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The impact of launcher turret vibrations control on the rocket launch

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Abstract. The paper discusses the missile lift-off from a launcher placed on a motor vehicle. The stimulation of the assembly vibration results from the input generated by the road and the missile firing. The unfavourable input generated in the course of assembly operation has an impact on the characteristics of the initial missile flight parameters. The aim of the paper is to apply the hybrid vibroisolation system of the launcher turret in order to improve the conditions of the missile launch from the assembly. The active system reducing the occurring disturbance allows to control the launcher turret vibrations. Owing to that it is possible influence effectively the initial missile flight parameters.

Key words: missile, vibration, vibroisolation, initial flight parameters.

Denotations

$y_v,\ \dot{y}_v,\ \ddot{y}_v$ – displacement, velocity and linear acceleration of the turret in the vertical direction,

$\vartheta_v,\ \dot{\vartheta}_v,\ \ddot{\vartheta}_v$ – displacement, velocity and angular acceleration of the turret in the inclination motion,

$\varphi_v,\ \dot{\varphi}_v,\ \ddot{\varphi}_v$ – displacement, velocity and angular acceleration of the turret in the tilting motion,

$\xi_{p1},\ \dot{\xi}_{p1},\ \ddot{\xi}_{p1}$ – displacement, velocity and linear acceleration of the missile in the motion along the launcher guide,

$\varphi_{p1},\ \dot{\varphi}_{p1},\ \ddot{\varphi}_{p1}$ – displacement, velocity and angular acceleration of the missile in the motion round the longitudinal axis,

$\vec{r}_{p1}(r_{p1x_v}, r_{p1y_v}, r_{p1y_v})$ – vector determining the missile centre of inertia,

$\vec{V}_{p1}(V_{p1x_v}, V_{p1y_v}, V_{p1z_v})$ – vector of the missile linear velocity,

$V_{p1},\ \gamma_{p1},\ \chi_{p1}$ – absolute value and direction angles of the missile linear velocity vector,

$\vec{\omega}_{p1}(\omega_{p1x_p}, \omega_{p1y_p}, \omega_{p1z_p})$ – missile angular velocity vector,

$\vec{\varepsilon}_{p1}(\varepsilon_{p1x_p}, \varepsilon_{p1y_p}, \varepsilon_{p1z_p})$ – missile angular acceleration vector,

$P_{ss1},\ M_{ss1}$ – load generated by the take-off engine,

$m_{p1},\ I_{p1x_p},\ I_{p1y_p},\ I_{p1z_p}$ – mass and moment of inertia of the missile,

g – acceleration of gravity,

$l_1,\ l_2,\ l_3,\ l_4,\ l_5,\ l_6,\ l_\xi,\ l_\eta,\ l_\zeta$ – direction cosines.

1. Introduction

A rocket taking off from a turret of a launcher located on a motor vehicle is the object of examination. The rocket is an anti-aircraft short-range missile with passive target-homing manner. Successful firing of the missile requires meeting certain safety standards and imposing some technical restraints. After leaving the assembly the missile continues motion towards the target. Although the assembly has no direct impact on the missile during its flight, the assembly interference in this flight phase occurs at the moment the missile leaves the launcher. In an instant the system undergoes natural degeneration, i.e. becomes divided into two independent systems, namely the anti-aircraft assembly and the missile. The initial kinematic flight parameters are determined at that moment. The flight path is shaped depending on the value of these parameters as well. The homing system of the short-range missile has little time for developing an effective flight trajectory. Unfavourable values of the initial flight parameters developed by the anti-aircraft assembly may lead to the failure of the launched missile. Therefore one must prevent the desired values from deviation. The application of the hybrid vibroisolation system [1] of the launcher turret contributes to the improvement of the conditions during the missile take-off from the assembly, and thus successfully influences the characteristics of the initial flight parameters. The aim of the paper is to present how the active turret vibration reduction system [2] operates effectively while firing the missiles and generating road-related disturbances.

2. The physical model of the self-propelled anti-aircraft missile assembly

The designed physical model of the anti-aircraft assembly was constructed on the basis of classical mechanics [3]. It consists of the inertial elements in the form of material points, rigid

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bodies and non-inertial elements with restitutive and dissipative properties. Therefore, the formulated physical model of the anti-aircraft assembly should be categorized as a discrete system. Generally, the constructed model is composed of ten material points, four rigid bodies, four objects variable in time, one mathematical point, sixteen non-inertial elements of the four control systems [4] realizing the target tracking process and four systems controlling the turret vibrations [5].

The formulated physical model of the self-propelled anti-aircraft missile assembly consists of the following, elementary objects as shown in Fig. 1:

- Motor vehicle,
- Operator and driver in their seats,
- Launcher consisting of:
  - Base,
  - Turret composed of a platform along with a system of four guides,
  - Hybrid vibroisolation system of the turret,
- Four missiles with the gyroscope tracking systems,
- Target.

Taking into consideration solely the vibrations [6, 7] resulting from the operation of the anti-aircraft assembly the number of degrees of freedom of the elaborated model in the general case amounts to forty one.

