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ANALYSIS OF FACTORS AFFECTING DESTABILIZATION OF A VISCOUS LIQUID FLOW IN CHANNELS

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An analysis of the influence of inertia forces and ponderomotive forces on the destabilization of the flow of viscous fluids in the hydrodynamic initial section is given. Cases of flow of viscous, anomalously viscous and electrically conductive liquids are considered; the degree of influence of mass forces on the destabilization of the flow is estimated. As applied to the flow in the hydrodynamic initial section, the degree of influence of inertia forces from convective acceleration and forces with a magnetic nature can be different. Inertia forces stimulate the accelerated movement of the fluid, and in the case of forces with a magnetic nature, ponderomotive forces contribute to deceleration, which is confirmed by the results of studies of the velocity field. Recommendations are given for calculating the length of the hydrodynamic initial section in the presence of mass forces with different nature.

Key words: hydrodynamic initial section, ponderomotive forces, length of the initial section, Hartmann number, unstabilized fluid flow.

1. Introduction

The problem associated with the destabilization of the flow in the field of action of body forces finds wide practical applications in machine-building hydromechanics. In addition to the forces of inertia from convective acceleration, as is known, forces of a magnetic nature (ponderomotive forces) can lead to destabilization of the flow. If the inertial forces can contribute to the acceleration of the flow, then the ponderomotive forces lead to the deceleration of the fluid flow. An analysis of the influence of this type of force on the nature of the flow can contribute to the process of "control" of the hydrodynamic characteristics of the flow, in particular, when solving problems of magnetohydrodynamics related to the design of flow meters, hydraulic drive seal systems, as well as in a number of other technological processes associated with the influence of a magnetic field on the energy characteristics of hydraulic equipment and heat and mass transfer processes.

Quite often, destabilization of the flow is observed in equipment associated with the processes and apparatuses of chemical technology, in the oil refining industry and in physicochemical mechanics. The factors influencing the flow destabilization can be the physicochemical and rheological properties of liquid media, the manifestation of inertia forces from convective acceleration, heat transfer conditions, and the influence of

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ponderomotive forces of a magnetic nature on the flow of an electrically conductive fluid [1-3]. A number of works by [4-6] are devoted to the study of these problems. At the same time, qualitative and quantitative estimates of flow destabilization in the field of magnetic forces have not been sufficiently studied so far. In this regard, in this paper, an attempt is made to analyze the flow hydrodynamics under a magnetic field in order to obtain characteristics associated with energy losses in the channels and the size of the flow destabilization zone in accordance with the forces of inertia and ponderomotive forces.

2. Analysis

In order to study the degree of joint influence of inertia forces from convective acceleration and ponderomotive forces, the flow in the channel is considered (Fig.1), which is described in the Cartesian coordinate system by the system of equations (2.1):

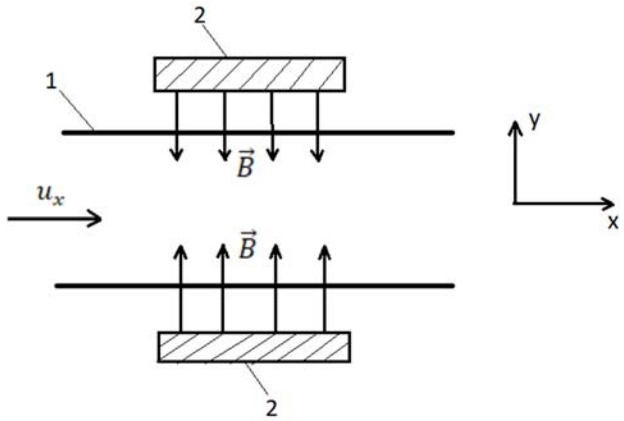

Fig.1. Channel scheme: 1 - channel, 2 - magnetic field source.

$$\begin{cases} \rho\left(u_x \dfrac{\partial u_x}{\partial x} + u_y \dfrac{\partial u_x}{\partial y}\right) = -\dfrac{\mathrm{d}p}{\mathrm{d}x} + \mu \dfrac{\partial^2 u_x}{\partial y^2} - \dfrac{\sigma B_0^2}{c^2}\dfrac{\partial u_x}{\partial y}, \\ \dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y} = 0. \end{cases} \tag{2.1}$$

In this equation, the influence of the magnetic field takes into account the last term

$$\left(\frac{\sigma B_0^2}{c^2}\frac{\partial u_x}{\partial y}\right).$$

When analyzing these equations, ponderomotive forces are considered as a quantity determined from the following expression:

$$\overrightarrow{F_m} = \left[\vec{j} \times \vec{B}\right] + \frac{\mu - \mu_0}{2\mu}\nabla \overrightarrow{B^2} \tag{2.2}$$

where μ is the magnetic permeability, μ_0 is the vacuum magnetic permeability, $\vec{B}$ is the magnetic induction, $\vec{j}$ is the current density.

