Analysis of vibration of printing unit of offset printing press

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There are several causes of vibration of offset printing presses. Major are cyclic work of feeders and deliveries, gaps with mechanisms for fixing plate and blanket in cylinders of printing pair and inappropriate stress between those cylinders. All mentioned above causes increased exploitation of bearings, gears driving cylinders and other parts of the printing press. The consequences of vibration are also breaking of paper web and such artefacts as doubling and streaking on the prints.

In this paper printing pair cylinders of lithographic printing press are described as a two-degree of freedom model. Solution of a system of differential equations, which describes this model shows influence of vibration of printing unit cylinders on the fluctuation of the ink film thickness on the prints (magnitude of streaking phenomenon).

Keywords and phrases: vibration, lithographic printing, offset printing, offset presses, printing unit, plate cylinder, blanket cylinder.

Introduction

Lithographic printing (offset printing) is one of the most popular printing technique. It is used for printing leaflets, posters, catalogues, newspapers and magazines.

The most important element of every lithographic printing press is printing unit, which basically consists of three cylinders: plate cylinder, offset blanket and impression cylinder.

While printing ink is put onto image, which lays on the printing plate (on the printing cylinder). After that, inked picture is transferred to the rubber blanket cylinder, where goes to the printing substrate from. Substrate is spread on the impression cylinder.

Lithographic printing presses full of moving masses, which may induce vibrations. To the most meaningful causes of vibrations of offset printing machines belongs elements of driving unit, cylinders of printing unit, rollers of inking and dampening units, feeders, deliveries and sheets transporters.

Vibrations increase dynamic loads and are the reasons of abnormally fast wear of bearings, gears and other parts of machine. It worsens precision of work of printing presses.

As the result of poor condition of printing press there may occur phenomenon called doubling on the prints. Usually it takes place in machines with several printing units. Ink film put on the printing substrate (paper) in the previous printing unit, in result of back-split, is partially transferred to the blanket of the following one. After that it goes once again on the substrate, but on the next sheet or section (on web-fed presses). Such transfer of ink in offset printing presses is normal phenomenon. However, worsening condition of bearings and gears disturbs synchronization of work of cylinders in consecutive printing units and systems transporting paper sheets. In the result of those disturbances, an image created by back-split of ink is misaligned with an image on the next sheet (section). The image is doubled, what reasonably decreases quality of the print.

Next serious problem caused by vibrations is phenomenon called streaking (banding). It is caused by radial displacement of plate and blanket cylinder in printing unit and fluctuation of ink film thickness in consequence of it. Both cylinders have canals with mechanisms for fixing plate and blanket, respectively.

While rolling those canals (Fig. 1) each other, it comes to sudden change of pressure between cylinders. It excites radial vibrations of the cylinders and there may appear stripes collateral to the cylinders axes on the prints.

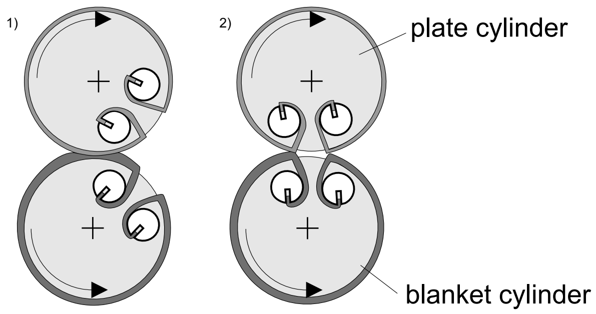

Fig. 1. Plate and blanket cylinders at the moment of canals rolling.

Vibrations of web-fed offset printing presses may also cause creasing and breaking of paper web, what seriously reduces productivity of the machines.

Model development

Printing cylinders are modelled as two-degree of freedom system, which consists of two masses m_1, m_2, three springs k_1, k_2, k_{12} and three dampers c_1, c_2, c_{12} (Fig. 2).

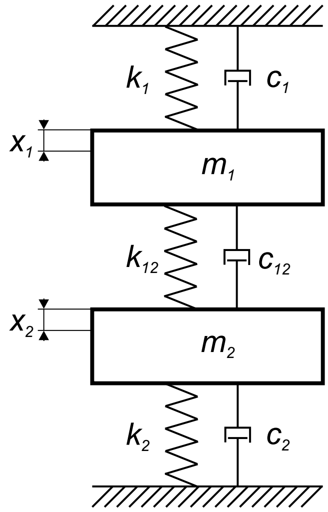

Fig. 2. Model of cylinders of printing unit.