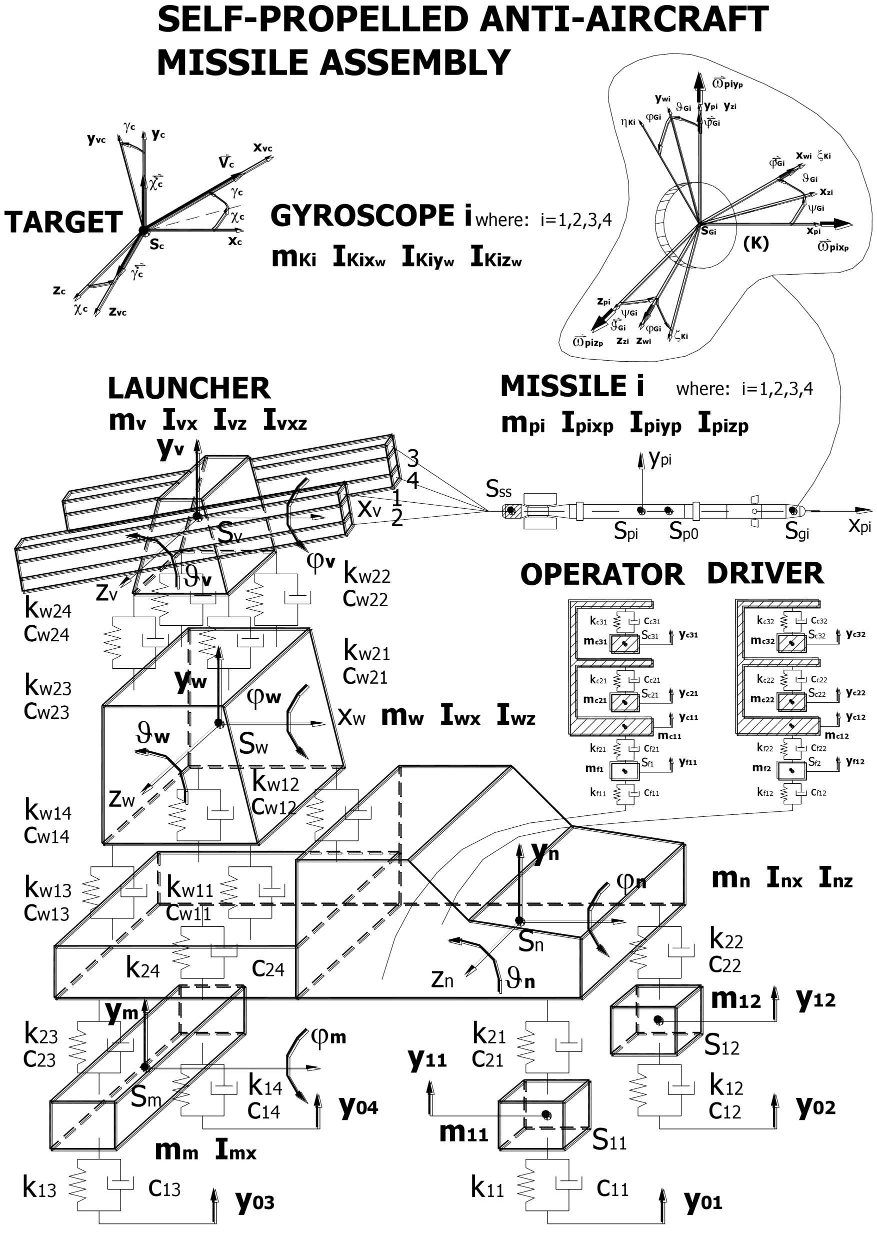

Fig. 1. Physical model of the self-propelled anti-aircraft missile assembly

The operation of the self-propelled anti-aircraft assembly is subjected to the comfort of the missile taking off from the launcher. The missile has a stiff connection to the guide before it is launched. The way it is attached makes it impossible for the missile to move towards the guide. The missile shell is in the shape of a cylinder, on which two guiding rings are housed. It is inserted into the tube guide with a certain interference. It means that the kinematic pair missile-guide form a specific type of fit. The missile touches the launcher guide with the guiding rings. It is between the rings and the inner wall of the guide that the slide connection is formed. The precision of the manufactured units under discussion guarantees the correctness of relative motion allowed by the structure of the discussed kinematic pair. The take-off engine, whose aim is to set the proper linear and angular missile velocity, is in the rear part of the shell. Therefore the missile on the launcher moves along the guide and at the same time round the longitudinal axis. The angular and linear relocation is realized in an unambiguously defined way. In the course of missile motion along the guide the guiding rings guarantee collinearity of the points located on the missile longitudinal axis with relation to the points located on the longitudinal axis of the guide. It is assumed that after leaving the guide by one of the guiding rings the coellinearity is also present. The system structure results from the accepted geometrical assumptions and determines the kinematic possibilities of the missile towards its mobility. Figures 2 and 3 present a momentary location of the missile during its motion towards the guide along with the geometric characteristics within the scope indispensable for conducting the analysis.

Several stages connected with the change of the parameters characterizing the missile inertia are taken into consideration in the course of the anti-aircraft assembly operation. These parameters include the missile mass, its moments of inertia and the location of the mass centre. In each of the distinguished stages one of the two properties is ascribed to the missile.

