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Nonlinear Properties of the Gotland Deep – Baltic Sea

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The properties of the nonlinear phenomenon in water, including sea water, have been well known for many decades. The feature of the non homogeneous distribution of the speed of sound along the depth of the sea is very interesting from the physical and technical point of view. It is important especially in the observation of underwater area by means of acoustical method (Grelowska *et al.*, 2013; 2014). The observation of the underwater space has been carried out for more than hundred years. In the second half of the twentieth century we observed very intense trend of development of the measuring methods of underwater sound speed. It was done mainly in the linear sound propagation aspect. At the end of 20th century nonlinear devices were invented. Thus, from this point of view, knowledge on the nonlinear properties of the sea water is the matter of interest. The phenomenon of nonlinear distortion of elastic waves, and the same the efficiency of nonlinear transfer of energy from the primary wave to the higher harmonic components depend on properties of the medium, especially on the material constant known as the nonlinearity parameter B/A. The Baltic Sea is a specific reservoir with untypically low salinity and low depth (Grelowska, 2000). In the paper results of investigation of nonlinear properties of the South and the Central Baltic by means of thermodynamic method are presented.

Keywords: nonlinear parameter of low saline seawater, properties of Baltic Sea.

1. Introduction

Devices based on the effect of nonlinear wave generation are used in investigation in liquid media in underwater acoustics as well as in medicine. They enable a significant improvement in image resolution (Grelowska, 2000; Kozaczka *et al.*, 2012; 2013) compared to conventional devices operating on the basis of linear acoustics.

The material constant characterizing nonlinear properties of the medium, commonly used in problems of nonlinear acoustics, is the parameter B/A. Definition of the parameter is based on the equation of state, and more precisely, on its Taylor series expansion in such a way that pressure variations in the medium are presented in the form of series describing density changes. It is assumed at the same time that propagation of a wave is a reversible adiabatic process and does not result in any change in chemical composition of the medium. The definition of B/A parameter used nowadays in nonlinear acoustics was introduced by F.E. Fox and W.A. Wallace (1954) who pointed out that the elastic wave in a liquid had to be treated as an adiabatic process. The quantity B/A, proportional to the ratio of quadratic and linear coefficients in the Taylor series, characterizes the effect of nonlinearity on the wave propagation velocity in fluid.

Expansion of the equation of state:

$$p = p(\rho, S) \tag{1}$$

in a Taylor series along isentrope $S = S_0$ leads to the virial equation of state in the form (Kozaczka, Grelowska, 1996):

$$p - P_0 = \left(\frac{\partial p}{\partial \rho}\right)_{S_0,\rho=\rho_0} (\rho - \rho_0) + \frac{1}{2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{S_0,\rho=\rho_0} (\rho - \rho_0)^2 + \frac{1}{6}\left(\frac{\partial^3 p}{\partial \rho^3}\right)_{S_0,\rho=\rho_0} (\rho - \rho_0)^3 + ... \tag{2}$$

Values of partial derivatives $(\partial p/\partial \rho)_{S,\rho_0}$, $(\partial^2 p/\partial \rho^2)_{S,\rho_0}$, etc. in the series (2) are determined for undisturbed medium (S_0, ρ_0). Equation (2) can be presented in the following form:

$$p-P_0 \approx A\left(\frac{\rho-\rho_0}{\rho_0}\right)+\frac{B}{2}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 +\frac{C}{6}\left(\frac{\rho-\rho_0}{\rho_0}\right)^3+... = A\left(\frac{\rho-\rho_0}{\rho_0}\right) \cdot\left[1+\frac{B}{2A}\left(\frac{\rho-\rho_0}{\rho_0}\right)+\frac{C}{6A}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right]+..., \tag{3}$$

where

$$A=\rho_0\left(\frac{\partial p}{\partial \rho}\right)_{S_0,\rho=\rho_0}=\rho_0 c_0^2, \tag{4}$$

$$B=\rho_0^2\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{S_0,\rho=\rho_0}=2\rho_0^2 c_0^3\left(\frac{\partial c}{\partial p}\right)_{S_0,\rho=\rho_0}, \tag{5}$$

$$C=\rho_0^3\left(\frac{\partial^3 p}{\partial \rho^3}\right)_{S_0,\rho=\rho_0} =2\rho_0^3 c_0^4\left[3\left(\frac{\partial c}{\partial p}\right)^2+c_0\left(\frac{\partial^2 c}{\partial p^2}\right)\right]_{S_0,\rho=\rho_0}, \tag{6}$$

Isentropic velocity of a wave with infinitesimally small amplitude is defined by (4).