When analyzing this equation, it is possible to solve the following problems:

- stabilized flow of Newtonian and non-Newtonian fluids, provided that there is no influence of the magnetic field and inertia forces from convective acceleration. Such flows, especially non-Newtonian fluids, are described quite fully in the literature. Table 1 presents some of the results of such studies:

Tab.1. Dependence of flow characteristics on the rheological properties of the liquid.

Rheological law and its author	Speed distribution law	Flow formula
Bingham $\tau = \tau_0 + \mu_b \frac{dv_x}{dr}$	$v_x(r) = \frac{R^2}{4\mu_b}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right] + \frac{R\tau_0}{b}\left[1-\left(\frac{r}{R}\right)^2\right]$	$Q = \frac{\pi R^4}{8\mu_b}\frac{\partial p}{\partial x}\left[1 - \frac{8}{3}\frac{\tau_0}{R\frac{\partial p}{\partial x}} + \frac{16}{3}\left(\frac{\tau_0}{R\frac{\partial p}{\partial x}}\right)^4\right]$
Ostwald – de Waele $\tau = K\left(\frac{dv_x}{dr}\right)^n$	$v_x(r) = \frac{nR}{n+1}\left(\frac{R}{2K}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]$	$Q = \frac{\pi n(3n+1)}{(n+1)^2}\left(\frac{1}{2K}\frac{\partial p}{\partial x}\right)^{\frac{1}{n}} R^{\frac{3n+1}{n}}$
Herschel – Barkley $\tau = \tau_0 + \left(\mu\frac{dv_x}{dr}\right)^n$	$v_x(r) = \frac{2}{\mu\frac{\partial p}{\partial x}\frac{n+1}{n}}\left[\left(R\frac{\partial p}{\partial x} - \tau_0\right)^{\frac{n+1}{n}}\right] + -\left(\frac{1}{2}\frac{\partial p}{\partial x} - \tau_0\right)^{\frac{n+1}{n}}$	$Q = \frac{4\pi}{\mu^{\frac{n+1}{n}}}\left(\frac{\partial p}{\partial x}\right)^{-1}\left\{\frac{R^2}{2}\left(\frac{R}{2}\frac{\partial p}{\partial x} - \tau_0\right)^{\frac{n+1}{n}} + -\frac{2n}{1+2n}\left(\frac{\partial p}{\partial x}\right)^{-1}\left[R\left(\frac{R}{2}\frac{\partial p}{\partial x} - \tau_0\right)^{\frac{1-2n}{2n}} + -\frac{2n}{n+3}\left(\frac{\partial p}{\partial x}\right)^{-1}\left(\frac{R}{2}\frac{\partial p}{\partial x} - \tau_0\right)^{\frac{1-3n}{3n}}\right]\right\}$
Casson $\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\mu_b\frac{dv_x}{dr}}$	$v_x(r) = \frac{1}{\mu_b}\left[\frac{1}{4}\frac{\partial p}{\partial x}\left(R^2 - r_0^2\right) + -\frac{4}{3}\left(\frac{\tau_0}{2}\frac{\partial p}{\partial x}\right)^{\frac{1}{2}}\left(\sqrt{R^3} - \sqrt{r_0^3}\right) + +\tau_0\left(R - r_0\right)\right]$	$Q = \frac{\pi}{\mu_b}\left(\frac{R^4}{8}\cdot\frac{\partial p}{\partial x}\right) - \frac{4}{7}\left(\frac{\tau_0}{2}\cdot\frac{\partial p}{\partial x}\right)^{\frac{1}{2}} R^3\sqrt{R} + +\frac{1}{3}\tau_0 R^3 - \frac{2}{21}\left(\frac{\partial p}{\partial x}\right)^{-3}\tau_0^4$

- unstabilized flow of viscous and anomalously viscous liquids in the absence of a magnetic field. Table 2 provides examples of such solutions for several non-Newtonian fluids.
- stabilized flow of electrically conductive liquids with a significant influence on the flow of ponderomotive forces. In this case, the equations of motion take the form:

Tab.2. Determination of the length of the hydrodynamic initial section.