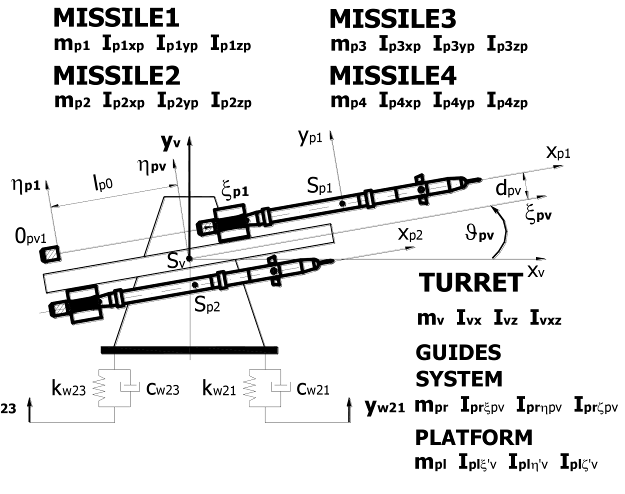

Fig. 2. The main view of the turret and missiles model

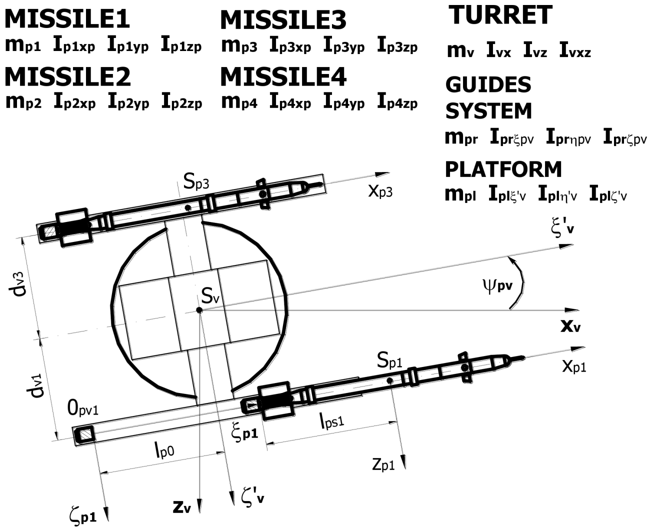

Fig. 3. The top view of the turret and missiles model

The missile has the features of either a rigid body or a system variable in time. The dual character of the missile results from the function it has as an element of the anti-aircraft assembly. The parameters characterizing the missile inertia change with time or remain stable. From the moment of target interception to the moment of starting the take-off engine the parameters do not change, and the missile is a rigid body. The next stage is connected with starting the take-off engine and with the missile motion along the launcher guide till the burn-out of the powder charge. The parameters change in a continuous mode, and the missile is a system variable in time. After the completion of this stage the missile continues motion along the guide, its parameters do not change, and the missile is a rigid body. The moment the engine is disconnected from the missile shell the parameters change in a discrete mode. For an instant the missile becomes a system variable in time. After detaching the take-off engine the missile loses contact with the assembly and moves independently of it.

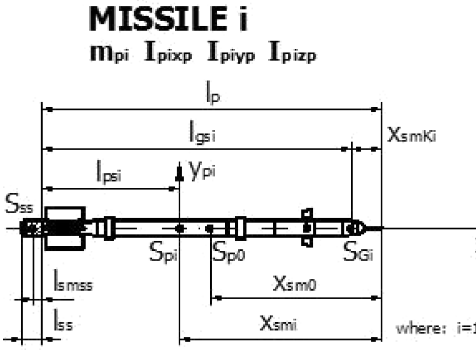

Fig. 4. Physical model of the missile

In the discussion concerning the anti-aircraft assembly analysis the missile is treated as an object variable in time [8, 9]. While formulating the model of the system the following assumptions are essential:

- The missile mass centre moves along the longitudinal axis of the shell,

- The only component of the reactive force is the rocket thrust,
- Mass and the missile moment of inertia are time function.

The formulated physical missile model along with its geometrical characteristics within the scope indispensable for conducting the analysis is presented in Fig. 4.

3. Mathematical model of the self-propelled anti-aircraft missile assembly

Indispensable coordinate systems, which enable unambiguous determination of the motion realized by particular anti-aircraft assembly units, were defined. On the basis of the assumed physical model the mathematical model of the self-propelled anti-aircraft assembly was elaborated. The model is determined by differential equations with common derivatives. These are conjugate equations. Analytic dependencies, which describe the assembly model in the general case consist of the motion equations of the system based on forty-one independent generalized coordinates, control units, kinematic dependencies, target motion equations, parameters defined by functions and twenty-one equations of static equilibrium. Generally, it is a non-linear model, determined, variable in time, dissipative and bounded. The assembly motion is examined in the three-dimensional Euclidean space.

On account of a large number of extensive equations of the self-propelled anti –aircraft missile assembly the article presents only the relative motion equations of the missiles, from which each makes a kinematic pair with the launcher guide. For determining missile motion defined as missile 1 towards the guide two independent coordinates were assumed: ξ_{p1}, φ_{p1}.

Motion equations:
(These are the two equations of the missile 1 motion towards the guide, from the 41 equations representing the motion of the self-propelled anti-aircraft assembly)

$$\begin{gathered}
m_{p1}\left[l_\xi^2+l_\eta^2+l_\zeta^2+\left(l_\xi^2+l_\eta^2\right)\vartheta_v^2\right. \\
\left.+\left(l_\eta^2+l_\zeta^2\right)\varphi_v^2\ -2l_\xi l_\zeta\vartheta_v\varphi_v\right]\ddot{\xi}_{p1} \\
+m_{p1}\left\{l_\eta\left(\xi_{p1}l_\xi+l_{\xi 1}\right)-l_\xi\left(\xi_{p1}l_\eta+l_{\eta 1}\right)\right. \\
-l_\zeta\left(\xi_{p1}l_\xi+l_{\xi 1}\right)\varphi_v \\
\left.+\left[l_\eta\left(\xi_{p1}l_\eta+l_{\eta 1}\right)+l_\xi\left(\xi_{p1}l_\xi+l_{\xi 1}\right)\right]\vartheta_v\right\}\ddot{\vartheta}_v \\
+m_{p1}\left\{l_\zeta\left(\xi_{p1}l_\eta+l_{\eta 1}\right)-l_\eta\left(\xi_{p1}l_\zeta+l_{\zeta 1}\right)\right. \\
-l_\xi\left(\xi_{p1}l_\zeta+l_{\zeta 1}\right)\vartheta_v \\
\left.+\left[l_\eta\left(\xi_{p1}l_\eta+l_{\eta 1}\right)+l_\zeta\left(\xi_{p1}l_\zeta+l_{\zeta 1}\right)\right]\varphi_v\right\}\ddot{\varphi}_v \\
+m_{p1}\left(l_\eta+l_\xi\vartheta_v-l_\zeta\varphi_v\right)\ddot{y}_v \\
+2m_{p1}\left[\left(l_\xi^2+l_\eta^2\right)\vartheta_v-l_\xi l_\zeta\varphi_v\right]\dot{\xi}_{p1}\dot{\vartheta}_v \\
+2m_{p1}\left[\left(l_\eta^2+l_\zeta^2\right)\varphi_v-l_\xi l_\zeta\vartheta_v\right]\dot{\xi}_{p1}\dot{\varphi}_v \\
+m_{p1}\left[l_\eta+l_\xi\vartheta_v-l_\zeta\varphi_v\right]g=P_{ss1},
\end{gathered} \tag{1}$$