The parameter of nonlinearity B/A can be defined based on relationships (4) and (5) as the ratio:

$$\frac{B}{A}=\frac{\rho_0}{c_0^2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{S,\rho=\rho_0}. \tag{7}$$

To use this definition as a point of departure for experimental determination of B/A value, it is necessary to measure pressure changes induced by adiabatic change of density which is a difficult task in view of low compressibility of liquids.

2. Nonlinearity parameter B/A of seawater

In this section we will present some remarks about the conditions of propagation of finite amplitude waves in the sea water, considering the significant impact of nonlinear and dissipative properties of the water for the distribution of elastic wave field and simultaneously the specificity of sea water and the differences between the characteristics of sea water and fresh water.

Much more attention is paid to research nonlinear properties of the medium in works from the area of nonlinear acoustics of liquid. It is justified because of the impact of nonlinear characteristics for the degree of distortion of the wave as well as the efficiency of the generation harmonics or a difference frequency wave. For this reason, there are developed methods of determination of the nonlinear parameter B/A. The first, still present and constituting a reference point in the study of nonlinearity of the medium, is the work of R.T. Beyer (1960). The author derived the formula allowing to determine the value of the parameter B/A based on measuring changes in the acoustic wave velocity due to changes in pressure and temperature changes:

$$\frac{B}{A}=2\rho_0 c_0\left(\frac{\partial c}{\partial p}\right)_{S_0}=2\rho_0 c_0\left(\frac{\partial c}{\partial p}\right)_T +\frac{2c_0 T\alpha_p}{\rho_0 C_p}\left(\frac{\partial c}{\partial T}\right)_p=\left(\frac{B}{A}\right)'+\left(\frac{B}{A}\right)'', \tag{8}$$

where $\alpha_p=(1/V)(\partial V/\partial T)_p$ is thermal expansion coefficient, V is volume, T – absolute temperature, C_p – specific heat capacity at constant pressure, S_0 – entropy. He also proposed a method of determining the nonlinear parameter B/A based on changes in the amplitude of the second harmonic. Both proposals gave rise to the division the methods such as thermodynamic and acoustic ones.

The thermodynamic methods mostly use the relation (8). In 1965, A.B. Coppens, R.T. Beyer and other (Coppens *et al.*, 1965) reported the results of measurements of the parameter B/A for a number of liquids, including fresh water and seawater. Changes in speed of the acoustic wave were measured by varying the temperature from 30°C to 80°C at the constant pressure 100 kPa, and by varying pressure in the range from 100 kPa to 140 MPa at the constant temperature of 30°C. In the following years the results of measurements obtained in a similar manner for a variety of liquids and liquid mixtures appeared in several papers. Among others, M.P. Hagelberg measured changes in the speed of sound and parameter B/A of water for a very wide range of pressure changes from 100 kPa to 1 GPa while temperature changed from 0°C to 80°C (Hagelberg *et al.*, 1967; Hagelberg, 1970). Value of the parameter B/A for different liquids determined experimentally can be found in a tabular form for example in (Shutilov, 1980; Beyer, 1997; Bjørnø, 2013). Both proposals, the thermodynamic method and the acoustic one, were used is determination of the parameter B/A in sea with low salinity (Kozaczka, Grelowska, 1994; 1999).

The accuracy of determining the value of B/A by means of thermodynamic method depends on the accuracy of measuring the changes in the speed of acoustic wave. These in turn are small, e.g. the acoustic wave velocity in sea water with a salinity of 35 PSU at atmospheric pressure when the temperature changes from 0°C to 30°C increases by 96.5 m/s, which is 6.66% while the increase in static pressure of 10 MPa corresponding the change in the depth of about 1000 m, increases speed of 16.5 m/sec, which is about 1.10% (at 35 PSU salinity and temperature 10°C). Method of measuring the phase shift between the transmitted signal and the received one, after passing through the test sample fluid, was used to increase the accuracy of measurement of changes in the acoustic wave velocity instead of direct measurement (Zhu *et al.*, 1983; Sehgal *et al.,* 1984; 1986). Ch.A. Cain (1986) developed a modification of the method of determining the change of the parameter B/A within the test sample. In this case, the nonlinear parameter is determined by measuring the phase shift of a sinusoidal signal with small amplitude as a result of interaction with the pulse of large amplitude, moving in the opposite direction.