Rheological law and its author	Length	Pressure loss	Velocity plot view	Author
Ostwald – de Waele law	$\mathcal{L}_{IS} = 0.122\,\mathrm{Re}\,D$	$\frac{\Delta p}{0.5\rho u_{cp}^2} = \frac{[2^{n+2}(1+3n)/n]^2}{\mathrm{Re}} \frac{L}{D} + 1.33$	-	M. Collins, W. Schowalter
Ostwald – de Waele law	$\mathcal{L}_{IS} = 0.101\,\mathrm{Re}\,D$	$\frac{\Delta p}{0.5\rho u_0^2} = \frac{2^{n+2}[(1+3n)/n]^2}{\mathrm{Re}} \frac{L}{D} + 1.22$	Parabola in the braking zone	Tomita
Shvedov – Bingham law	$\mathcal{L}_{IS} = 0.025\,\mathrm{Re}\,D$	$\frac{\Delta p}{\gamma} = \frac{u_0^2}{2g}(\omega_2 - \omega_1)$	Parabola in the near-wall region	L. Leibenson
Rheological law $[\tau_{ij}] = \mu_1[\dot{\gamma}_{ij}] + \mu_2[\dot{\gamma}_{ij}]^2 + \mu_3[\dot{\gamma}_{ij}]^3 + \ldots$	$\mathcal{L}_{IS} = \mathrm{A}H\mathrm{Re}$, $\mathrm{A} = \mathrm{const}$, $\mathrm{A} = f(\mu_1, \mu_2, \mu_3)$	$\Delta p = \frac{3}{2}\left(\frac{u_n - 2}{a^2}\right)^2 + \ldots - \mathrm{A}(2-L)$	$u = 2u_{cp}\left\{1 - \frac{r^2}{R^2} + \frac{\mu_{2n+1}}{\mu_1}\left(\frac{\mathrm{A}R}{2\mu}\right)^{2n}\left[\frac{2}{n+2}\left(1-\frac{r^2}{R^2}\right) + -\frac{1}{n+1}\left(1-\left(\frac{r}{R}\right)^{2n+2}\right)\right]\right\}$	Kapoor, Gupta

$$\rho\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right) = 0, \tag{2.3}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u_x}{\partial y^2} - \frac{\sigma B^2(x)}{c^2} u_x + \frac{\sigma E^*(x) B(x)}{c}. \tag{2.4}$$

The solution of these equations can be represented as follows:

$$\frac{d^2u_x}{dy^2}-\frac{\sigma B_0^2}{\mu}u_x=\frac{\sigma E_0B_0}{\mu}+\frac{1}{\mu}\frac{dp}{dx}. \tag{2.5}$$

This equation is written under the assumption that the inertial forces from convective acceleration are small compared to the surface forces arising in the magnetic field. It is assumed that the local velocity is the component of only the transverse coordinate y. This equation is a differential equation of the second kind. Its solution should be sought as a sum of two functions $u(y)=\mathrm{u}_0(y)+u_1(y)$, where $\mathrm{u}_0(y),u_1(y)$ are partial solutions of the inhomogeneous differential equation. In order to determine the function $\mathrm{u}_0(y)$, we write the characteristic equation in the following form:

$$\frac{d^2u_x}{dy^2}+0\frac{du_x}{dy}-\frac{\sigma B_0^2}{\mu}u_x=0. \tag{2.6}$$

In this case, the following conditions are met:

$$\lambda^2+0\lambda-\frac{\sigma B_0^2}{\mu}=0,\quad \begin{cases}\lambda=-B_0\sqrt{\dfrac{\sigma}{\mu}},\\ \lambda=B_0\sqrt{\dfrac{\sigma}{\mu}}.\end{cases} \tag{2.7}$$

As is known [7, 8], the following expression can serve as a solution to this equation:

$$u_0(y)=c_1e^{-\lambda y}+c_2e^{\lambda y}=c_1\exp\left\{-B_0\sqrt{\frac{\sigma}{\mu}}y\right\}+c_2\exp\left\{B_0\sqrt{\frac{\sigma}{\mu}}y\right\}. \tag{1.8}$$

A particular solution to Eq.(2.5), that is, the definition of the function $u_1(y)$ can be obtained based on the equation:

$$0\,y+\frac{\sigma E_0B_0}{\mu}+\frac{1}{\mu}\frac{dp}{dx}=\frac{d^2u_x}{dy^2}-\frac{\sigma B_0^2}{\mu}u_x. \tag{2.9}$$

The solution in this case can be sought in the form of the following function:

$$u_1(y)=0\,y+\mathrm{A},\quad u_1^{\prime}=0,\quad u_1^{\prime\prime}=0. \tag{2.10}$$