When exciting force $F_0h(t)$ is applied, model can be described by a system of differential equations:

$$\begin{cases} m_1\ddot{x}_1+(c_1+c_{12})\dot{x}_1+(k_1+k_{12})x_1-c_{12}\dot{x}_2-k_{12}x_2=F_0h(t) \\ m_2\ddot{x}_2+(c_2+c_{12})\dot{x}_2+(k_2+k_{12})x_2-c_{12}\dot{x}_1-k_{12}x_1=0 \end{cases} \quad (1)$$

where: m_1, m_2 — masses of plate and blanket cylinders, respectively,
c_1, c_2, c_{12} — dampening coefficients of dampers,
k_1, k_2, k_{12} — spring constants,
$h(t)$ — applied nondimential exciting force,
F_0 — maximal magnitude of exciting force.

We introduce new parameters to the system of equations (1):

$$2h_1=\frac{c_1+c_{12}}{m_1},\ 2h_2=\frac{c_2+c_{12}}{m_2},\ \omega_1^2=\frac{k_1+k_{12}}{m_1},$$
$$\omega_2^2=\frac{k_2+k_{12}}{m_2},\ 2h_{01}=\frac{c_1}{m_1},\ 2h_{02}=\frac{c_2}{m_2},$$
$$\omega_{01}^2=\frac{k_1}{m_1},\ \omega_{02}^2=\frac{k_2}{m_2},\ 2h_{10}=\frac{c_{12}}{m_1} \quad (2)$$
$$2h_{20}=\frac{c_{12}}{m_2},\ \omega_{10}^2=\frac{k_{12}}{m_1},\ \omega_{20}^2=\frac{k_{12}}{m_2}.$$

Then we obtain an expression:

$$\begin{cases} \ddot{x}_1+2h_1\dot{x}_1+\omega_1^2x_1-2h_{10}\dot{x}_2-\omega_{10}^2x_2=\dfrac{F_0}{m_1}h(t) \\ \ddot{x}_2+2h_2\dot{x}_2+\omega_2^2x_2-2h_{20}\dot{x}_1-\omega_{20}^2x_1=0 \end{cases} \quad (3)$$

Introducing to the system (1) new state variables:

$$x_1=z_1,\ \dot{x}_1=z_2,\ x_2=z_3,\dot{x}_2=z_4 \quad (4)$$

the system of four equations is formed. It can be written as follows:

$$\dot{\mathbf{z}}(t)=\hat{\mathbf{A}}\mathbf{z}(t)+\mathbf{b}(t) \quad (5)$$

where: $\mathbf{z}(t)$ — vector of state,
$\hat{\mathbf{A}}$ — matrix of a system

$$\hat{\mathbf{A}}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(\omega_{01}^2+\omega_{10}^2) & -(2h_{01}+2h_{10}) & \omega_{10}^2 & 2h_{10} \\ 0 & 0 & 0 & 1 \\ \omega_{20}^2 & 2h_{20} & -(\omega_{02}^2+\omega_{20}^2) & -(2h_{02}+2h_{20}) \end{bmatrix}$$

$$\mathbf{z}(t)=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \qquad \mathbf{b}(t)=\begin{bmatrix} 0 \\ \dfrac{F_0h(t)}{m_1} \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

In case when $c_1=c_2, k_1=k_2$ and $m_1=m_2$, we have:

$$2h_0=2h_{01}=2h_{02},\ \omega_0^2=\omega_{01}^2=\omega_{02}^2,$$
$$2h_{00}=2h_{10}=2h_{20},\ \omega_{00}^2=\omega_{10}^2=\omega_{20}^2 \quad (7)$$

and then matrix $\hat{\mathbf{A}}$ can be written in the following way:

$$\hat{\mathbf{A}}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(\omega_0^2+\omega_{00}^2) & -(2h_0+2h_{00}) & \omega_{00}^2 & 2h_{00} \\ 0 & 0 & 0 & 1 \\ \omega_{00}^2 & 2h_{00} & -(\omega_0^2+\omega_{00}^2) & -(2h_0+2h_{00}) \end{bmatrix} \quad (8)$$

Using Laplace transform for state variables ($\mathbf{Z}(s)=L[\mathbf{z}(t)$, $\mathbf{B}(s)=L[\mathbf{b}(t)]$), we obtain:

$$s\mathbf{Z}(s)=\hat{\mathbf{A}}\mathbf{Z}(s)+\mathbf{B}(s)+\mathbf{z}_0 \quad (9)$$

where: $\mathbf{z}_0$ — initial conditions.

