ARCHIVES OF MECHANICS
Arch. Mech. **73** (3), 237–251, 2021, DOI: 10.24423/aom.3764

Wave propagation in nonlocal orthotropic micropolar elastic solids

D. X. TUNG

Faculty of Civil Engineering, Hanoi Architectural University, Km 10 Nguyen Trai Str., Thanh Xuan, Hanoi, Vietnam, e-mail: tungdx2783@gmail.com

The presented paper is concerned with the propagation of Rayleigh waves in an orthotropic nonlocal micropolar elastic half-space. The main aim of the paper is to derive dispersion equations of Rayleigh wave as well as Stroh formalism for the orthotropic nonlocal micropolar medium. Based on the obtained dispersion equation, the effect of material, nonlocality parameter on the Rayleigh wave propagation is considered through some numerical examples.

Key words: dispersion equation, nonlocal, local, micropolar, orthotropic.

1. Introduction

The nonlocal theories of elasticity have been proposed by Eringen and Edelen [1, 2], which state that the stress at any reference point within a continuous body is not only a function of strain at that point, but also a function of the strain fields at all other points of the body. If L denotes the external characteristic length and l the internal characteristic length, then in the region $L/l \gg 1$, classical field theories predict sufficiently accurate results. On the other hand, when $L/l \approx 1$, local theories fail and we must use either atomic or nonlocal theories that can account for the long-range interatomic attractions [3]. The applications of a nonlocal theory also help to explain, predict the physical phenomena at small length scales and bridge continuum mechanics with the deformation mechanisms existing at nanoscales, and for this reason it has been extensively used within the field of nanomechanics [4, 5].

Khurana and Tomar [6,7] have investigated the reflection/transmission phenomenon due to incident longitudinal displacement wave at the stress-free flat boundary and plane discontinuity separating the two distinct nonlocal micropolar solids in perfect contact of a micropolar solid half-space. They have proved that there exist four basic plane waves in a nonlocal micropolar elastic medium, and two of them are independent compressional waves and remaining two are coupled transverse waves. Recently, Rayleigh-type waves in nonlocal micropolar solid half-space have been considered by Khurana and Tomar [8]. Frequency equations of these Rayleigh type modes and their conditions of existence have been derived. Chakraborty [9] has made an effort to understand and quantify the effect of nonlocal elasticity on the wave propagation response of laminated composite layered media. However, no investigation has been carried out so far for the case when the half-space (or laminated composite layered media) is the orthotropic nonlocal micropolar medium, to the best knowledge of the authors.

Therefore, the aim of the present paper is twofold: firstly, to present the Stroh formalism for the orthotropic nonlocal micropolar medium, it is the base tool for investigating the reflection and transmission problems as well as a propagation wave in laminated composite layered media; secondly, to obtain the dispersion equation of the Rayleigh wave in the orthotropic nonlocal micropolar medium, the condition for existence of Rayleigh type modes, the effect of parameters material, nonlocality on the Rayleigh wave modes are shown.

2. Basic equations

For two-dimensional geometry (planex_1x_2), the plane strains are related to the displacement field u_1, u_2 and microrotation ϕ. The constitutive equations are given as [9]

$$\begin{aligned} &t_{11} = Q_{11}u_{1,1} + Q_{12}u_{2,2}, \quad t_{12} = Q_{78}u_{1,2} + Q_{77}u_{2,1} - \kappa_{12}\phi, \\ &t_{21} = Q_{88}u_{1,2} + Q_{78}u_{2,1} - \kappa_{21}\phi, \quad t_{22} = Q_{12}u_{1,1} + Q_{22}u_{2,2}, \\ &M_{13} = B_{66}\phi_{,1}, \quad M_{23} = B_{44}\phi_{,2}, \end{aligned} \tag{2.1}$$

where t_{ij}, M_{ij} (σ_{ij}, m_{ij}) are the force and couple stress tensors in an local (nonlocal) micropolar elastic medium, which have relations [3, 8, 9]

$$\begin{aligned} &t_{11} = (1-\epsilon^2\nabla^2)\sigma_{11}, \quad t_{21} = (1-\epsilon^2\nabla^2)\sigma_{21}, \quad t_{22} = (1-\epsilon^2\nabla^2)\sigma_{22}, \\ &M_{13} = (1-\epsilon^2\nabla^2)m_{13}, \quad M_{23} = (1-\epsilon^2\nabla^2)m_{23} \end{aligned} \tag{2.2}$$

and ϵ $(= e_0a)$ is the nonlocal parameter (e_0 is the nonlocal constant and a is the internal characteristic length). The present nonlocal model can be compared to the Born-Karman model of lattice dynamics where the nearest neighbour interactions are accounted [10]. This is in accordance with atomic theory of lattice dynamics and experimental observations on the phonon dispersion [11]. Therefore, the choice of the value of parameter $e_0 \approx 0.39$ is taken such that the e_0 provides the perfect matching of phonon dispersion curves at the end of the Brillouim zone [3, 9]. The symbols Q_{ij}, κ_{ij}, B_{ij} are the local micropolar constants.

The equations of motion for a micropolar elastic solid are expressed by [12, 13]

$$\begin{aligned} &\sigma_{11,1} + \sigma_{21,2} - \rho\ddot{u}_1 = 0, \qquad \sigma_{12,1} + \sigma_{22,2} - \rho\ddot{u}_2 = 0, \\ &m_{13,1} + m_{23,2} + \sigma_{12} - \sigma_{21} - \rho j\ddot{\phi} = 0. \end{aligned} \tag{2.3}$$

Substituting (2.1) into (2.3) and taking into account (2.2), we have

$$
\begin{aligned}
&Q_{11}u_{1,11}+(Q_{12}+Q_{78})u_{2,12}+Q_{88}u_{1,22}-\kappa_{21}\phi_{,2} = \rho\ddot{u}_1-\rho\epsilon^2(\ddot{u}_{1,11}+\ddot{u}_{1,22}),\\
&(Q_{12}+Q_{78})u_{1,12}+Q_{77}u_{2,11}+Q_{22}u_{2,22}-\kappa_{12}\phi_{,1} = \rho\ddot{u}_2-\rho\epsilon^2(\ddot{u}_{2,11}+\ddot{u}_{2,22}),\\
&B_{66}\phi_{,11}+B_{44}\phi_{,22}+(\kappa_{21}-\kappa_{12})\phi+\kappa_{21}u_{1,2}+\kappa_{12}u_{2,1} = \rho j\ddot{\phi}-\rho j\epsilon^2(\ddot{\phi}_{,11}+\ddot{\phi}_{,22}).
\end{aligned}
\tag{2.4}
$$

Remark 1. The Stroh formalism for the nonlocal orthotropic micropolar medium.