$$I_{p1x_p}\left(\ddot{\varphi}_{p1}+l_1\ddot{\vartheta}_v+l_2\ddot{\varphi}_v\right)=M_{ss1}, \tag{2}$$

where

$$\begin{gathered}
l_\xi=\cos\vartheta_{pv}\cos\psi_{pv}, \\
l_\eta=\sin\vartheta_{pv}, \\
l_\zeta=-\cos\vartheta_{pv}\sin\psi_{pv}, \\
l_{\xi 1}=\left(l_{ps1}-l_{p0}\right)l_\xi-d_{pv}\sin\vartheta_{pv}\cos\psi_{pv}+d_{v1}\sin\psi_{pv}, \\
l_{\eta 1}=\left(l_{ps1}-l_{p0}\right)l_\eta+d_{pv}\cos\vartheta_{pv}, \\
l_{\zeta 1}=\left(l_{ps1}-l_{p0}\right)l_\zeta+d_{pv}\sin\vartheta_{pv}\sin\psi_{pv}+d_{v1}\cos\psi_{pv},
\end{gathered}$$

Kinematic dependencies complete the motion equations:
(These are equations representing the kinematic dependencies relating to missile 1 motion)

Mass Centre location S_{p1} of missile 1 within the system of coordinates $0_v x_v y_v z_v$:

$$\vec{r}_{p1}\left(r_{p1x_v}, r_{p1y_v}, r_{p1y_v}\right) \tag{3}$$

$$\begin{gathered}
r_{p1x_v}=\xi_{p1}l_\xi+l_{\xi 1}-\left(\xi_{p1}l_\eta+l_{\eta 1}\right)\vartheta_v, \\
r_{p1y_v}=\left(\xi_{p1}l_\xi+l_{\xi 1}\right)\vartheta_v-\left(\xi_{p1}l_\zeta+l_{\zeta 1}\right)\varphi_v+\xi_{p1}l_\eta+l_{\eta 1}+y_v, \\
r_{p1z_v}=\left(\xi_{p1}l_\eta+l_{\eta 1}\right)\varphi_v+\xi_{p1}l_\zeta+l_{1i}.
\end{gathered}$$

The coordinates of the velocity vector of the mass centre S_{p1} of missile 1within the system of coordinates $0_v x_v y_v z_v$:

$$\vec{V}_{p1}\left(V_{p1x_v}, V_{p1y_v}, V_{p1z_v}\right), \tag{4}$$

$$\begin{gathered}
V_{p1x_v}=\left(l_\xi-l_\eta\vartheta_v\right)\dot{\xi}_{p1}-\left(\xi_{p1}l_\eta+l_{\eta 1}\right)\dot{\vartheta}_v, \\
V_{p1y_v}=\left(l_\eta+l_\xi\vartheta_v-l_\zeta\varphi_v\right)\dot{\xi}_{p1}+\left(\xi_{pi}l_\xi+l_{\xi 1}\right)\dot{\vartheta}_v \\
-\left(\xi_{p1}l_\zeta+l_{\zeta 1}\right)\dot{\varphi}_v+\dot{y}_v, \\
V_{p1z_v}=\left(l_\zeta+l_\eta\varphi_v\right)\dot{\xi}_{p1}+\left(\xi_{p1}l_\eta+l_{\eta 1}\right)\dot{\varphi}_v.
\end{gathered}$$

The module and direction angles of the velocity vector of the mass centre S_{p1} and missile 1

$$V_{p1}=\sqrt{V_{p1x_v}^2+V_{p1y_v}^2+V_{p1z_v}^2}, \tag{5}$$

$$\sin\gamma_{p1}=\frac{V_{p1y_v}}{V_{p1}},$$

$$\sin\chi_{p1}=-\frac{V_{p1z_v}}{V_{p1}\cos\gamma_{p1}}.$$

The coordinates of the angular velocity vector of missile 1 in the system of coordinates $S_{p1}x_{p1}y_{p1}z_{p1}$:

$$\vec{\omega}_{p1}\left(\omega_{p1x_p}, \omega_{p1y_p}, \omega_{p1z_p}\right), \tag{6}$$

$$\begin{gathered}
\omega_{p1x_p}=\dot{\vartheta}_v l_1+\dot{\varphi}_v l_2+\dot{\varphi}_{p1}, \\
\omega_{p1y_p}=\dot{\vartheta}_v l_3+\dot{\varphi}_v l_4, \\
\omega_{p1z_p}=\dot{\vartheta}_v l_5+\dot{\varphi}_v l_6.
\end{gathered}$$

The coordinates of the angular acceleration vector of the missile 1 in the system of coordinates $S_{p1}x_{p1}y_{p1}z_{p1}$:

$$\vec{\varepsilon}_{p1}\left(\varepsilon_{p1x_p}, \varepsilon_{p1y_p}, \varepsilon_{p1z_p}\right), \tag{7}$$

$$\begin{gathered}
\varepsilon_{p1x_p}=\ddot{\vartheta}_v l_1+\ddot{\varphi}_v l_2+\ddot{\varphi}_{p1}, \\
\varepsilon_{p1y_p}=\ddot{\vartheta}_v l_3+\ddot{\varphi}_v l_4, \\
\varepsilon_{p1z_p}=\ddot{\vartheta}_v l_5+\varphi_v l_6,
\end{gathered}$$

where:

$$\begin{gathered}
l_1=-\cos\vartheta_{pv}\sin\psi_{pv}, \quad l_2=\cos\vartheta_{pv}\cos\psi_{pv}, \\
l_3=\sin\vartheta_{pv}\sin\psi_{pv}, \quad l_4=-\sin\vartheta_{pv}\cos\psi_{pv}, \\
l_5=\cos\psi_{pv}, \quad l_6=\sin\psi_{pv}.
\end{gathered}$$

4. The system stabilizing the launcher turret

The hybrid system stabilizing the launcher turret [10, 11] was introduced in order to improve the launching conditions. Stabilizing the turret in the presence of the road-related input as well as resulting from launching is considered. Too intense turret vibrations may lead to exceeding the acceptable values of parameters characterizing the missile take-off process. As a result the missile may miss the chance of reaching the target before it leaves the launcher guide. The missile develops the initial flight parameters at the take-off. Whether the missile manages to elaborate the flight trajectory necessary for destroying the target depends on the characteristics of these parameters. Therefore the turret should isolate the missile from undesirable vibrations as effectively as possible [12, 13]. Four control devices installed in series in the turret platform suspension [14] were used to reduce the vibrations. All the four control systems operate independently of one another. Each of them stabilizes only one point of the platform suspension attachment, Fig. 5.