Equation (9) is transformed to the form:

$$(s\hat{\mathbf{I}} - \hat{\mathbf{A}})\mathbf{Z}(s) = \mathbf{B}(s) + \mathbf{z}_0 \tag{10}$$

where $\hat{\mathbf{I}}$ is an identity matrix of order four.

If matrix $s\hat{\mathbf{I}} - \hat{\mathbf{A}}$ is not a singular one (determinant $|s\hat{\mathbf{I}} - \hat{\mathbf{A}}|$ is not equal to zero), there exists matrix $(s\hat{\mathbf{I}} - \hat{\mathbf{A}})^{-1}$. Equation (10) can be then written as follows:

$$\mathbf{Z}(s) = (s\hat{\mathbf{I}} - \hat{\mathbf{A}})^{-1}\mathbf{B}(s) + (s\hat{\mathbf{I}} - \hat{\mathbf{A}})^{-1}\mathbf{z}_0 \tag{11}$$

When we calculate $\mathbf{Z}_S$ from equation (11), using inverse Laplace transform we can obtain vector of state $\mathbf{z}(t)$. However, this manner needs calculating of matrix $(s\hat{\mathbf{I}} - \hat{\mathbf{A}})^{-1}$. To solve this problem, we use well-known equation:

$$e^{\hat{\mathbf{A}}t} = L^{-1}\left[(s\hat{\mathbf{I}} - \hat{\mathbf{A}})^{-1}\right] \tag{12}$$

Then we obtain:

$$\begin{aligned} \mathbf{z}(t) &= e^{\hat{\mathbf{A}}t} * \mathbf{b}(t) + e^{\hat{\mathbf{A}}t}\mathbf{z}_0 = \\ &= e^{\hat{\mathbf{A}}t}\mathbf{z}_0 + \int_0^t e^{\hat{\mathbf{A}}(t-\tau)}\mathbf{b}(\tau)d\tau \end{aligned} \tag{13}$$

Matrix $e^{\hat{\mathbf{A}}t}$ can be calculated from Sylvester's equation:

$$e^{\hat{\mathbf{A}}t} = \sum_{k=1}^{4} \hat{\mathbf{B}}_k e^{s_k t} \qquad \hat{\mathbf{B}}_k = \prod_{\substack{j=1 \\ j\neq k}}^{4} \frac{\hat{\mathbf{A}} - s_j\hat{\mathbf{I}}}{s_k - s_j} \tag{14}$$

where s_1, s_2, s_3, s_4 are eigenvalues of matrix $\hat{\mathbf{A}}$ or solutions of equation

$$\left|s\hat{\mathbf{I}} - \hat{\mathbf{A}}\right| = 0 \tag{15}$$

The fourth degree characteristic equation in general can be written in the following way:

$$s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \tag{16}$$

where: $a_4 = 1 \quad a_3 = 2(h_{01} + h_{02} + h_{10} + h_{20})$,

$$\begin{aligned} a_2 &= 4h_{02}h_{10} + 4h_{01}h_{02} + 4h_{01}h_{20} + \omega_{01}^2 + \\ &+ \omega_{02}^2 + \omega_{10}^2 + \omega_{20}^2 \\ a_1 &= h_{20}\omega_{01}^2 + h_{01}\omega_{02}^2 + h_{10}\omega_{02}^2 + h_{02}\omega_{01}^2 + \\ &+ h_{02}\omega_{10}^2 + h_{01}\omega_{20}^2 \\ a_0 &= \omega_{01}^2\omega_{02}^2 + \omega_{01}^2\omega_{20}^2 + \omega_{10}^2\omega_{02}^2 + \omega_{10}^2\omega_{20}^2 - \\ &- \omega_{10}^2\omega_{20}^2 \end{aligned}$$

are coefficients of characteristic equation.

Solutions of equation (16) have complicated analytical form, but they can be easily calculated numerically for concrete parameters of vibrating system.