For the waves propagating in the plane $x_2 = 0$, we take the form of relevant components of displacement and microrotation fields as [14]

$$
\begin{aligned}
&u_1 = U_1(y)\mathrm{e}^{ik(x_1-ct)}, \quad u_2 = U_2(y)\mathrm{e}^{ik(x_1-ct)}, \quad \phi = k\Phi(y)\mathrm{e}^{ik(x_1-ct)},\\
&t_{21} = ikT_1(y)\mathrm{e}^{ik(x_1-ct)}, \quad t_{22} = ikT_2(y)\mathrm{e}^{ik(x_1-ct)},\\
&M_{23} = iM(y)\mathrm{e}^{ik(x_1-ct)}; \qquad y = kx_2,
\end{aligned}
\tag{2.5}
$$

where $U_m(y)$, $T_m(y)$ $(m = 1, 2)$, $\Phi(y)$ and $M(y)$ are unknown functions to be determined, k is the wavenumber, c is the speed of wave propagation in the positive direction of x_1-axis.

From (2.1)–(2.4) and (2.5) one can see that the unknown functions $U_m(y)$, $T_m(y)$ $(m = 1, 2)$, $\Phi(y)$ and $M(y)$ are the solution of the differential equation.

$$
\boldsymbol{\xi}'(y) = i\mathbf{N}\boldsymbol{\xi}(y),
\tag{2.6}
$$

where $\xi = [U_1\ U_2\ \Phi\ T_1\ T_2\ M]^T$, the prime indicates the differentiation with respect to y and

$$
\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3 & \mathbf{N}_2 \end{bmatrix}.
\tag{2.7}
$$

The matrices $\mathbf{N}_k$ are given by

$$
\begin{aligned}
&\mathbf{N}_1 = \begin{bmatrix} 0 & -\frac{Q_{78}}{Q_{88}} & -\frac{i\kappa_{21}}{Q_{88}} \\ -\frac{Q_{12}}{Q_{22}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad
\mathbf{N}_2 = \begin{bmatrix} \frac{1}{Q_{88}} & 0 & 0 \\ 0 & \frac{1}{Q_{22}} & 0 \\ 0 & 0 & \frac{1}{k^2B_{44}} \end{bmatrix},\\
&\mathbf{N}_3 = \begin{bmatrix} N_3^{11} & 0 & 0 \\ N_3^{21} & N_3^{22} & N_3^{23} \\ 0 & N_3^{32} & N_3^{33} \end{bmatrix}, \qquad
\mathbf{N}_4 = \begin{bmatrix} 0 & N_4^{12} & N_4^{13} \\ N_4^{21} & 0 & 0 \\ N_4^{31} & 0 & 0 \end{bmatrix}.
\end{aligned}
\tag{2.8}
$$

where the elements N_3^{ij}, N_4^{ij} are given in Appendix. Equation (2.6) is called the Stroh formalism (see [15]). It is the base tool for investigating the reflection and transmission problem as well as the propagation wave in laminated composite layered media [14]. In order to find the solution of Eq. (2.6) we have to solve its characteristic equation

$$|\mathbf{N} - p\mathbf{I}| = 0 \tag{2.9}$$

to calculate characteristic values p of matrix $\mathbf{N}$, here $\mathbf{I}$ is the identity matrix of six order.

We note that $\mathbf{N}_3, \mathbf{N}_4$ are not symmetric real matrices, therefore we cannot employ the method of first integrals [16, 17] in order to obtain the explicit dispersion equation of the wave. In the next section, the explicit dispersion equation is only derived for the special case of the nonlocal orthotropic micropolar medium.

3. The dispersion equations of Rayleigh wave in the nonlocal orthotropic micropolar medium

For the waves propagating in the plane $x_2 = 0$; with their amplitudes decaying in x_2 positive direction (surface wave), we take the form of relevant components of displacement and microrotation fields as [18]

$$\begin{cases} u_1 = Ae^{-\xi y}e^{ik(x_1-ct)}, \\ u_2 = Be^{-\xi y}e^{ik(x_1-ct)}, \\ \phi = Dke^{-\xi y}e^{ik(x_1-ct)}, \end{cases} \qquad y = kx_2, \tag{3.1}$$

here A, B, D are scalar constants, k is the wavenumber, ξ is real, positive quantity, c is a speed of wave propagation.

Substituting (3.1) into (2.4), we get

$$\begin{aligned} &[Q_{88}\xi^2 - Q_{11} + \rho c^2 + \rho\epsilon^2 k^2 c^2(1-\xi^2)]A - i[(Q_{12}+Q_{78})\xi]B + \kappa_{21}\xi D = 0, \\ &[(Q_{12}+Q_{78})i\xi]A - [Q_{22}\xi^2 - Q_{77} + \rho c^2 + \rho\epsilon^2 k^2 c^2(1-\xi^2)]B + \kappa_{12}iD = 0, \\ &\kappa_{21}\xi A - \kappa_{12}iB \\ &\quad -[B_{44}k^2\xi^2 - B_{66}k^2 + (\kappa_{21}-\kappa_{12}) + \rho jk^2c^2 - \rho j\epsilon^2 k^4 c^2(1-\xi^2)]D = 0. \end{aligned} \tag{3.2}$$

The necessary condition for the existence of a non-trivial solution of A, B and D for above system equations is vanishing of the determinant of the corresponding coefficients matrix, which yields a full cubic equation for ξ^2. Using Mathematica, analytic roots are given. However, it is quite a complex form, and we have solved it numerically. An approximate analytic root is obtained when the medium is nonlocal isotropic micropolar elastic (see [8])

$$\begin{aligned} &Q_{11} = Q_{22} = \lambda + 2\mu + \kappa, \quad Q_{12} = \lambda, \quad Q_{78} = \mu, \quad Q_{77} = Q_{88} = \mu + \kappa, \\ &\kappa_{12} = -\kappa_{21} = \kappa, \quad B_{44} = B_{66} = \gamma. \end{aligned} \tag{3.3}$$