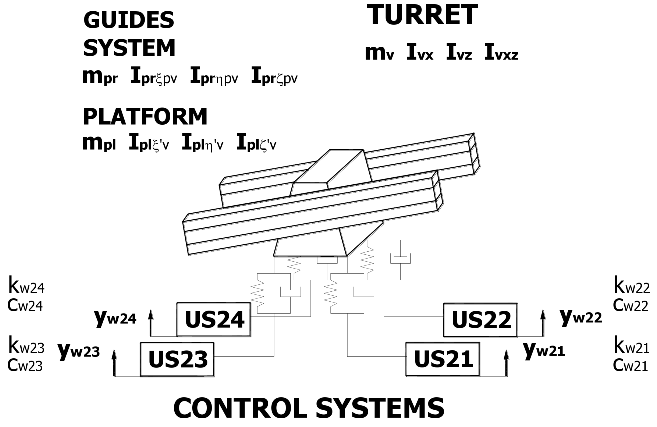

Fig. 5. Launcher turret stabilizing system

The regulation is conducted within a closed system. The control system structure is presented on the example of the US21 device, Fig. 6. The control loop is made by the passive suspension in the form of the Voight-Kelvin model along with the controlled performing system in the form of electro-hydraulic motor operator, two acceleration sensors installed in the turret platform and the base as well as the computer.

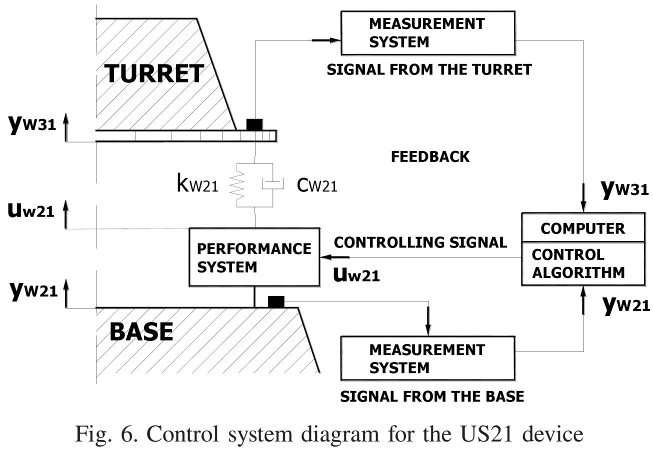

Fig. 6. Control system diagram for the US21 device

Making use of the double integrator the measurement system transmits the signal realized from the turret and the forcing signal from the base of the computer which on their basis determines the controlling signal. The formulation of the signal by the computer runs according to the assumed control algorithm. The four analogical control systems provide both linear and angular stability of the launcher turret in space.

The assumed control algorithm realized in the process of simulation takes the following form for the US21 device For the US22 device

$$\begin{aligned} u_{w21} &= k_{s21}y_{w21} + k_{s31}y_{w31}, \\ u_{w22} &= k_{s22}y_{w22} + k_{s32}y_{w32}, \\ \dot{u}_{w21} &= k_{s21}\dot{y}_{w21} + k_{s31}\dot{y}_{w31}, \\ \dot{u}_{w22} &= k_{s22}\dot{y}_{w22} + k_{s32}\dot{y}_{w32}. \end{aligned} \tag{8}$$

For the US23 device For the US24 device

$$\begin{aligned} u_{w23} &= k_{s23}y_{w23} + k_{s33}y_{w33}, \\ u_{w24} &= k_{s24}y_{w24} + k_{s34}y_{w34}, \\ \dot{u}_{w23} &= k_{s23}\dot{y}_{w23} + k_{s33}\dot{y}_{w33}, \\ \dot{u}_{w24} &= k_{s24}\dot{y}_{w24} + k_{s34}\dot{y}_{w34}, \end{aligned} \tag{9}$$

where

y_{w21}, y_{w22}, y_{w23}, y_{w24} – signals from the base,
y_{w31}, y_{w32}, y_{w33}, y_{w34} – signals from the turret,
$k_{s21} = k_{s22}$, k_{s23}, $k_{s24} = 1$ – control coefficients,
k_{s31}, k_{s32}, k_{s33}, $k_{s34} = 40$

5. Numerical simulation

The computer program in the Borland C++ system, which made it possible to conduct a numerical simulation of the formulated system operation, was edited on the basis of the elaborated mathematical model of the self-propelled anti-aircraft missile assembly. Owing to that it is possible to present an analysis of the effective operation of the active turret vibrations reduction system while launching missile 1 and generating road disturbances. The missile 1 takes off at the moment $t = 1$ s, and the road-related input in the form of a transverse hump, as in Fig. 7, affects the motor vehicle from the moment $t = 0$ s.

Road-related input.

Selected systems replies were presented. The examination aims at presenting the missile motion during the operation of the assembly considering in particular the characteristics of the initial flight parameters of the missile 1. The analysis is limited solely to the phenomena characteristic of mechanical reaction and is applied in the evaluation of their influence on the missile performance.