In the eighties, there were publications in which the expression was derived binding nonlinear parameter B/A and thermodynamic constants (Endo, 1982; 1988; Sharma, 1983). B.K. Sharma (1983) reported a relationship between B/A and acoustic parameters:

$$\frac{B}{A} = 2\gamma K'' + 2K, \tag{9}$$

where $K'' = K' - K$, K'' is isochoric acoustic parameter, $K = -\left(\frac{1}{\alpha_p}\right)\left(\frac{\partial \ln c}{\partial T}\right)_p$ – the Rao's parameter, $K' = \left(\frac{1}{\beta_T}\right)\left(\frac{\partial \ln c}{\partial p}\right)_T$ – the Carnevale's and Litovitz's parameter, $\beta_T = -\left(\frac{1}{V}\right)\left(\frac{\partial V}{\partial p}\right)_T$ – the coefficient of isothermal compressibility, $\gamma = \frac{C_p}{C_V}$, C_p – specific heat at constant pressure, C_V – specific heat at constant volume.

H. Endo (1988) derived the following formula:

$$\begin{aligned}\frac{B}{A} = (K_T - 1) + (\gamma - 1)\left[2K_T - 1 - \frac{3}{\alpha_p \beta_T}\left(\frac{\partial \beta_T}{\partial T}\right)_p\right] \\ + (\gamma - 1)^2\left\{K_T + 1 + \frac{1}{\alpha_p}\left[\frac{1}{T} + \frac{3}{\alpha_p}\left(\frac{\partial \alpha_p}{\partial T}\right)_p \right.\right. \\ \left.\left. -\frac{3}{\beta_T}\left(\frac{\partial \beta_T}{\partial T}\right)_p - \frac{1}{C_p}\left(\frac{\partial C_p}{\partial T}\right)_p\right]\right\},\end{aligned} \tag{10}$$

where

$$K_T = \left[\frac{\partial (1/\beta_T)}{\partial p}\right]_T.$$

The same author (Endo, 1988) showed the method of determining a parameter B/A of seawater and the results of calculations the parameter for a range of temperature from 0°C to 35°C, salinity from 25 to 40 PSU and pressure from 0 to 100 MPa. A comparison of the results obtained by him using the thermodynamic method (Endo, 1982) and the acoustic method (relationship (8)) indicated that the difference is less than 3% for the temperature change from 0° to 35°C and a pressure changes from 0 to 80 MPa.

In 1991 F.D. Cotaras and C.L. Morfey developed polynomial expressions for the coefficient of nonlinearity β ($\beta = B/2A + 1$) for both seawater and fresh water (Cotaras, Morfey, 1991). The seawater relations are for salinities of 30–40 PSU, temperatures 0–40°C and pressure 1–800 bars absolute (1–80 MPa). The fresh water relations are for temperatures 0–90°C and pressure 1–1000 bars absolute (1–100 MPa). There are also presented polynomial expressions for the term $\Lambda \equiv \beta/(\rho c^5)^{1/2}$ which arises in nonlinear geometrical acoustics. The precision of the fits of the polynomials to the underlying data is better than 1 PSU in all cases. However, the uncertainty in the seawater β relation is ±3% for realistic ocean temperatures and pressures. The information is published in form of Technical Report under Grant sponsored by Office of the Chief of Naval Research in Virginia (US).

3. Impact of material constants of seawater for B/A parameter in the Baltic Sea

The values of the parameter B/A for seawater, reported in the literature, were obtained predominantly for oceanic high-salinity water. This paper contains the investigation results for nonlinearity for low-salinity seawater, especially for the Baltic Sea. The parameter B/A has been determined using the thermodynamic method according to the relation (8) by R.T. Beyer.

The changes in the speed of sound and thermodynamic parameters with the changes of pressure, temperature and salinity were determined by applying the relations recommended by UNESCO for computation of fundamental properties of seawater. The following relations were used in the computations (Fofonoff, Millard, 1983):

1. Equation of state for seawater $\rho(S, T, p)$;
2. Speed of sound in seawater $c(S, T, p)$;
3. Specific heat of seawater $C_p(S, T, p)$;
4. Thermal expansion coefficient $\alpha(S, T, p)$.

The range of validity of these relations is as follows:
S: from 0 to 40 PSU (Practical Salinity Unit);
T: from 0 to 40°C;
p: form 0 to 100 MPa.