We note that for the surface waves to be Rayleigh wave, the quantities ξ_1, ξ_2, ξ_3 must be real and positive. When the expressions (3.1) of displacement and microrotation fields are rewritten as

$$\begin{cases} u_1 = \sum_{j=1}^{3} A_j e^{-\xi_j y} e^{ik(x_1-ct)}, \\ u_2 = \sum_{j=1}^{3} B_j e^{-\xi_j y} e^{ik(x_1-ct)}, \\ \phi = \sum_{j=1}^{3} D_j k e^{-\xi_j y} e^{ik(x_1-ct)}. \end{cases} \tag{3.4}$$

For each ξ_j, from (3.2), we find A_j, B_j, D_j and that have relations $A_j = \alpha_j D_j$ and $B_j = \beta_j D_j, (j = 1, 2, 3)$ with

$$\begin{aligned} \alpha_j &= \frac{\kappa_{12}(Q_{12}+Q_{78})\xi_j + \kappa_{21}\xi_j[Q_{22}\xi_j^2 - Q_{77} + \rho c^2 + \rho\xi_j^2 k^2 c^2(1-\xi_j^2)]}{\Delta_j}, \\ \beta_j &= \frac{\kappa_{21}\xi_j^2(Q_{12}+Q_{78})i - \kappa_{12}i[Q_{88}\xi_j^2 - Q_{11} + \rho c^2 + \rho\xi_j^2 k^2 c^2(1-\xi_j^2)]}{\Delta_j}, \\ \Delta_j &= -[Q_{88}\xi_j^2 - Q_{11} + \rho c^2 + \rho\xi_j^2 k^2 c^2(1-\xi_j^2)] \\ &\quad \times [Q_{22}\xi_j^2 - Q_{77} + \rho c^2 + \rho\xi_j^2 k^2 c^2(1-\xi_j^2)] - (Q_{12}+Q_{78})^2\xi_j^2. \end{aligned} \tag{3.5}$$

Since the boundary surface of the half-space is mechanically stress free, therefore all the components of force and couple stresses must vanish, that is, $t_{12} = t_{22} = 0$ and $M_{23} = 0$ at the surface $x_2 = 0$ lead to

$$\begin{cases} \xi_1 D_1 + \xi_2 D_2 + \xi_3 D_3 = 0, \\ \xi_1^* D_1 + \xi_2^* D_2 + \xi_3^* D_3 = 0, \\ \xi_1^{**} D_1 + \xi_2^{**} D_2 + \xi_3^{**} D_3 = 0, \end{cases} \tag{3.6}$$

where $\xi_j^* = Q_{78}\alpha_j\xi_j - Q_{77}i\beta_j$ and $\xi_j^{**} = Q_{12}i\alpha_j - Q_{22}\xi_j\beta_j$, $(j = 1, 2)$.

The determinant of the coefficient matrix must vanish, which yields

$$\det \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \\ \xi_1^* & \xi_2^* & \xi_3^* \\ \xi_1^{**} & \xi_2^{**} & \xi_3^{**} \end{bmatrix} = 0. \tag{3.7}$$

This is the dispersion equation for the propagation of Rayleigh type waves in nonlocal micropolar elastic solid half-space. As you know, the dispersion Eq. (3.7) is also implicit because the determination of the analytic solutions ξ_i of (3.2) is impossible. However, when the medium is nonlocal orthotropic elastic $Q_{77} = Q_{78} = Q_{88} = Q_{66}$ and $\kappa_{12} = \kappa_{21} = 0$, the explicit dispersion equation is presented, namely. The system equations (2.4) reduce to

$$\begin{aligned} Q_{11}u_{1,11} + (Q_{12}+Q_{66})u_{2,12} + Q_{66}u_{1,22} &= \rho\ddot{u}_1 - \rho\epsilon^2(\ddot{u}_{1,11} + \ddot{u}_{1,22}), \\ Q_{66}u_{2,11} + (Q_{12}+Q_{66})u_{1,22} + Q_{22}u_{2,22} &= \rho\ddot{u}_2 - \rho\epsilon^2(\ddot{u}_{2,11} + \ddot{u}_{2,22}). \end{aligned} \tag{3.8}$$

The solutions of (3.8) with their amplitudes decaying in x_2 direction are the form of

$$\begin{cases} u_1 = Ae^{-\xi y}e^{ik(x_1-ct)}, \\ u_2 = Be^{-\xi y}e^{ik(x_1-ct)}, \end{cases} \qquad y = kx_2. \tag{3.9}$$

Substituting (3.9) into (3.8) we have system equations for A and B

$$\begin{cases} [Q_{66}\xi^2 - Q_{11} + \rho c^2 + \rho\epsilon^2 k^2 c^2(1-\xi^2)]A - i[(Q_{12}+Q_{66})\xi]B = 0, \\ -i[(Q_{12}+Q_{66})\xi]A + [Q_{22}\xi^2 - Q_{66} + \rho c^2 + \rho\epsilon^2 k^2 c^2(1-\xi^2)]B = 0. \end{cases} \tag{3.10}$$

The necessary condition for the existence of a non-trivial solution of A and B for above system equations is vanishing of the determinant of the corresponding coefficients matrix, which yields

$$\begin{aligned} &[(Q_{66} - \rho c^2 k^2 \epsilon^2)\xi^2 + \rho c^2(1+k^2\xi^2) - Q_{11}] \\ &\quad \times [(Q_{22} - \rho c^2 k^2\epsilon^2)\xi^2 + \rho c^2(1+k^2\xi^2) - Q_{66}] + (Q_{12}+Q_{66})^2\xi^2 = 0. \end{aligned} \tag{3.11}$$