After setting (2.10) in (2.9), we obtain an expression for A:

$$0-\frac{\sigma B_0^2}{\mu}\mathrm{A}=\frac{\sigma E_0B_0}{\mu}+\frac{1}{\mu}\frac{dp}{dx}, \tag{2.11}$$

which takes the following form:

$$\mathrm{A} = -\frac{E_0}{B_0} - \frac{1}{\sigma B_0^2}\frac{dp}{dx} \tag{2.12}$$

and essentially takes into account the influence of the transverse magnetic field on the flow. Thus, based on the above relations, the velocity component $u_1(y)$ can be represented as a function of electromagnetic parameters and pressure drop:

$$u_1(y) = -\frac{E_0}{B_0} - \frac{1}{\sigma B_0^2}\frac{dp}{dx}. \tag{2.13}$$

Using the expressions $\mathrm{u}_0(y)$ and $u_1(y)$ we can obtain a general dependence for the speed u, which takes the form:

$$u(y) = c_1 \exp\left\{-B_0\sqrt{\frac{\sigma}{\mu}}y\right\} + c_2 \exp\left\{B_0\sqrt{\frac{\sigma}{\mu}}y\right\} - \frac{E_0}{B_0} - \frac{1}{\sigma B_0^2}\frac{dp}{dx} \tag{2.14}$$

or

$$u(y) = c_1 \exp\left\{-Ha\frac{y}{L}\right\} + c_2 \exp\left\{Ha\frac{y}{L}\right\} - \frac{\sigma E_0 B_0 + \dfrac{dp}{dx}}{\sigma B_0^2}, \tag{2.15}$$

where c_1 and c_2 are constants of integration determined from the boundary conditions.

I. As boundary conditions, the conditions of fluid adhesion on the channel walls and the conditions for the value of the maximum velocity on the axis are taken, that is, these conditions can be written in the following form:

$$-\text{ conditions on the wall}: \quad y = \frac{H}{2}, \quad u(y) = 0 \tag{2.16}$$

$$-\text{ conditions on the axis}: \quad y = 0, \quad u(y) = u_{\max}. \tag{2.17}$$

Substitution of these conditions into Eq.(2.15) allows us to obtain two algebraic expressions for the quantities c_1 and c_2, namely;

$$0 = c_1 \exp\left\{-Ha\frac{H}{2L}\right\} + c_2 \exp\left\{Ha\frac{H}{2L}\right\} - \frac{\sigma E_0 B_0 + \dfrac{dp}{dx}}{\sigma B_0^2}, \tag{2.18}$$

$$u_{\max} = c_1 + c_2 - \frac{\sigma E_0 B_0 + \dfrac{dp}{dx}}{\sigma B_0^2}. \tag{2.19}$$

The constants are calculated as follows:

$$c_1 = -c_2 + u_{\max} + \frac{\sigma E_0 B_0 + \dfrac{dp}{dx}}{\sigma B_0^2} = -c_2 + u_{\max} + \Phi, \tag{2.20}$$

where

$$\Phi = \frac{\sigma E_0 B_0 + \dfrac{dp}{dx}}{\sigma B_0^2}. \tag{2.21}$$

Substitute (2.20) in (2.18):

$$0 = \exp\left\{-Ha\frac{H}{2L}\right\}\left(-c_2 + u_{\max} + \Phi\right)c_2 \exp\left\{Ha\frac{H}{2L}\right\} - \Phi,$$

$$0 = c_2\left(\exp\left\{Ha\frac{H}{2L}\right\} - \exp\left\{-Ha\frac{H}{2L}\right\}\right) + u_{\max}\exp\left\{-Ha\frac{H}{2L}\right\} + \Phi\left(\exp\left\{-Ha\frac{H}{2L}\right\} - 1\right),$$

$$c_2 = -\frac{u_{\max}\exp\left\{-Ha\dfrac{H}{2L}\right\} + \Phi\left(\exp\left\{-Ha\dfrac{H}{2L}\right\} - 1\right)}{2sh\left\{Ha\dfrac{H}{2L}\right\}}. \tag{2.22}$$

To determine c_1, we substitute (2.22) into (2.20):

$$c_1 = \frac{u_{\max}\exp\left\{Ha\dfrac{H}{2L}\right\} + \Phi\left(\exp\left\{Ha\dfrac{H}{2L}\right\} - 1\right)}{2sh\left\{Ha\dfrac{H}{2L}\right\}}. \tag{2.23}$$

Thus, using expressions (2.22) and (2.23), which determine c_1 and c_2 after substituting them into (2.15), we obtain the final expression for the distribution of velocities at the end of the hydrodynamic initial section in the flow under the action of a transverse magnetic field:

$$u(y) = \frac{u_{\max}\exp\left\{Ha\dfrac{H}{2L}\right\} + \Phi\left(\exp\left\{Ha\dfrac{H}{2L}\right\} - 1\right)}{2sh\left\{Ha\dfrac{H}{2L}\right\}}\exp\left\{-Ha\frac{y}{L}\right\} + \\ -\frac{u_{\max}\exp\left\{-Ha\dfrac{H}{2L}\right\} + \Phi\left(\exp\left\{-Ha\dfrac{H}{2L}\right\} - 1\right)}{2sh\left\{Ha\dfrac{H}{2L}\right\}}\exp\left\{Ha\frac{y}{L}\right\} - \Phi. \tag{2.24}$$

The advantage of this expression is that the local flow velocity at the end of the initial section is determined as a function of the maximum velocity on the channel axis.