In case when $c_1 = c_2$, $k_1 = k_2$, $m_1 = m_2$ characteristic equation can be presented in the form (16), where:

$$\begin{aligned} a_4 &= 1,\ a_3 = 4(h_0 + h_{00}) \\ a_2 &= 4h_0^2 + 8h_0h_{00} + 2\omega_0^2 + 2\omega_{00}^2 \\ a_1 &= 4h_0\omega_0^2 + 4h_{00}\omega_0^2 + 4h_0\omega_{00}^2 \end{aligned}$$

Then we can write equation as follows:

$$\begin{aligned} &(s^2 + 2h_0 s + \omega_0^2)(s^2 + 2(h_0 + 2h_{00})s + \\ &+ \omega_0^2 + 2\omega_{00}^2) = 0 \end{aligned} \tag{17}$$

In this case, solutions of equation (17) can be presented in the form:

$$\begin{aligned} s_{1,2} &= -h_0 \pm i\sqrt{\omega_0^2 - h_0^2} \\ s_{3,4} &= -h_0 - 2h_{00} \pm i\sqrt{\omega_0^2 + 2\omega_{00}^2 - (h_0 + 2h_{00})^2} \end{aligned} \tag{18}$$

Real parts of these solutions represent damping coefficients of eigenvibrations of a system, whereas imaginary parts represent its eigenfrequencies.

$$\begin{aligned} T_1 &= \frac{2\pi}{\sqrt{\omega_0^2 - h_0^2}} \\ T_3 &= \frac{2\pi}{\sqrt{\omega_0^2 + 2\omega_{00}^2 - (h_0 + 2h_{00})^2}} \end{aligned} \tag{19}$$

For excitation force an integral (13) was calculated as follows:

$$\int_0^t e^{s_k(t-\tau)} e^{i\omega\tau} d\tau = \frac{e^{i\omega t} - e^{s_k t}}{(-s_k + i\omega)} \tag{20}$$

Solution was written in the form:

$$\begin{aligned} &\mathbf{z}(t) = \sum_{k=1}^{4} \hat{\mathbf{B}}_k e^{s_k t}\mathbf{z}_0 + \int_0^t \sum_{k=1}^{4} \hat{\mathbf{B}}_k e^{s_k(t-\tau)}\mathbf{b}(\tau)d\tau = \\ &\sum_{k=1}^{4} \hat{\mathbf{B}}_k e^{s_k t}\mathbf{z}_0 + \sum_{k=1}^{4} \hat{\mathbf{B}}_k \mathbf{b}_k(t), \end{aligned} \tag{21}$$

where vector $\mathbf{b}_k(t)$ has only one element different from zero.

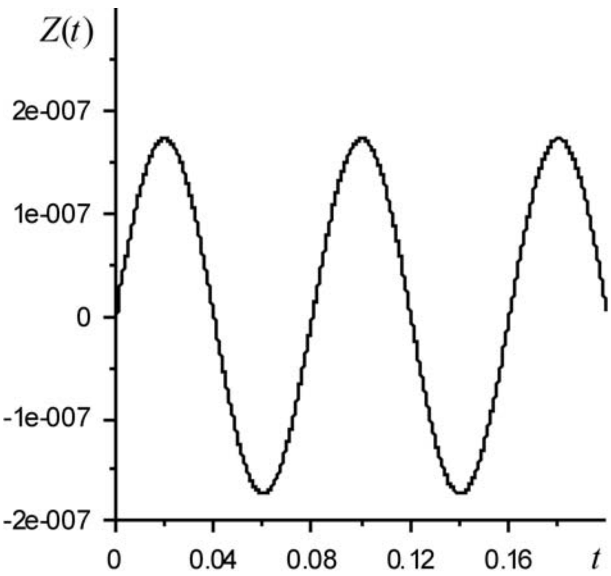

Fig. 3. Fluctuation of ink film thickness as a result of cylinder vibrations for excitation h(t) = sinωt.

