harmonic progressive oblique wave train originating at $x \to -\infty$ is incident on the step, and is partially reflected and partially transmitted, the time harmonic progressive waves from negative infinity can be represented by the velocity potentials $\mathrm{Re}\left\{\varphi_-^{\mathrm{inc}}\left(x,y\right)\exp(i\vartheta_1 z - i\sigma t)\right\}$ where

$$\varphi_-^{\mathrm{inc}}\left(x,y\right) = \frac{\cosh k_0\left(h - y\right)}{\cosh k_0 h}\exp\left(i\mu_1 x\right), \tag{3.18}$$

with $\vartheta_1 = k_0\sin\alpha, \quad \mu_1 = k_0\cos\alpha$.

If the resulting motion is described by the velocity potential $\mathrm{Re}\left\{\varphi\left(x,y\right)\exp(i\vartheta_1 z - i\sigma t)\right\}$, then φ satisfies

$$\nabla^2\varphi - \vartheta_1^2\varphi = 0 \quad \text{in the fluid region}, \tag{3.19}$$

and the conditions from Eqs (2.3) to (2.7).

The behavior of $\varphi\left(x,y\right)$ at infinity gives

$$\varphi\left(x,y\right) \to \begin{cases} T_2\exp\left(-Ky + im_1 x\right) & \text{as} \quad x \to +\infty \\ \dfrac{\cosh k_0\left(h - y\right)}{\cosh k_0 h}\left\{\exp\left(i\mu_1 x\right) + R_2\exp(-i\mu_1 x)\right\} & \text{as} \quad x \to -\infty \end{cases} \tag{3.20}$$

where $m_1^2 = K^2 - \vartheta_1^2$ and T_2, R_2 are the unknown transmission and reflection coefficients to be determined.

A solution for $\varphi(x,y)$ satisfying Eqs (3.19), (2.3), (2.4), (2.5) and (3.20) can be represented as

$$\varphi(x,y) \to \begin{cases} T_2 \exp(-Ky + im_1 x) + \int_0^\infty D(k)(k\cos ky - K\sin ky)\exp(-\xi_1 x)\,dk, & x \ge 0, \\ \dfrac{\cosh k_0(h-y)}{\cosh k_0 h}\{\exp(i\mu_1 x) + R_2\exp(-i\mu_1 x)\} + \sum_1^\infty C_n \cos k_n(h-y)\exp(s_n' x), & x \le 0 \end{cases} \tag{3.21}$$

where $s_n' = \sqrt{k_n^2 + \vartheta_1^2}, \xi_1 = \sqrt{k^2 + \vartheta_1^2}$.

Using Eq.(3.21) in Eqs (3.1) and (3.2), we find

$$\begin{aligned} f(y) &= im_1 T_2 \exp(-Ky) - \int_0^\infty \xi_1 D(k)\,(k\cos ky - K\sin ky)dk, \quad 0<y<\infty, \\ &= i\mu_1(1-R_2)\frac{\cosh k_0(h-y)}{\cosh k_0 h} + \sum_1^\infty s_n' C_n \cos k_n(h-y), \quad 0<y<h \;. \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} &T_2 \exp(-Ky) + \int_0^\infty D(k)\,(k\cos ky - K\sin ky)dk = \\ &= (1+R_2)\frac{\cosh k_0(h-y)}{\cosh k_0 h} + \sum_1^\infty C_n \cos k_n(h-y), \quad 0<y<h \;. \end{aligned} \tag{3.23}$$

Use of Havelock's [19] inversion theorem in Eq.(3.22) produces

$$T_2 = -\frac{2iK}{m_1}\int_0^h f(y)\exp(-Ky)\,dy \,, \tag{3.24}$$

$$s_n' C_n = \frac{4k_n}{2k_n h + \sin 2k_n h}\int_0^h f(y)\cos k_n(h-y)dy \,, \tag{3.25}$$

$$i\mu_1(1-R_2) = \frac{4k_0 \cosh k_0 h}{2k_0 h + \sinh 2k_0 h}\int_0^h f(y)\cos k_0(h-y)\,dy \,, \tag{3.26}$$

$$D(k) = -\frac{2}{\pi\xi_1\left(k^2 + K^2\right)}\int_0^h f(y)(k\cos ky - K\sin ky)dy \;. \tag{3.27}$$