The computer program enables the simulation of the anti-aircraft missile assembly operation from the moment the target tracking process is started by the missile located on the launcher. The moment is selected by determining the initial target location towards the assembly for the time $t = 0$ s. The target performs independent motion, which can be treated as a defensive manoeuvre of the pilot. The launcher turret configuration results from the initial location of the platform and the system of guides: $\psi_{pv} = 45$ deg and $\vartheta_{pv} = 45$ deg. During the missile take-off along the guide some characteristic moments can be differentiated that appear during the motion. The interpretation of the particular characteristic moments is the following:

- Point 1 – the take-off engine starts work, moment of time: $t = 1$ s.
- Point 2 – the take–off engine finishes work, moment of time: $t = 1.07$ s.
- Point 3 – the missile leaves the launcher, moment of time: the time depends, among others, on the system dynamics.

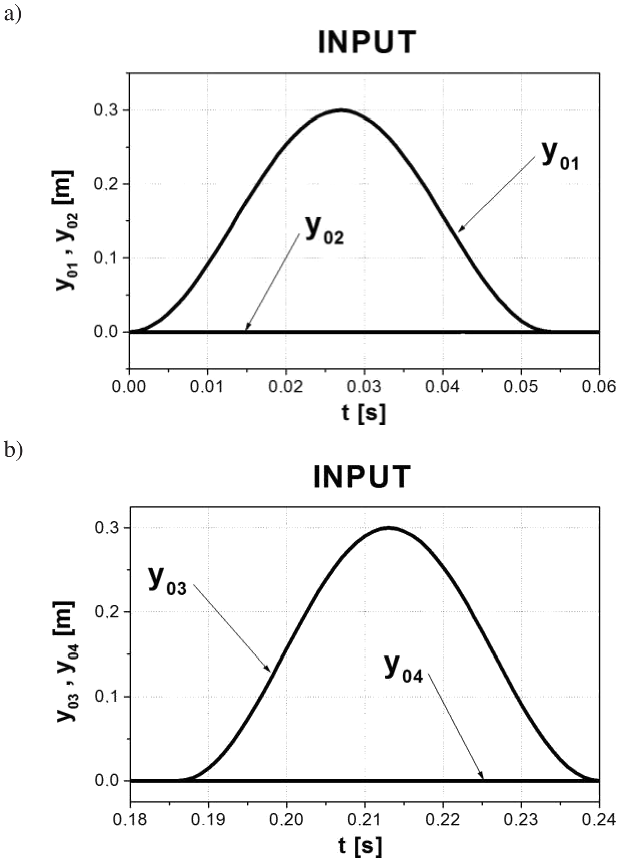

Fig. 7. Kinematic excitation affecting for: a) front and b) rear wheels of the vehicle

The turret motion is extremely important for the missile, which forms a kinematic pair with the guide. Both the linear and angular turret vibrations affect the course of variability of the kinematic values such as the vector of location of the missile mass centre, velocity and the missile angular acceleration. These vectors shape the characteristics of the initial kinematic parameters of the missile flight. The characteristics, among others, determines the trajectory realized by the missile.

The analysis focused on a hypothetical system, the parameters of which are close to those of the antiaircraft missile assembly "Poprad" and the missiles "Grad". The selected values of the system parameters are as follows:

a) Parameters describing the inert elements of the self-propelled vehicle:

- Vehicle body

$$m_n = 1780 \text{ kg}, \qquad I_{nx} = 872 \text{ kgm}^2,$$
$$I_{nz} = 2620 \text{ kgm}^2.$$

- Front axle along with the wheels

$$m_{11} = 57 \text{ kg}, \qquad m_{12} = 57 \text{ kg}.$$

- Rear axle along with the wheels

$$m_m = 157 \text{ kg}, \qquad I_{mx} = 70 \text{ kgm}^2.$$

b) Parameters describing the inert elements of the launcher:

b1) Base

$$m_w = 198 \text{ kg}, \qquad I_{wx} = 30 \text{ kgm}^2,$$
$$I_{wy} = 8.3 \text{ kgm}^2, \qquad I_{wz} = 30 \text{ kgm}^2,$$

b2) Turret

$$m_v = m_{pl} + m_{pr},$$
$$I_{vx} = \left(I_{pl\xi'_v} + I_{pr\xi_{pv}} \cos^2\vartheta_{pv} + I_{pr\eta_{pv}} \sin^2\vartheta_{pv}\right)\cos^2\psi_{pv} + \left(I_{pl\zeta'_v} + I_{pr\zeta_{pv}}\right)\sin^2\psi_{pv},$$
$$I_{vz} = \left(I_{pl\xi'_v} + I_{pr\xi_{pv}} \cos^2\vartheta_{pv} + I_{pr\eta_{pv}} \sin^2\vartheta_{pv}\right)\sin^2\psi_{pv} + \left(I_{pl\zeta'_v} + I_{pr\zeta_{pv}}\right)\cos^2\psi_{pv},$$
$$I_{vxz} = \left(I_{pl\xi'_v} + I_{pr\xi_{pv}} \cos^2\vartheta_{pv} + I_{pr\eta_{pv}} \sin^2\vartheta_{pv} - I_{pl\zeta'_v} - I_{pr\zeta_{pv}}\right)\cos\psi_{pv}\sin\psi_{pv}.$$

- Platform

$$m_{pl} = 34.2 \text{ kg}, \qquad I_{pl\xi'_v} = 0.96 \text{ kgm}^2,$$
$$I_{pl\eta'_v} = 0.7 \text{ kgm}^2, \qquad I_{pl\zeta'_v} = 1.27 \text{ kgm}^2.$$

- Four-guide system

$$m_{pr} = 85.2 \text{ kg}, \qquad I_{pr\xi_{pv}} = 23.11 \text{ kgm}^2,$$
$$I_{pr\eta_{pv}} = 32.23 \text{ kgm}^2, \qquad I_{pr\zeta_{pv}} = 9.86 \text{ kgm}^2.$$

c) Parameters describing the inert elements of the missile:

- Parameters constant in time

$$m_{p0} = 10.4 \text{ kg}, \qquad I_{p0x_p} = 0.00733 \text{ kgm}^2,$$
$$I_{p0y_p} = 1.7 \text{ kgm}^2, \qquad I_{p0z_p} = 1.7 \text{ kgm}^2,$$
$$x_{sm0} = 0.75 \text{ m}.$$