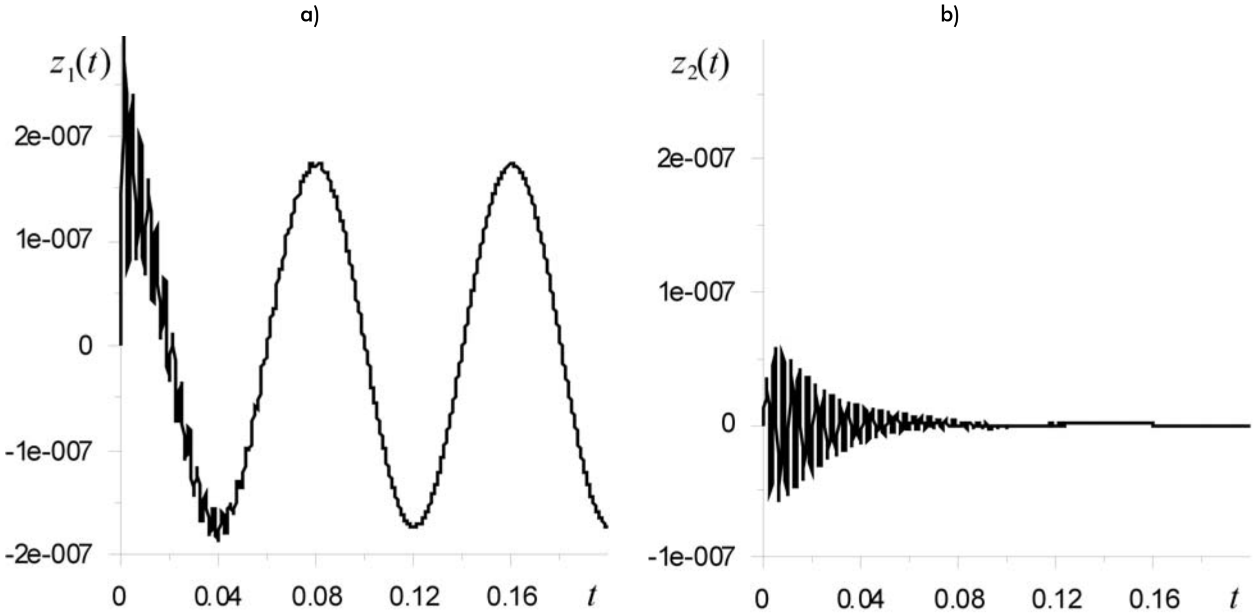

Fig. 4. Cylinders displacement z1(t) (a) and z2(t) (b) for excitation h(t) = sinωt.

$$\mathbf{b}_k(t) = \begin{bmatrix} 0 \\ \dfrac{F_0}{m_1} \dfrac{e^{i\omega t} - e^{s_k t}}{(-s_k + i\omega)} \\ 0 \\ 0 \end{bmatrix} \qquad (22)$$

Real part Re[$\mathbf{z}(t)$] of solution (21) corresponds to real part of excitation Re[$h(t)$] = Re[$e^{i\omega t}$] = cos(ωt) and the imaginary part Im[$\mathbf{z}(t)$] corresponds to excitation Im[$h(t)$] = sin(ωt).

Numerical analysis was pursued for parameters:

$$\omega = 78{,}5\,\mathrm{s}^{-1},\ m_1 = m_2 = 210\ \mathrm{kg},$$
$$k_1 = k_2 = 750\cdot 10^6\,\mathrm{N\,m}^{-1}, k_{12} = 7{,}5\cdot 10^4\,\mathrm{N\,m}^{-1},$$
$$c_1 = c_2 = 1.9\cdot 10^4\ \mathrm{N\,s^2 m}^{-1},$$
$$c_{12} = 7.5\cdot 10^4\,\mathrm{N\,s^2 m}^{-1},\ \ F_0 = 129{,}5\,\mathrm{kg\ m\ c}^{-2}$$

There is presented an evolution of magnitude for excitation force (Fig. 3).

As the result of cylinders vibrations, particularly in consequence of varying distance between their axes, it comes to fluctuations of ink film thickness transferred between them and to the printing substrate. If those fluctuations are smaller than ±2 μm (1 μm = 10^{-6} m), they will be unseenable for human eye. In our case ink film thickness varies in range of , so the cross stripes on the prints will not be noticeable.

Using equations (18), we calculated solutions of characteristic equation and periods of vibrations:

$$s_{1,2} = -45.24 \pm i1889.28$$
$$s_{3,4} = -402.38 \pm i1846.68$$
$$T_1 = 0.0033\,[s]\ \ T_3 = 0.0034\,[s]$$

They can be observed in Fig. 4.

Summary

There is model presented in this paper, which shows influence of vibrations of plate and blanket cylinders on fluctuations of ink film thickness transferred to the printing substrate. Solution of system of differential equations describing that model shows that exciting force $h(t)$ = sin(ωt) by analyzed set of parameters will not cause big enough fluctuations of ink film thickness, so the streaking phenomenon would be noticeable to human eye.

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