Using Eqs (3.24), (3.25), (3.27) in Eq.(3.23) and from Eq.(3.26), we find

$$\int_0^h F_2(u) M_2(y,u)\,du = \frac{\cosh k_0(h-y)}{\cosh k_0 h}, \quad 0<y<h, \tag{3.28}$$

$$\int_0^h F_2(y)\frac{\cosh k_0(h-y)}{\cosh k_0 h}\,dy = C_2 \tag{3.29}$$

where

$$F_2(u) = -\frac{f(u)}{1+R_2}\frac{4k_0\cosh^2 k_0 h}{\mu_1(2k_0h+\sinh 2k_0h)},$$

$$M_2(y,u) = \frac{\mu_1(2k_0h+\sinh 2k_0h)}{k_0\cosh^2 k_0h}\left[\frac{iK}{2m_1}\exp(-K\{y+u\})+\right. \tag{3.30}$$

$$\left.+\sum_1^\infty \frac{k_n\cos k_n(h-y)\cos k_n(h-u)}{s_n'(2k_nh+\sin 2k_nh)}+\frac{1}{2\pi}\int_0^\infty \frac{(k\cos ky-K\sin ky)(k\cos ku-K\sin ku)}{\xi_1(k^2+K^2)}dk\right],$$

$$C_2 = -i\frac{1-R_2}{1+R_2}.$$

It may be noted that the function $F_2(y)$ and the constant C_2 are real. The integral Eq.(3.28) is to be solved by $(N+1)$ multi-term Galerkin approximations of $F_2(y)$ in terms of ultra spherical Gegenbauer polynomials $C_{2n}^{1/6}(y/h)$ by noting the behavior of $F_2(y)\sim(h-y)^{-1/3}$ as $y\to h-0$ given by (cf. Kanoria *et al*. [20])

$$F_2(y) = \sum_{n=0}^N a_n' f_n(y), \quad 0<y<h$$

where

$$f_n(y) = -\frac{d}{dy}\exp(-Ky)\int_y^h \exp(Kt)\hat{f}_n(t)\,dt, 0<y<h,$$

with

$$\hat{f}_n(y) = \frac{2^{7/6}\Gamma(1/6)(2n)!}{\pi\Gamma(2n+1/3)h^{1/3}(h^2-y^2)^{1/3}}C_{2n}^{1/6}(y/h).$$

The unknown coefficients a_n' $(n=0,1,2,\cdots,N)$ are obtained by solving the system of linear equations

$$\sum_{n=0}^{N} a_n^{'} \mathfrak{R}_{nm}^{'} = d_m^{'}, \qquad m = 0, 1, 2, \cdots, N$$

where

$$\mathfrak{R}_{nm}^{'} = \frac{\cos\alpha\left(2k_0 h + \sinh 2k_0 h\right)}{\cosh^2 k_0 h}\left[4\left(-1\right)^{n+m}\left\{\sum_{r=1}^{\infty}\frac{4k_r\cos^2 k_r h}{s_r^{'}(2k_r h + \sin 2k_r h)}\frac{J_{2n+1/6}\left(k_r h\right)J_{2m+1/6}\left(k_r h\right)}{\left(k_r h\right)^{1/3}} + \right.\right.$$

$$\left.\left. + \frac{2}{\pi}\int_0^{\infty}\frac{k^2 J_{2n+1/6}\left(kh\right)J_{2m+1/6}\left(kh\right)}{\left(k^2+K^2\right)\left(k^2+\vartheta_1^2\right)^{1/2}\left(kh\right)^{1/3}}dk\right\} + \frac{iK}{2m_1}\frac{I_{2n+1/6}\left(Kh\right)I_{2m+1/6}\left(Kh\right)}{\left(Kh\right)^{1/3}}\right],$$

$$d_m^{'} = \frac{I_{2m+1/6}\left(k_0 h\right)}{\left(k_0 h\right)^{1/6}}.$$

Once $a_n^{'}(n = 0, 1, 2, \cdots, N)$ are solved, the real constant C_2 can be determined from Eq.(3.29)

$$C_2 = \sum_{n=0}^{N} a_n^{'} d_n^{'} . \tag{3.33}$$

Then R_2 can be found using Eq.(3.30) and T_2 can be found from Eq.(3.24) using Eq.(3.31) as

$$R_2 = \frac{i + C_2}{i - C_2}, \tag{3.34}$$

$$T_2 = \frac{iBK\cos\alpha}{\left(Kh\right)^{1/6}}\sum_{n=0}^{N} a_n^{'} I_{2n+1/6}\left(Kh\right) \tag{3.35}$$

where

$$B = \frac{\left(1 + R_2\right)\left(2k_0 h + \sinh 2k_0 h\right)}{2m_1\cosh^2 k_0 h}.$$

4. Numerical results

Multi-term Galerkin approximations are used to obtain the numerical estimate for $|R_1|, |T_1|$ and $|R_2|, |T_2|$. In the numerical computations we take at most six terms to produce fairly accurate numerical estimates for $|R_1|, |T_1|$ and $|R_2|, |T_2|$.