This equality leads to the quadratic equation in ξ as

$$\begin{aligned} &(Q_{22} - \rho c^2 k^2\epsilon^2)(Q_{66} - \rho c^2 k^2 \epsilon^2)\xi^4 + [(Q_{12}+Q_{66})^2 + (Q_{66} - \rho c^2 k^2\epsilon^2) \\ &\quad \times (\rho c^2(1+k^2\epsilon^2) - Q_{66}) + (Q_{22} - \rho c^2 k^2 \epsilon^2)(\rho c^2(1+k^2\epsilon^2) - Q_{11})]\xi^2 \\ &\quad + [\rho c^2(1+k^2\epsilon^2) - Q_{11}][\rho c^2(1+k^2\epsilon^2) - Q_{66}] = 0. \end{aligned} \tag{3.12}$$

Remark 2. The frequency at which the imaginary parts become real is called the cut-off frequency, ω_c [9, 11]. The values of these frequencies can easily be obtained by substituting $\xi = 0$ in Eq. (3.12). Thus, the two cut-off frequencies are $\omega_{c1} = k\sqrt{Q_{11}/\rho'}$ and $\omega_{c2} = k\sqrt{Q_{66}/\rho'}$ where $\rho' = \rho(1+k^2\epsilon^2)$. When the wavenumbers become infinite at a particular frequency, which is referred here as the escape frequency, ω_e. Beyond this frequency, the wavenumbers are purely imaginary, i.e., evanescent modes. The expressions for the escape frequencies can be obtained by forcing the coefficient of ξ^4 equal to 0 in Eq. (3.12) [9, 11], which gives $\omega_{e1} = \epsilon^{-1}\sqrt{Q_{22}/\rho}$ and $\omega_{e2} = \epsilon^{-1}\sqrt{Q_{66}/\rho}$.

We choose the solutions ξ_1, ξ_2 real, positive of Eq. (3.12)

$$\begin{cases} u_1 = (A_1 e^{-\xi_1 y} + A_2 e^{-\xi_2 y})e^{ik(x_1-ct)}, \\ u_2 = (B_1 e^{-\xi_1 y} + B_2 e^{-\xi_2 y})e^{ik(x_1-ct)}, \end{cases} \tag{3.13}$$

where $B_k = \alpha_k A_k$ and

$$\alpha_k = \frac{i[Q_{11} - Q_{66}\xi_k^2 - \rho c^2 - \rho c^2 k^2 \epsilon^2(1-\xi_k^2)]}{(Q_{12}+Q_{66})\xi_k} \qquad (k = 1, 2). \tag{3.14}$$

From (3.12), using the Viète theorem, we have

$$\begin{aligned}
\xi_1^2+\xi_2^2 = &-\frac{[(Q_{12}+Q_{66})^2+(Q_{66}-\rho c^2k^2\epsilon^2)(\rho c^2(1+k^2\epsilon^2)-Q_{66})]}{(Q_{22}-\rho c^2k^2\epsilon^2)(Q_{66}-\rho c^2k^2\epsilon^2)}\\
&-\frac{[(Q_{22}-\rho c^2k^2\epsilon^2)(\rho c^2(1+k^2\epsilon^2)-Q_{11})]}{(Q_{22}-\rho c^2k^2\epsilon^2)(Q_{66}-\rho c^2k^2\epsilon^2)},\\
\xi_1^2\xi_2^2 = &\frac{[\rho c^2(1+k^2\epsilon^2)-Q_{11}][\rho c^2(1+k^2\epsilon^2)-Q_{66}]}{(Q_{22}-\rho c^2k^2\epsilon^2)(Q_{66}-\rho c^2k^2\epsilon^2)}.
\end{aligned} \tag{3.15}$$

For true Rayleigh-type surface waves, the quantities ξ_1, ξ_2 must be positive. For to be positive and real, we must have

$$0<\rho c^2<\min\left(\frac{Q_{11}}{1+k^2\epsilon^2},\frac{Q_{66}}{1+k^2\epsilon^2}\right). \tag{3.16}$$

It is noted that, in order to obtain the condition for existence of Rayleigh type modes, we use (3.15) and the determinant Δ^* of the quadratic equation (3.12) in ξ

$$\begin{aligned}
\Delta^* = &[(Q_{12}+Q_{66})^2+Q'_{66}(X'-Q_{66})+Q'_{22}(X'-Q_{22})]\\
&-4Q'_{22}Q'_{66}(X'-Q_{11})(X'-Q_{66})+(Q_{12}+Q_{66})^4\\
&+[Q'_{22}(X'-Q_{11})-Q'_{66}(X'-Q_{66})]^2\\
&+2(Q_{12}+Q_{66})^2[Q'_{22}(X'-Q_{11})+Q'_{66}(X'-Q_{66})],
\end{aligned} \tag{3.17}$$

where

$$X'=\rho c^2+\rho c^2k^2\epsilon^2, \qquad Q'_{66}=Q_{66}+\rho c^2k^2\epsilon^2, \qquad Q'_{22}=Q_{22}+\rho c^2k^2\epsilon^2. \tag{3.18}$$

Using the stress-free boundary condition gives:

$$t_{12}=t_{22}=0 \quad \text{at } x_2=0. \tag{3.19}$$

That leads to

$$\begin{cases}(\alpha_1+\xi_1)e^{-\xi_1 y}A_1+(\alpha_2+\xi_2)e^{-\xi_2 y}A_2=0,\\ (Q_{12}-Q_{22}\alpha_1\xi_1)e^{-\xi_1 y}A_1+(Q_{12}-Q_{22}\alpha_2\xi_2)e^{-\xi_2 y}A_2=0.\end{cases} \tag{3.20}$$

The solvability of (3.20), we obtain

$$(Q_{22}\xi_1\xi_2+Q_{12})(\alpha_2-\alpha_1)+(Q_{22}\alpha_1\alpha_2+Q_{12})(\xi_2-\xi_1)=0. \tag{3.21}$$

Using (3.14) and (3.15), we have the relations

$$\alpha_2-\alpha_1=-\frac{Q_{11}-X'+Q'_{66}\xi_1\xi_2}{(Q_{12}+Q_{66})\xi_1\xi_2}(\xi_2-\xi_1), \qquad \alpha_1\alpha_2=\frac{Q_{11}-X'}{Q'_{22}\xi_1\xi_2}. \tag{3.22}$$