- Parameters variable in time

$$m_{pi} = m_{p0} + m_{ssi},$$
$$I_{pix_p} = I_{p0x_p} + I_{ssix_p},$$
$$I_{piy_p} = I_{p0y_p} + I_{ssiy_p} + m_{p0}\left(x_{smi} - x_{sm0}\right)^2 + m_{ssi}\left(l_p - x_{smi} + l_{smss}\right)^2,$$
$$I_{piz_p} = I_{p0z_p} + I_{ssiz_p} + m_{p0}\left(x_{smi} - x_{sm0}\right)^2 + m_{ssi}\left(l_p - x_{smi} + l_{smss}\right)^2,$$

$$x_{smi} = \frac{m_{p0}x_{sm0} + m_{ssi}\left(l_p + l_{smss}\right)}{m_{pi}},$$

$$l_{psi} = l_p - x_{smi}$$

d) Load generated by the take-off engine of the missile:

- Thrust force of the nozzle number 6 $P_{ssd6} = 667$ N.
- Thrust force of the nozzle number 1, 2, 3, 4 and 5 $P_{ssd2} = 616$ N.
- Angle defining the thrust forces direction of the nozzle number 1, 2, 3, 4, and 5 $\alpha_{ss} = 0.1745$ rad.
- Position of the axis of the nozzle number 1, 2, 3, 4, and 5

$$r_{ss} = 0.023 \text{ m},$$

$$P_{ssi} = \begin{cases} 0 & \text{for} \quad 0 \le t \le t_{pi0} \\ P_{ssd6} + 5P_{ssd2}\cos\alpha_{ss} & \text{for} \quad t_{pi0} < t \le t_{pi0} + t_{ss} \\ 0 & \text{for} \quad t > t_{pi0} + t_{ss} \end{cases}$$

$$M_{ssi} = \begin{cases} 0 & \text{for} \quad 0 \le t \le t_{pi0} \\ 5P_{ssd2}r_{ss}\sin\alpha_{ss} & \text{for} \quad t_{pi0} < t \le t_{pi0} + t_{ss} \\ 0 & \text{for} \quad t > t_{pi0} + t_{ss} \end{cases}$$

Figures 8 to 18 present the comparison of the turret and missile motion in the case of applying the hybrid (with the reduction) and passive (without the reduction) vibroisolation system.

Relocation, velocity and linear acceleration of the launcher turret in the vertical motion.