II. It should be noted that in [9-15] a solution of a similar problem is presented for slightly different boundary conditions. Thus, it is assumed that the solution of Eq.(2.3), transformed into (2.15), should be found using the following boundary conditions:

$$-\text{ conditions on the upper wall: } \quad y=\frac{H}{2}, \quad u(y)=0\ , \tag{2.25}$$

$$-\text{ conditions on the bottom wall: } \quad y=-\frac{H}{2}, \quad u(y)=0. \tag{2.26}$$

Thus, using these boundary conditions in equation (2.15), the problem is reduced to determining the values c_1 and c_2 from the expressions:

$$0=c_1 \exp\left\{-Ha\frac{H}{2L}\right\}+c_2 \exp\left\{Ha\frac{H}{2L}\right\}-\Phi, \tag{2.27}$$

$$0=c_1 \exp\left\{Ha\frac{H}{2L}\right\}+c_2 \exp\left\{-Ha\frac{H}{2L}\right\}-\Phi. \tag{2.28}$$

The relation between c_1 and c_2 in this case takes the form:

$$c_1=-c_2+\frac{\Phi}{ch\left\{Ha\frac{H}{2L}\right\}}, \tag{2.29}$$

$$c_2=-c_1+\frac{\Phi}{ch\left\{Ha\frac{H}{2L}\right\}}. \tag{2.30}$$

Based on (2.29) and (2.30) we conclude that:

$$c_1=c_2=c_0. \tag{2.31}$$

We substitute (1.31) into (1.29):

$$c_0=\frac{\Phi}{2ch\left\{Ha\frac{H}{2L}\right\}}. \tag{2.32}$$

Thus, in this problem setting, the expression for the velocity distribution after substituting (2.31) and (2.32) into (2.15) takes the form:

$$u(y)=\frac{\Phi}{2ch\left\{Ha\frac{H}{2L}\right\}}\left(\exp\left\{-Ha\frac{y}{L}\right\}+\exp\left\{Ha\frac{y}{L}\right\}\right)-\Phi=\Phi\left(\frac{ch\left\{Ha\frac{y}{L}\right\}}{ch\left\{Ha\frac{H}{2L}\right\}}-1\right). \tag{2.33}$$

Using the boundary conditions (2.17), which are characteristic for the maximum speed, we select the value of the maximum speed in expression (2.33):

$$u_{\max}=\Phi\left(\frac{1}{ch\left\{Ha\frac{H}{2L}\right\}}-1\right)=\frac{1-ch\left\{Ha\frac{H}{2L}\right\}}{ch\left\{Ha\frac{H}{2L}\right\}}, \tag{2.34}$$

We rewrite Eq.(2.33) using (2.34):

$$u(y)=\Phi\frac{\left(ch\left\{Ha\frac{y}{L}\right\}-1\right)+\left(1-ch\left\{Ha\frac{H}{2L}\right\}\right)}{ch\left\{Ha\frac{H}{2L}\right\}}=u_{\max}+\Phi\frac{ch\left\{Ha\frac{y}{L}\right\}-1}{ch\left\{Ha\frac{H}{2L}\right\}}. \tag{2.35a}$$

Equation (2.35) is a consequence of the solution of the problem presented in [9-11, 16]. At the same time, on the basis of our studies, the expression for the velocity diagram can be obtained by substituting (2.34) into (2.24):

$$u(y)=\left[\frac{\left(1-ch\left\{Ha\frac{H}{2L}\right\}\right)\exp\left\{Ha\frac{H}{2L}\right\}}{2sh\left\{Ha\frac{H}{2L}\right\}ch\left\{Ha\frac{H}{2L}\right\}}+\frac{\exp\left\{Ha\frac{H}{2L}\right\}-1}{2sh\left\{Ha\frac{H}{2L}\right\}}\right]\Phi\exp\left\{-Ha\frac{y}{L}\right\}+$$
$$-\left[\frac{\left(1-ch\left\{Ha\frac{H}{2L}\right\}\right)\exp\left\{-Ha\frac{H}{2L}\right\}}{2sh\left\{Ha\frac{H}{2L}\right\}ch\left\{Ha\frac{H}{2L}\right\}}+\frac{\exp\left\{-Ha\frac{H}{2L}\right\}-1}{2sh\left\{Ha\frac{H}{2L}\right\}}\right]\Phi\exp\left\{Ha\frac{y}{L}\right\}-\Phi= \tag{2.35b}$$
$$=\frac{\Phi}{sh\left\{Ha\frac{H}{L}\right\}}\left[2sh\left\{\frac{Ha}{L}\left(\frac{H}{2}+y\right)\right\}-1\right].$$