Substituting (3.22) into (3.21), that reduces to

$$\begin{aligned}(3.23)\qquad & \xi_1\xi_2[(Q_{11}+Q_{66})Q_{12}Q'_{22}-Q'_{22}(Q_{12}Q'_{66}+Q_{22}(Q_{11}-X'))]\\ &+(Q_{12}+Q_{66})Q_{22}(Q_{11}-X')-Q'_{22}Q_{12}(Q_{11}-X')-Q'_{22}Q_{22}Q'_{66}\xi_1^2\xi_2^2=0.\end{aligned}$$

Using $\xi_1^2\xi_2^2$ from (3.15), Eq. (3.23) may be written

$$\begin{aligned}(3.24)\qquad & (Q_{66}-X')[(Q_{12}+Q_{66})Q_{12}Q'_{22}-Q'_{22}(Q_{12}Q'_{66}+Q_{22}(Q_{11}-X'))]\\ &+(Q_{22}X'-Q_{12}\rho c^2k^2\epsilon^2)\sqrt{Q'_{22}Q'_{66}}\sqrt{(Q_{11}-X')(Q_{66}-X')}.\end{aligned}$$

In order to proceed it is convenient to use to computational purposes, dimensionless material parameters defined by [19]

$$(3.25)\qquad x=\frac{\rho c^2}{Q_{78}},\quad ep=k^2\epsilon^2,\quad \alpha=\frac{Q_{22}}{Q_{11}},\quad \gamma=\frac{Q_{66}}{Q_{11}},\quad \delta=1-\frac{Q_{12}^2}{Q_{11}Q_{22}}.$$

Equation (3.24) has another form

$$\begin{aligned}(3.26)\qquad & (1-x(1+ep))\Big(\frac{\alpha}{\gamma}+x.ep\Big)\Big(\frac{\alpha x(1+ep)}{\gamma}-\frac{\alpha\delta}{\gamma^2}\Big)\\ &+\Big(\frac{\alpha}{\gamma}x(1+ep)-\sqrt{\frac{\alpha(1-\delta)}{\gamma^2}}x.ep\Big)\\ &\times\sqrt{\Big(\frac{\alpha}{\gamma}+x.ep\Big)\Big(1+x.ep\Big)}\sqrt{\Big(\frac{1}{\gamma}-x(1+ep)\Big)(1-x(1+ep))}=0.\end{aligned}$$

This is explicit dispersion equation of Rayleigh waves propagating in nonlocal orthotropic halfspace. Especially, when the medium is classical elastic solid, namely $Q_{ij}\equiv C_{ij}$ and $\epsilon=0$, Eq. (3.26) reduces to Eq. (3.5) that is presented by Vinh and Ogden in [19]

$$(3.27)\qquad \sqrt{\alpha}(1-x)\Big(x-\frac{\delta}{\gamma}\Big)+x\sqrt{1-x}\sqrt{1-\gamma x}=0.$$

4. Numerical results and discussions

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results taking (see [8])

$$\begin{aligned}&Q_{11}=0.088\times10^{11}\,\mathrm{N/m^2},\quad Q_{12}=0.4\times10^{11}\,\mathrm{N/m^2},\quad Q_{22}=10^{11}\,\mathrm{N/m^2},\\ &Q_{77}=0.4\times10^{11}\,\mathrm{N/m^2},\quad Q_{78}=4\times10^{11}\,\mathrm{N/m^2},\quad Q_{88}=0.12\times10^{11}\,\mathrm{N/m^2},\\ (4.1)\quad &B_{44}=0.1\times10^{-9}\,\mathrm{N},\quad B_{66}=0.5\times10^{-9}\,\mathrm{N},\\ &\kappa_{12}=0.1\times10^{11}\,\mathrm{N/m^2},\quad \kappa_{21}=0.5\times10^{11}\,\mathrm{N/m^2},\\ &\rho=1740\,\mathrm{kg/m^{-3}},\quad j=2\times10^{-20}\,\mathrm{m^2},\quad e_0=0.39,\quad a=0.421\times10^{-9}\,\mathrm{m}.\end{aligned}$$

First, we can find the non-dimensional speed of the Rayleigh wave $x=\rho c^2/Q_{78}$

by solving equation from (3.2). For this computational purpose, we use dimensionless parameters

$$
\begin{aligned}
&e_0 = jk_0^2, \quad e_1 = \frac{Q_{11}}{Q_{78}}, \quad e_2 = \frac{Q_{22}}{Q_{78}}, \quad e_3 = \frac{Q_{12}}{Q_{78}}, \quad e_4 = \frac{Q_{77}}{Q_{78}},\\
&e_5 = \frac{Q_{88}}{Q_{78}}, \quad e_6 = \frac{B_{44}}{jQ_{78}}, \quad e_7 = \frac{B_{66}}{jQ_{78}}, \quad e_8 = k_0^2\epsilon_0^2.
\end{aligned} \tag{4.2}
$$

In Fig. 1 we depict the comparison between the nonlocal and local micropolar for amplitudes of x_1; x_2; x_3 (correspond to read; green; blue curve) against the

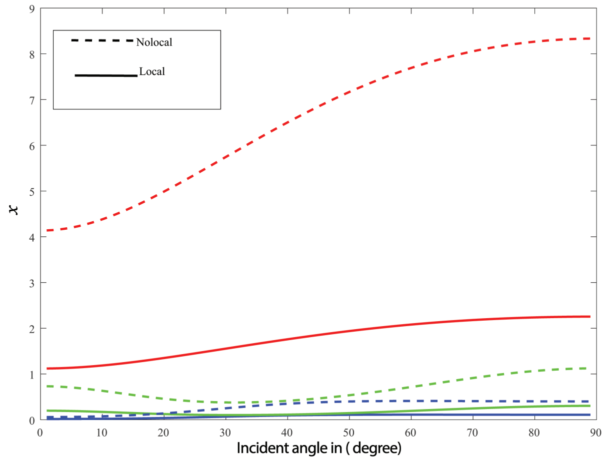
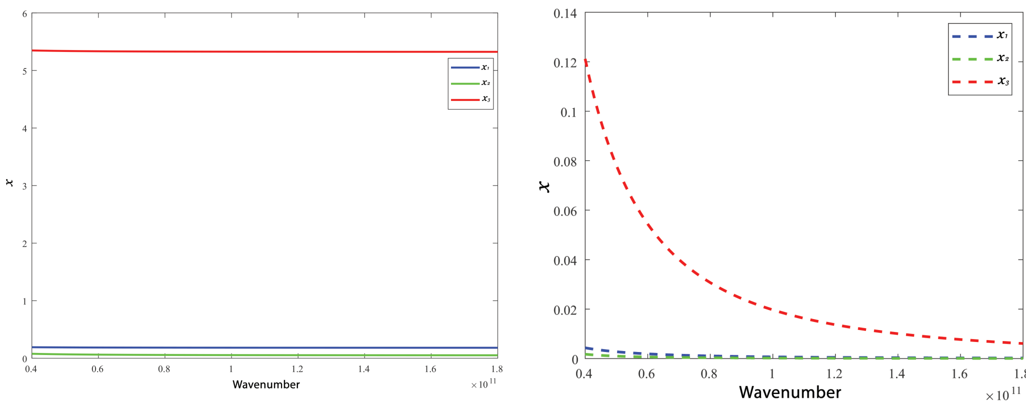