We display a representative set of numerical estimates for $|R_1|, |T_1|$ and $|R_2|, |T_2|$ in Tabs 1 and 2, taking $N = 0$, 1, 2, 3, 4 and 5 in the $\left(N + 1\right)$ - term Galerkin approximations and some particular values of the different parameters.

It is observed from Tabs 1 and 2 that the computed results for $|R_1|, |T_1|$ and $|R_2|, |T_2|$ converge very rapidly with N, and for $N \geq 3$ an accuracy of almost six decimal places is observed. It appears that the present method numerical procedure for the numerical computations of reflection and transmission coefficients is quite efficient. We also note from these tables that for normal incidence of the wave train $\left(\alpha = 0^0\right)$, $|R_1| = |R_2| = |R|$ and $|T_1 T_2| = 1 - |R|^2$. Similar observations were pointed out by Newman [10]. For normal incidence of the surface wave train, the results are compared with Newman [10] results and a very good agreement is achieved.

Table 1.

$Kh = 0.5, \alpha = 0^0$				
	$\|R_1\|$	$\|R_2\|$	$\|T_1\|$	$\|T_2\|$
0	0.499321	0.499321	0.831563	0.913674
1	0.497135	0.497135	0.830125	0.912582
2	0.495763	0.495763	0.829743	0.909163
3	0.495754	0.495754	0.829685	0.909054
4	0.495754	0.495754	0.829685	0.909054
5	0.495754	0.495754	0.829685	0.909054
$Kh = 0.5, \alpha = 30^0$				
N	$\|R_1\|$	$\|R_2\|$	$\|T_1\|$	$\|T_2\|$
0	0.431136	0.399764	0.808523	1.02439
1	0.430171	0.399432	0.804461	1.02356
2	0.429813	0.398671	0.801358	1.021435
3	0.429686	0.398543	0.801196	1.021104
4	0.429686	0.398542	0.801196	1.021103
5	0.429686	0.398542	0.801196	1.021103

Table 2.

$Kh = 1.5, \alpha = 0^0$				
N	$\|R_1\|$	$\|R_2\|$	$\|T_1\|$	$\|T_2\|$
0	0.091135	0.091135	0.929412	1.074878
1	0.090356	0.090356	0.928354	1.074233
2	0.089934	0.089934	0.925132	1.072554
3	0.089842	0.089842	0.924971	1.072389
4	0.089842	0.089842	0.924971	1.072389
5	0.089842	0.089842	0.924971	1.072389
$Kh = 1.5, \alpha = 30^0$				
N	$\|R_1\|$	$\|R_2\|$	$\|T_1\|$	$\|T_2\|$
0	0.054351	0.036159	0.907255	1.138113
1	0.053124	0.034862	0.904326	1.135326
2	0.051226	0.034455	0.901538	1.134623
3	0.051013	0.034398	0.901384	1.134539
4	0.051013	0.034397	0.901383	1.134539
5	0.051013	0.034397	0.901383	1.134539

5. Conclusion

The method of multi-term Galerkin approximations in terms of ultra spherical Gegenbauer polynomials has been utilized here to obtain very accurate numerical estimates for the reflection and transmission coefficients in the water wave scattering problem of obliquely incident surface wave train on an obstacle in the form of a step between the regions of finite and infinite depth. By choosing only five terms in the Galerkin approximations, we achieve almost six figure accuracy in the numerical estimates for the reflection and transmission coefficients. The numerical results are illustrated in tables. For normal incidence of the surface wave train, the results are compared with the known results available in the literature and a very good agreement is achieved.

Nomenclature

g – gravity
h – depth of the shallow water
K – wave number
R_1, R_2 – reflection coefficients
T_1, T_2 – transmission coefficient
t – time
x – horizontal distance
y – vertical distance
φ – velocity potential
σ – wave frequency

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