The inconvenience of using this equation is that it includes $sh\left\{Ha\frac{H}{L}\right\}$. To simplify this dependence, the sh function can be represented as a Taylor series of the following notation:

$$sh\left\{Ha\frac{H}{L}\right\}=Ha\frac{H}{2L}, \tag{2.36}$$

$$2sh\left\{\frac{Ha}{L}\left(\frac{H}{2}+y\right)\right\}=\frac{Ha}{L}\left(\frac{H}{2}+y\right). \tag{2.37}$$

Then the velocity distribution law is simplified to the form:

$$u(y)=\frac{\Phi}{Ha}\frac{Ha(H+2y)-2L}{H}. \tag{2.38}$$

Thus, finally, after simple algebraic transformations, we have obtained a formula for calculating the velocity field for a stabilized flow in the field of magnetic forces, which includes, in addition to the hydraulic characteristics, also the characteristics of the magnetic field $\vec{B}$ and $\vec{E}$:

$$u(y)=\frac{2\left(\sigma E_0 B_0+\frac{dp}{dx}\right)}{\sigma B_0^2 H}(y)+\frac{\sigma E_0 B_0+\frac{dp}{dx}}{\sigma B_0^3 L}\left(\frac{\sigma}{\eta}\right)^{-\frac{1}{2}}\frac{L}{H}\left(B_0 H\left(\frac{\sigma}{\eta}\right)^{\frac{1}{2}}-2\right). \tag{2.39}$$

The results of such an analysis allow us to formulate the problem of an unstabilized flow of a viscous fluid in a transverse magnetic field for a hydrodynamic initial section, assuming that the diagram at the inlet of the initial section is close to rectangular, and at the outlet it corresponds to the curve of Eq.(2.39). As shown by the analysis of the presented solution and our experiments, one of the features of the considered flow is the influence of the magnetic field on the "stagnation" of the flow.

Electrically conductive fluid were used as working fluids in the experiment. Figure 2 shows the rheological characteristics of the fluids used at a temperature $18°C$.

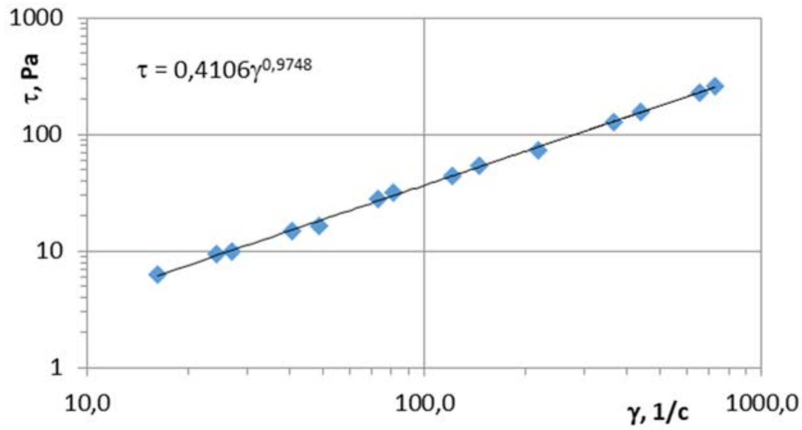

Fig.2. Dependency plot $\tau=f(\dot{\gamma})$.

The hydrodynamic parameters of the flow in the initial section are significantly affected by various factors, among which it is necessary to note the intensity of the magnetic field. With a steady flow of liquids, the existence of a magnetic field with a relatively large Hartmann number (*Ha*) contributes to the appearance of a quasi-solid flow zone, that is, the appearance of the magnetic plasticity effect, which is especially significant for non-Newtonian liquids [4]. There are a number of theories that explain this phenomenon. In accordance with some of them, the interaction of hydrodynamic and magnetic flows is explained as follows: when a liquid with electrically conductive properties moves through a channel, an EMF is induced in it due to a magnetic field. Any changes in the flow velocity lead to the appearance of closed currents, which, in turn, distort the lines of the magnetic field applied to the flow. Thus, the liquid, as it were, drags along the magnetic lines of force, that is, the process of interaction of electromagnetic forces with the forces that determine the flow of the liquid is observed. In addition, electromagnetic forces, as usual, are directed against the movement of the fluid, therefore, the total electromagnetic forces, along with the forces of viscous friction, impede the movement of the flow. The velocity profile becomes more filled as a result of such an interaction. As the results of the experiment showed, for the initial section there is a ratio between the Re number and the *Ha* number. Such a ratio is shown in Fig.3, which makes it possible to evaluate for the initial section the influence of the ratio of inertial forces with forces with a magnetic nature.