Changes in the nonlinearity parameter B/A of oceanic water as a function of temperature and static pressure by Endo (1984) are shown in Figs. 1–2 (dashed lines). The thermodynamic method presented by Endo (1984) was used to obtain changes in the parameter B/A for the Baltic Sea. Calculations were made based on salinity and temperature values measured as function of depth.

The impact of salinity on the parameter B/A is shown in Fig. 1. In that case, parameter B/A was determined for salinity 7 PSU, it means typical value in the upper layer of the Southern and the Central Baltic. The diminution in B/A value by lowering the salinity is evident, as well as the impact of the depth of the sea (Fig. 2). The impact of the depth of the sea is rather small. But due to smaller salinity in the Baltic Sea in comparison to the ocean the nonlinear distortion will be weaker.

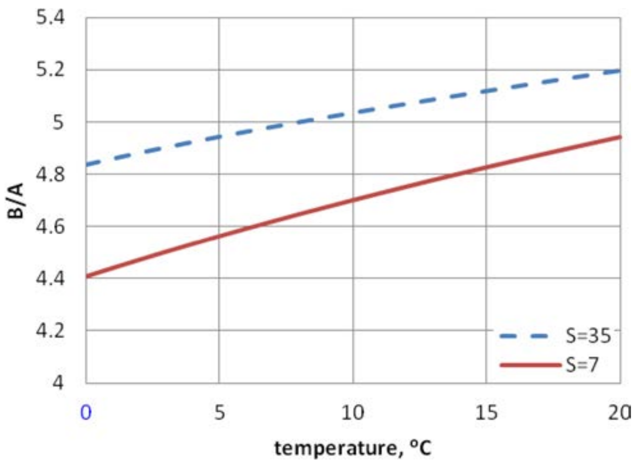

Fig. 1. Changes in nonlinearity parameter B/A in sea water at the surface as a function of temperature for salinity S = 7 PSU and S = 35 PSU.

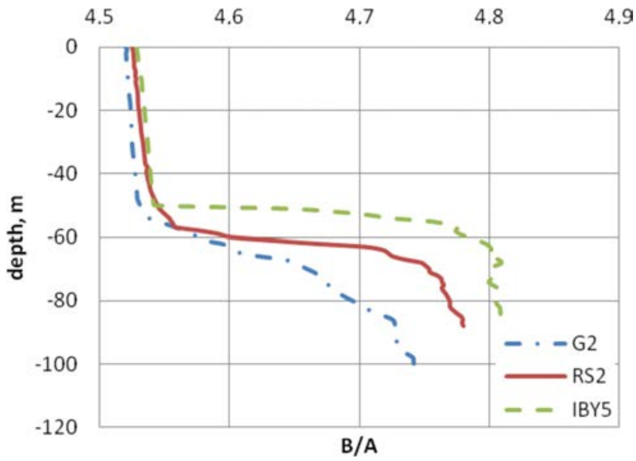

Fig. 3. Vertical distribution of the nonlinearity parameter in winter at Gdansk Deep (G2), Slupsk Furrow (RS2) and Bornholm Deep (IBY5).

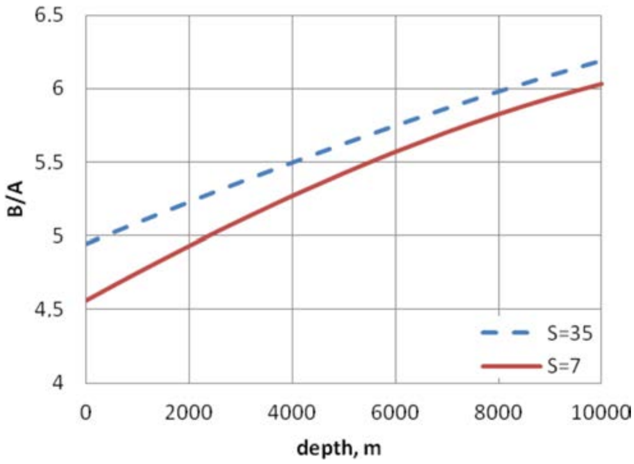

Fig. 2. Changes in nonlinearity parameter B/A in sea water of temperature 5°C as a function of depth for salinity S = 7 PSU and S = 35 PSU.