FIG. 1. Variation of the non-dimensional speeds of Rayleigh wave x on the incident angle θ_0 for the local micropolar, nonlocal micropolar.



(a) Local

(b) Nonlocal

FIG. 2. Variation of the non-dimensional speeds of Rayleigh wave x on wavenumber k for the local micropolar, nonlocal micropolar.

incident angle θ_0. We have seen that at each angle of incidence the amplitudes of x for the nonlocal orthotropic micropolar solid are bigger than the corresponding values for the orthotropic local micropolar solid.

Figure 2 shows the variation of these roots against the wavenumber k for the orthotropic local micropolar, nonlocal orthotropic micropolar. Only those roots that satisfy the condition for existence of Rayleigh type waves given by (3.16) correspond to true Rayleigh type waves. It is also clear from this figure that the Rayleigh type wave is dispersive in the nonlocal micropolar solid, while it is

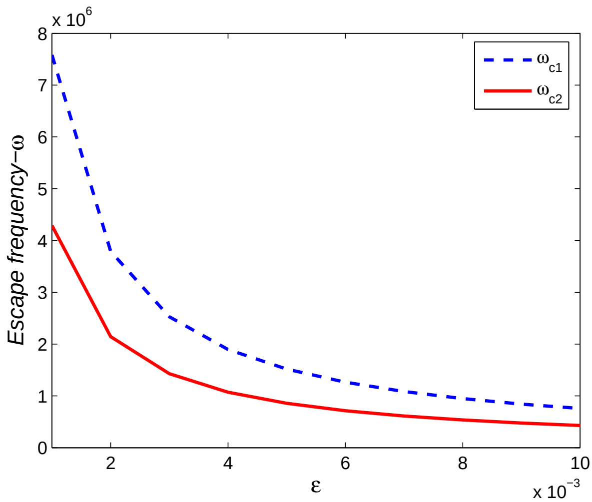


Fig. 3. Escape frequency variation with nonlocality parameter, ϵ.

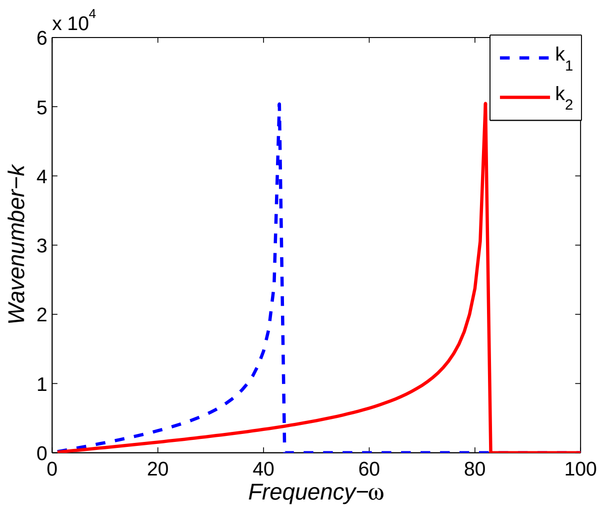
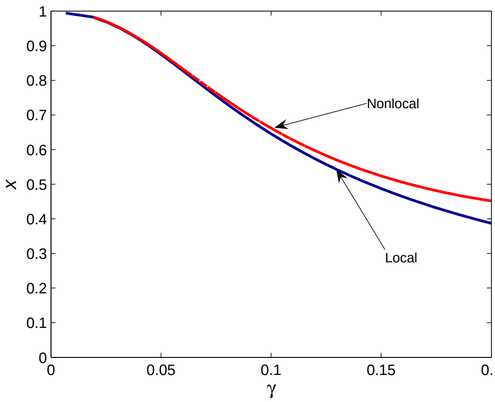


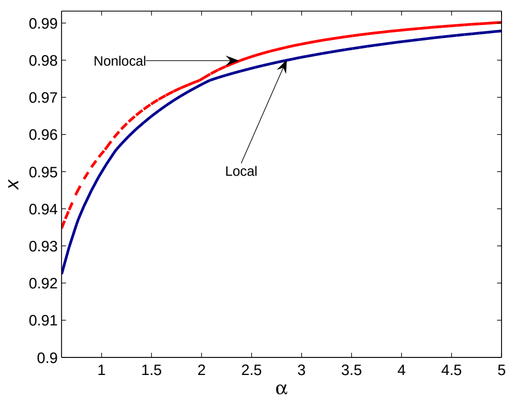
Fig. 4. Wavenumber k variation with frequency, ω.

non-dispersive in local micropolar solid. It can be seen that the Rayleigh wave is highly dispersive at the low wavenumber ranging between $0.4.10^{11} < k < 1.10^{11}$. However, it has been seen that for the high wavenumber $k > 1.10^{11}$; the Rayleigh wave is poorly dispersive in a nonlocal orthotropic micropolar solid.

Figure 3 shows the variation of escape frequencies with the nonlocal parameter $\epsilon = e_0 a$. The value of escape frequencies decreases with an increase in the scale parameter ϵ. At higher values of ϵ, escape frequencies approach to very small values.