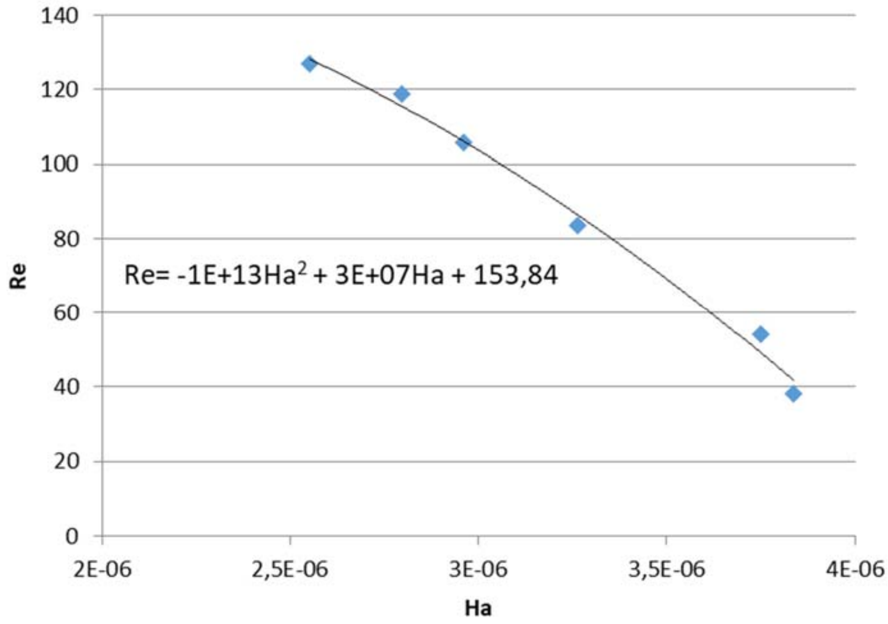

Fig.3. Dependency plot $\mathrm{Re} = f\left(Ha\right)$.

During the fluid flow, the development of the velocity profile is observed in the initial section. At significant *Ha* numbers, the decrease in the core of the flow in the initial section will occur until the pressure forces are balanced by the volume forces arising from the interaction of induced currents and the magnetic field (in some viscoplastic fluids, the viscosity increases under the action of a magnetic field). In the case of equilibrium of these forces, shear stresses at the boundary of the quasi-solid flow zone will practically be absent, therefore, further development of the flow will not be observed. As the number *Ha* increases, the distance from the channel entrance at which this effect will be observed decreases. In the case under consideration, the length of the initial section is a function not only of the Re number, but also of the *Ha* number, that is, according to [17], [18], [19], [20]:

$$\mathcal{L}_m = \frac{\mathcal{L}_{IS}}{Ha} = \frac{\mathrm{const}\,\mathrm{Re}\, d}{Ha}. \tag{2.40}$$

An analysis of studies on determining the length of the initial section in electrically conductive liquids showed that the length of the initial section can be determined by the formula:

$$\mathcal{L}_{IS} = 0.16\ \mathrm{Re}\, d + 3.8\, Ha\, d. \tag{2.41}$$

where *0.16* and *3.8* are constants obtained experimentally (*0.16* for the case when there is no magnetic field). Features of pressure change along the length of the hydrodynamic initial section depending on the *Ha* number are shown in Fig.4.

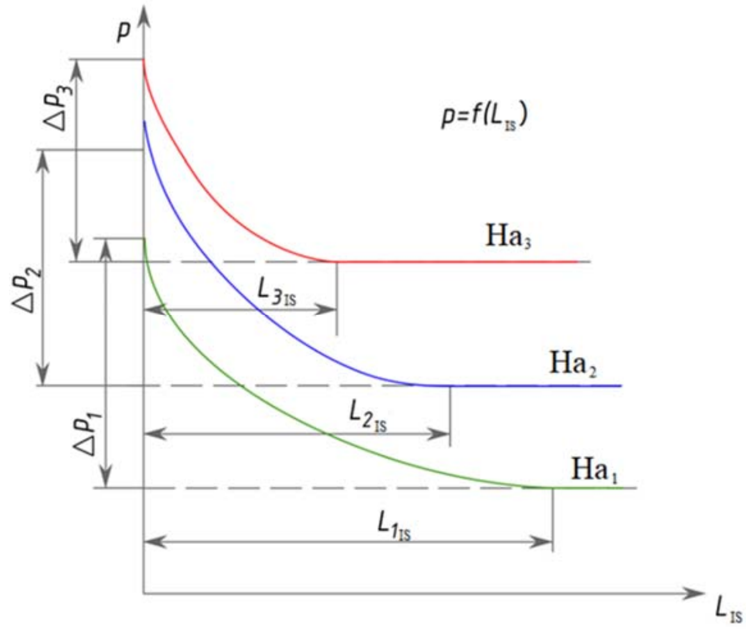

Fig.4. Dependency plot $p = f(\mathcal{L}_{IS})$ where $Ha_1 < Ha_2 < Ha_3$.