It is known that sound propagation in shallow water is much more complex than in the ocean. Propagation of the wave of infinitesimal amplitude depends mainly on changes in the sound speed distribution. Propagation of the finite amplitude wave depends also on spatial distribution of the nonlinear parameter B/A as well as on the attenuation coefficient. All of them are influenced by variation in temperature and salinity, but the first and the second depend strongly on temperature whereas the third – the attenuation coefficient – on salinity.

The field of temperature and salinity in the Baltic Sea is inhomogeneous. This applies both to the spatial distribution and changes during the time (year). The differences in the B/A distribution at the same time determined for three main stations in the Southern Baltic: Bornholm Deep, Slupsk Furrow and Gdansk Deep are shown in Fig. 3, while the distribution the parameter B/A at the same place, at the Gotland Deep in May and in October of the same year are given in Fig. 4. The measurements of temperature and salinity as a function of depth were done by using CTD Neil Brown sensor whereas speed of sound was calculated according to formula given in (Fofonoff, Millard, 1983) and nonlinear parameter B/A using formula (8).

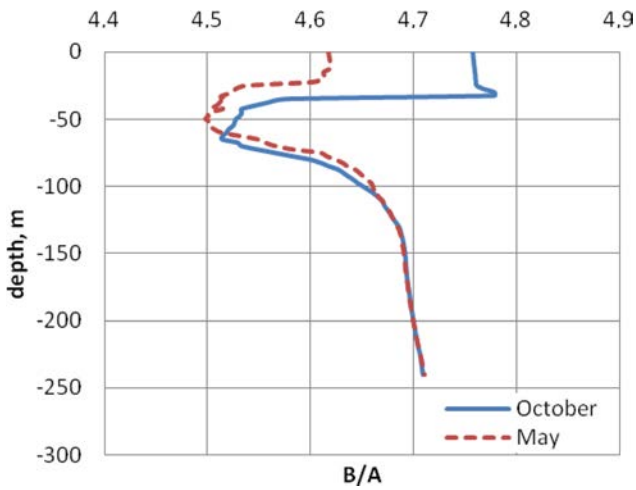

Fig. 4. Vertical distribution of the nonlinearity parameter at Gotland Deep in May and in October.

We can trace the interdependence between distribution of hydrological parameters and the acoustical parameters basing on data collected along the cross-section of the Gotland Deep, marked in Fig. 5, in May and in October of the same year. Vertical distributions of temperature, salinity, sound speed and parameter B/A in May and in October are show in Fig. 6 and Fig. 7.

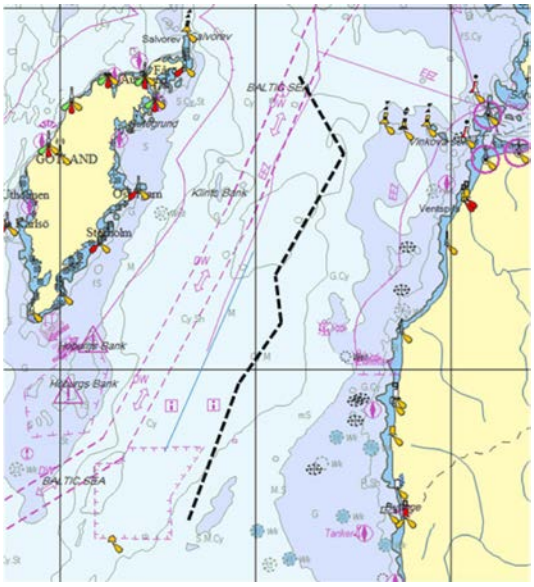

Fig. 5. Transect along the Gotland Deep region.