(a) Effect of γ: $\alpha = 0.6$, $\delta = 0.2$, $\epsilon = 10^{-10}$



(b) Effect of α: $\gamma = 0.1$, $\delta = 0.5$, $\epsilon = 10^{-10}$

FIG. 5. Effect of γ, α parameter on the non-dimensional speed of Rayleigh wave x for nonlocal and local orthotropic half-space.

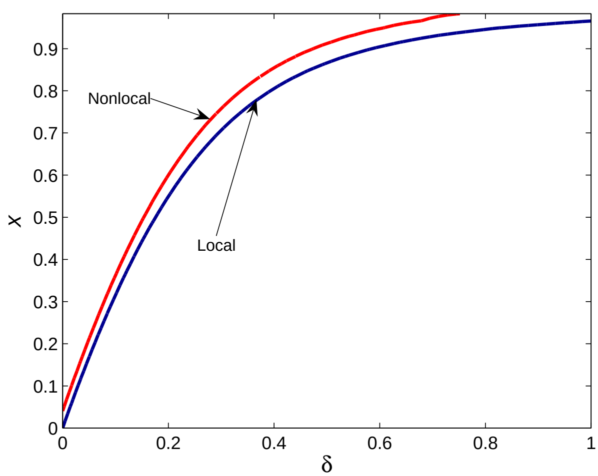
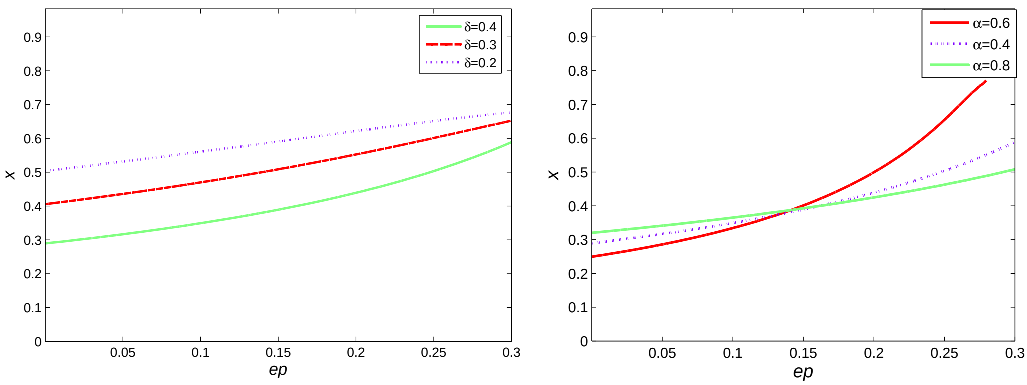


FIG. 6. Effect of δ parameter on the non-dimensional speed of Rayleigh wave x for nonlocal and local orthotropic half-space with $\gamma = 0.1$, $\alpha = 0.3$, $\epsilon = 10^{-10}$.

The variation of the cut-off frequencies with wavenumber k variation with frequency, ω are shown in Fig. 4. This figure shows that, the cut-off frequency k_1 appears at $\omega \approx 42$ THz, and the one k_2 emerges at $\omega \approx 82$ THz. From Eqs. (3.26) and (3.27), the comparison of the effect of γ, α and δ parameter on the non-dimensional speed of the Rayleigh wave x for the nonlocal and local orthotropic half-space are shown graphically in Figs. 5 and 6. The pattern is similar for the nonlocal and the local solid. However, at each value of the parameter, the value of x for the nonlocal solid is bigger than one for the local solid.



(a) $\alpha = 0.06$ and $\gamma = 0.12$ (b) $\delta = 0.2$ and $\gamma = 0.12$

Fig. 7. Variation of non-dimensional speed x of Rayleigh wave against non-dimensional constant *ep*.

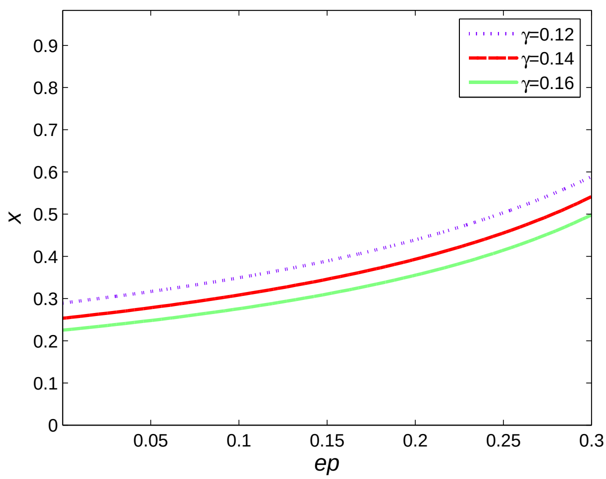


Fig. 8. Variation of non-dimensional speed against non-dimensional constant *ep* when $\alpha = 0.06$ and $\delta = 0.2$.

Using the explicit dispersion equation (3.26) for the case of nonlocal orthotropic medium, the variation of non-dimensional speed x against non-dimensional constant $ep = k^2\epsilon^2$ for different values of γ, α and δ parameters are plotted in Figs. 7 and 8. It is interesting that the shape of curve for the different values of γ is almost unchanged in Fig. 8.

5. Conclusions

In this paper, we have studied the propagation of Rayleigh waves in a nonlocal orthotropic micropolar elastic half-space. The few conclusions drawn from this analysis may be explained as follows.

(i) The Stroh formalism is presented for the nonlocal orthotropic micropolar medium. That is the base tool for investigating the reflection and transmission problem and the propagation wave in layered media.

(ii) The dispersion equation of the Rayleigh wave in the nonlocal orthotropic micropolar medium is found. For a special case, when the half-space is nonlocal orthotropic elastic, the explicit dispersion equation of Rayleigh wave has been derived. This equation reduces to Eq. 3.5 in [19] when we neglect the nonlocal effect from the model.

(iii) Phase speeds of these waves are computed numerically and their variation against the incident angle θ_0, non-dimensional frequency dimensionless parameter are presented graphically. For orthotropic nonlocal half-space, the comparisons have been made between the phase speeds of Rayleigh wave through nonlocal, local and different parameters half-spaces.

The present numerical study might provide more relevant information about the wave propagation in nonlocal orthotropic micropolar elastic solids. These results are recorded and used as the input data of the inverse problem (non-destructive evaluation).