Thus, the length of the initial segment in a magnetic field is a function of the ratio of ponderomotive forces to viscous forces.

It should be noted that in the study of the hydrodynamic initial section in a magnetic field [7, 18, 21], the length of the initial section was assumed to be:

$$\mathcal{L}_{IS} = 0.5\,\mathrm{Re}\frac{R}{Ha^2} \tag{2.42}$$

and from the inlet to the boundary value zone, which differs from the stabilized flow by less than *1*%. In this case, the order of the Reynolds and Hartmann numbers is:

$$\mathrm{Re} \sim 10^5; \;\; Ha \sim 10^2. \tag{2.43}$$

Therefore, [18, 21] confirm the effect of magnetic plasticity, the presence of which significantly affects the length of the hydrodynamic initial section.

As mentioned earlier, the magnetic field in the hydrodynamic initial section contributes to the deceleration of the flow. The degree of deceleration can be estimated on the dependence of the flow rate of the fluid passing through the initial section on the magnetic field induction, which is shown in Fig.5.

Thus, the above analysis of the influence of mass forces of different nature allows us to conclude that it is possible to “control” the flow under various conditions of a destabilized flow, in particular, depending on the magnitude of the forces acting on the flow, to “regulate” the length of the hydrodynamic initial section.

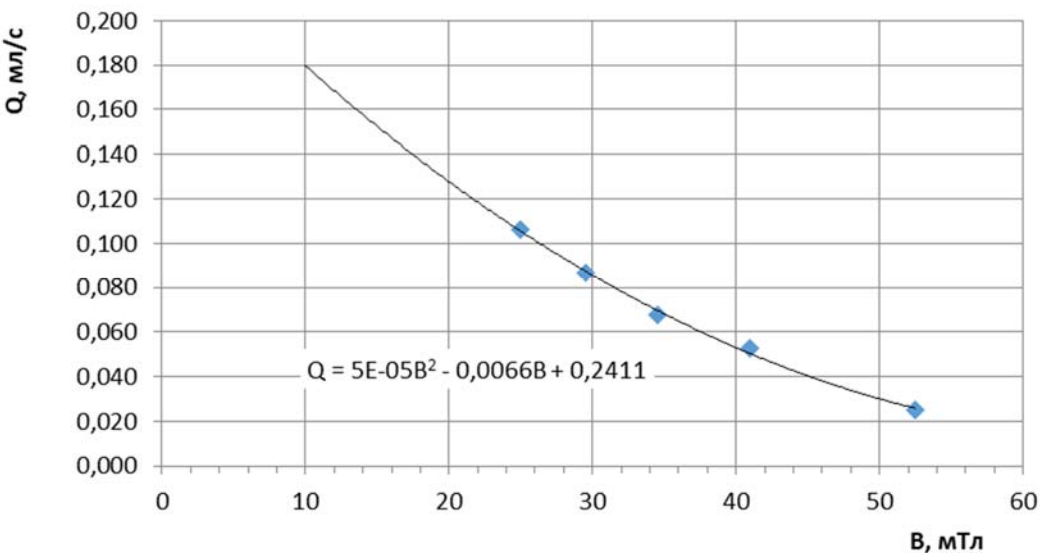

Fig.5. Dependency plot $Q = f(B)$.

3. Conclusion.

Depending on the rheological properties of the electrically conductive liquid, a conclusion about the characteristics of the flow in the initial section and the factors affecting them can be drawn. One of the important points is the effect of flow deceleration in the hydrodynamic initial section with an increase in the Hartmann criterion. This effect can have a significant impact on the pressure loss. In a general case, the pressure loss in the initial section can be represented by the sum of two components:

$$\Delta p = \Delta p_1 + \Delta p_2 \tag{3.1}$$

where Δp_1 corresponds to the steady flow losses and Δp_2 is a function of the Reynolds and Hartmann criteria for an unstabilized flow. These pressure losses can be considered as pressure losses for a stabilized flow over a fictitious channel length ΔL :

$$\Delta L = mHh \tag{3.2}$$

where m is the correction factor, H is the magnetic field strength, h is the channel width.

Nomeclature

$\boldsymbol{B}$ – magnetic induction
c – speed of light
D – channel diameter
$\boldsymbol{E}$ – electric field strength
H – liquid head
Ha – Hartmann number
$\boldsymbol{j}$ – current density
$\mathcal{L}_{IS}$ – length of the initial section
p – pressure

Q – fluid flow
R – channel radius
Re – Reynolds number
r – distance from center
η – automodel variable
ρ – liquid density
$\boldsymbol{u}$ – fluid flow velocity
$\boldsymbol{u_{cp}}$ – average flow velocity
μ – magnetic permeability
μ_0 – vacuum magnetic permeability
σ – conductivity
τ – shear stress in the flow
τ_0 – static shear stress

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