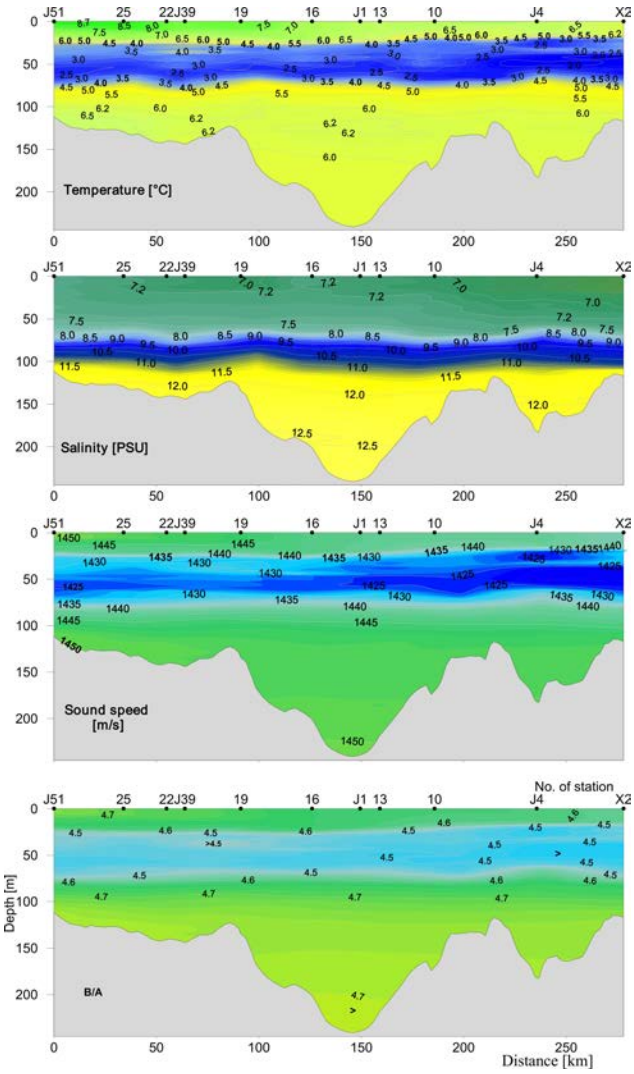

Fig. 6. Vertical distribution of temperature, salinity, sound speed and B/A in May at the Gotland Deep along transect shown in Fig. 5.

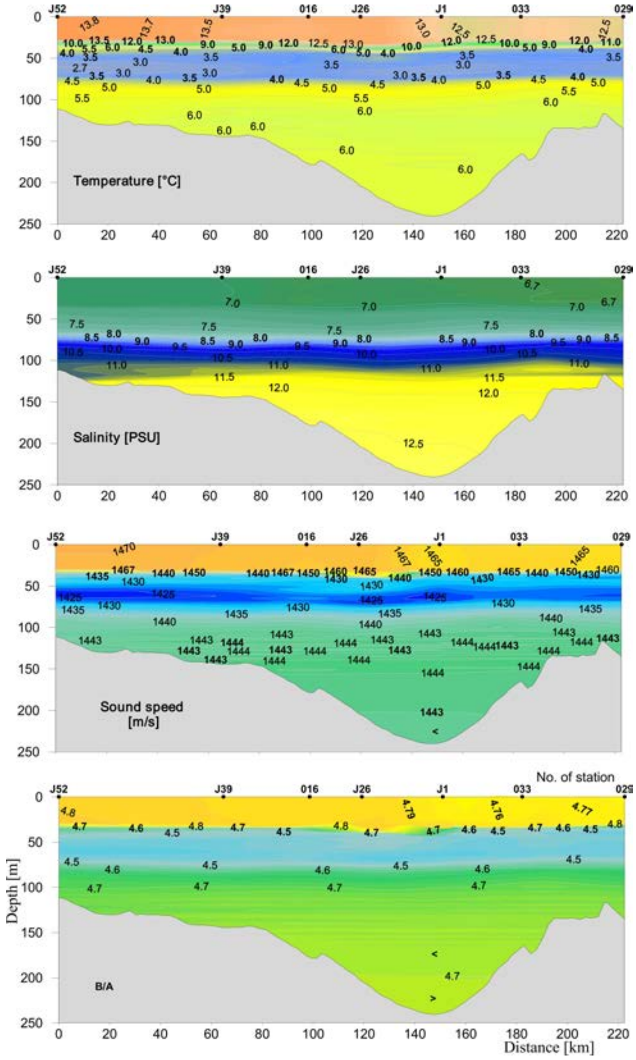

Fig. 7. Vertical distribution of temperature, salinity, sound speed and B/A in October at the Gotland Deep along transect shown in Fig. 5.

4. Conclusions

Making observation of the changes of the speed of sound in the deepest region of the Baltic Sea and also temperature and salinity we can estimate B/A parameter as a function of the depth. It is complementary parameter of the sea water important especially in the case of the application of the devices basing on the nonlinear interaction of the waves of the higher intensity as parametric sonars. The distribution of nonlinear parameter B/A in the sea water characterizes the special efficiency of the energy transformation from primary waves to the difference frequency wave and also to higher harmonics. This paper gives some contribution to properties of the sea water in the area of the Gotland Deep. In a real environment because of internal waves (Rybak, Serebryany, 2011) the field of acoustic nonlinearity can be subject to fluctuations.

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