Appendix

The elements of matrices $\mathbf{N}_3$ and $\mathbf{N}_4$

$$N_3^{11} = \Big[\rho c^2 - Q_{11} + \frac{Q_{12}^2}{Q_{22}} + \rho c^2 k^2 \epsilon^2 \Big(1 + \frac{Q_{78}Q_{12}}{Q_{22}Q_{88}}\Big)\Big]/\delta_1,$$

$$N_3^{21} = \Big[\rho c^2 k^2 \epsilon^2 \frac{iQ_{12}}{Q_{22}}\Big]/\delta_2, \qquad N_3^{22} = \Big[\rho c^2 - Q_{77} + \frac{Q_{78}^2}{Q_{88}} + \rho c^2 k^2 \epsilon^2\Big]/\delta_2,$$

$$N_3^{23} = -i\Big[\kappa_{12} - \frac{\kappa_{21}Q_{78}}{Q_{88}}\Big]/\delta_2, \quad N_3^{32} = i\Big[\kappa_{12} - \frac{\kappa_{21}Q_{78}}{Q_{88}} - \rho j c^2 \epsilon^2 k^4 i\Big]/\delta_3,$$

$$N_3^{33} = \left[\rho j k^2 c^2 + \kappa_{21} - \kappa_{12} + \frac{\kappa_{21}^2}{Q_{88}} - B_{66}k^2\right]/\delta_3,$$
$$N_4^{12} = \left[\frac{\rho c^2 k^2 \epsilon^2 Q_{78} i}{Q_{88}Q_{22}} - \frac{Q_{12}}{Q_{22}}\right]/\delta_1, \quad N_4^{13} = -\left[\frac{\kappa_{21} i \rho c^2 k^2 \epsilon^2}{Q_{88}B_{44}k^2}\right]/\delta_1,$$
$$N_4^{21} = -\left[\frac{Q_{78}}{Q_{88}}\right]/\delta_2, \quad N_4^{31} = \left[\frac{\kappa_{21} i}{Q_{88}}\right]/\delta_3,$$
$$\delta_1 = 1 - \frac{\rho c^2 k^2 \epsilon^2}{Q_{88}}, \quad \delta_2 = 1 - \frac{\rho c^2 k^2 \epsilon^2}{Q_{22}}, \quad \delta_3 = 1 - \frac{\rho c^2 j k^2 \epsilon^2}{B_{44}}.$$

References

1. A.C. Eringen, D.G.B. Edelen, *On nonlocal elasticity*, International Journal of Engineering Science, **10**, 3, 233–248, 1972.

2. A.C. Eringen, *On differential-equations of nonlocal elasticity and solutions of screw dislocation and surface waves*, Journal of Applied Physics, **54**, 9, 4703–4710, 1983.

3. A.C. Eringen, *Nonlocal Continuum Field Theories*, Springer, New York, 2001.

4. N. Challamel, D. Zorica, T. Atanackovic, D. Spasic, *On the fractional generalization of Eringen's nonlocal elasticity for wave propagation*, Comptes Rendus Mecanique, **341**, 298–303, 2013.

5. S. Narendar, S. Gopalakrishnan, *Nonlocal continuum mechanics formulation for axial, flexural, shear and contraction coupled wave propagation in single walled carbon nanotubes*, Latin American Journal of Solids and Structures, **9**, 497–513, 2012.

6. S. Tomar, A. Khurana, *Reflection of plane longitudinal waves from the stress-free boundary of a nonlocal, micropolar solid half-space*, Journal of Mechanics of Materials and Structures, **85**, 1, 95–107, 2013.

7. S. Tomar, A. Khurana, *Waves at interface of dissimilar nonlocal micropolar elastic half-spaces*, Mechanics of Advanced Materials and Structures, **26**, 10, 825–833, 2018.

8. S. Tomar, A. Khurana, *Rayleigh-type waves in nonlocalmicropolar solid half-space*, Ultrasonics, **73**, 162–168, 2016.

9. A. Chakraborty, *Wave propagation in anisotropic media with nonlocal elasticity*, International Journal of Solids and Structures, **44**, 5723–5741, 2007.

10. N. Challamel, L. Rakotomanana, L.L. Marrec, *A dispersive wave equation using nonlocal elasticity*, Comptes Rendus Mecanique, **337**, 591–595, 2009.

11. S. Gopalakrishnan, S. Narender, *Wave Propagation in Nanostructures: Nonlocal Continuum Mechanics Formulations*, Springer, New York, 2013.

12. A.C. Eringen, *Plane waves in nonlocal micropolar elasticity*, International Journal of Engineering. Science, **22**, 8–10, 1113–1121, 1984.

13. S. Tomar, A. Khurana, *Wave propagation in nonlocal microstretch solid*, Applied Mathematical Modelling, **40**, 11–12, 5858–5875, 2016.

14. P.C. Vinh, V.T.N. Anh, D.X. Tung, N.T. Kieu, *Homogenization of very rough interfaces for the micropolar elasticity theory*, Applied Mathematical Modelling, **54**, 467–482, 2017.

15. A.N. Stroh, *Steady state problems in anisotropic elasticity*, Journal Mathematical Physics, **41**, 317, 687–708, 1962.

16. M. Destrade, *The explicit dispersion equation for surface acoustic waves in monoclinic elastic crystals*, Journal of the Acoustical Society of America, **109**, 1398–1402, 2001.

17. V.G. Mozhaev, *Some new ideas in the theory of surface acoustic waves in anisotropic media*, IUTAM Symposium on Anisotropy, Inhomogeneity and Nonlinearity in Solid Mechanicsm, D.F. Parker and A.H. England [eds.], Kluwer Academic Publishers, pp. 455–462, 1995.

18. D.X. Tung, *Dispersion equation of Rayleigh waves in transversely isotropic nonlocal piezoelastic solids half-space*, Vietnam Journal of Mechanics, VAST, **41**, 4, 363–371, 2019.

19. P.C. Vinh, R.W. Ogden, *Formulas for the Rayleigh wave speed in orthotropic elastic solids*, Archives of Mechanics, **56**, 3, 247–265, 2004.

Received February 11, 2021; revised version June 6, 2021.
Published online June 30, 2021.