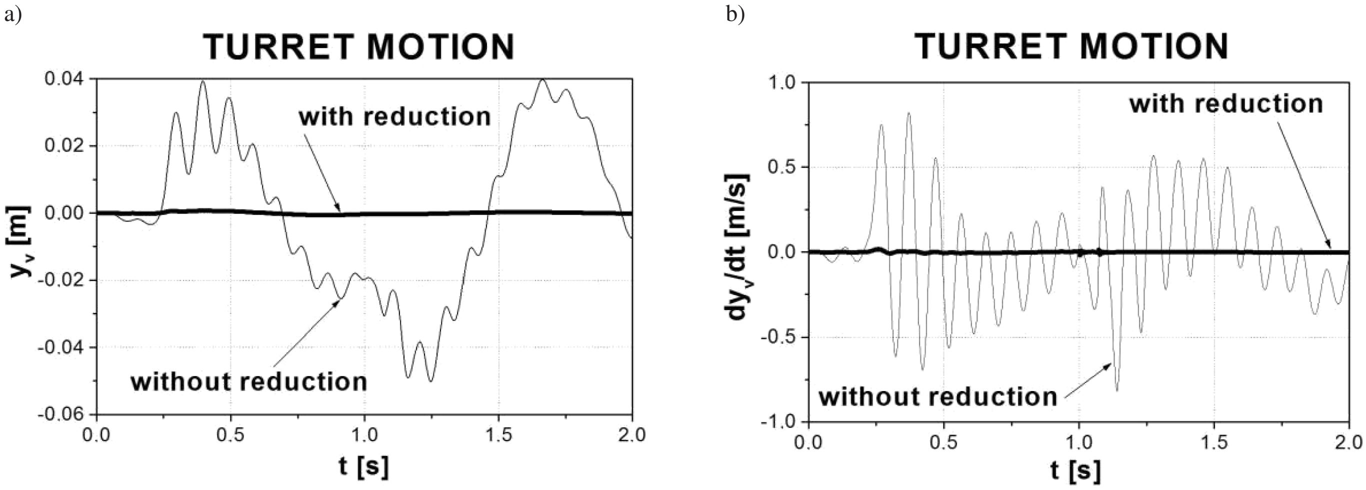

Fig. 8. a) Linear turret relocation, b) linear velocity of the turret

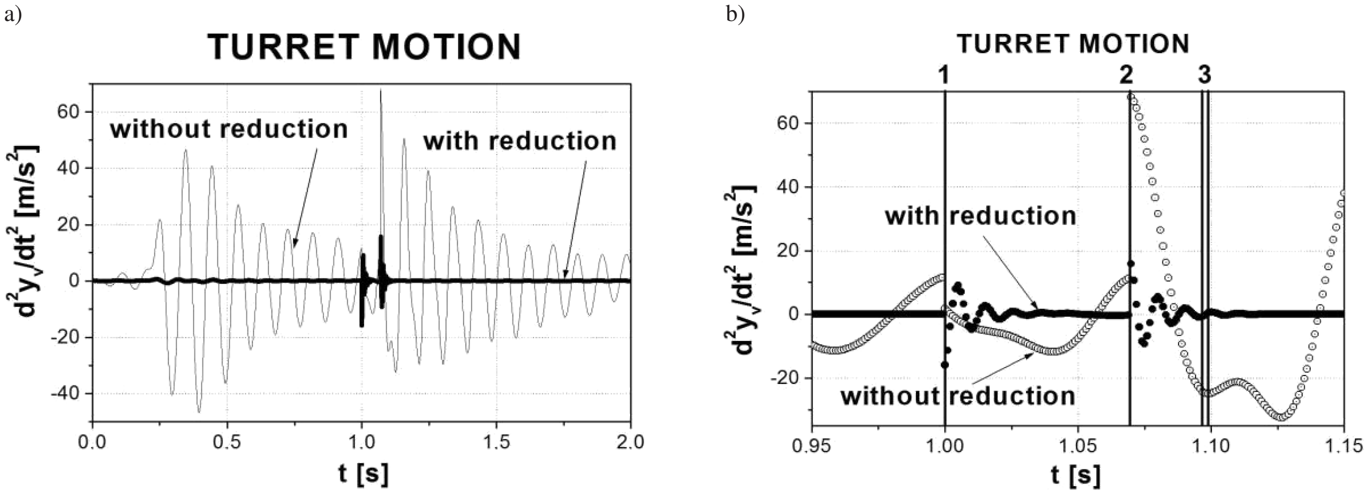

Fig. 9. a) Linear acceleration of the turret, b) linear acceleration of the turret at the take-off of the missile

Relocation, velocity and angular acceleration of the turret in the inclination motion.

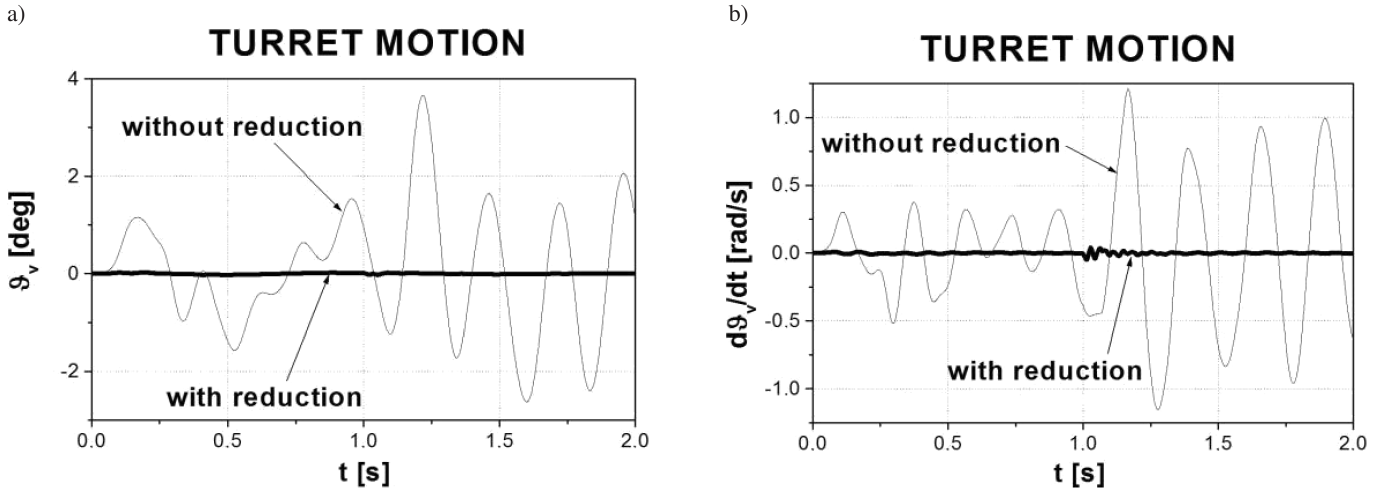

Fig. 10. a) Angular relocation of the turret in the inclination motion, b) angular velocity of the turret in the inclination motion

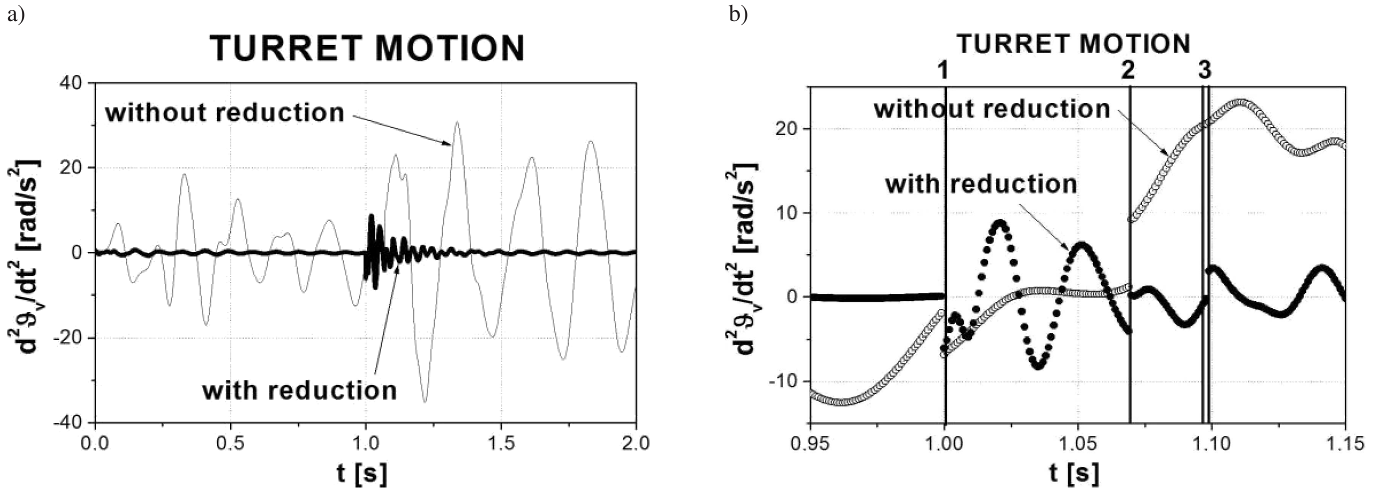

Fig. 11. a) Angular acceleration of the turret in the inclination motion, b) angular acceleration of the turret in the inclination motion at the take-off of missile 1

Relocation, velocity and angular acceleration of the turret in the inclination motion.

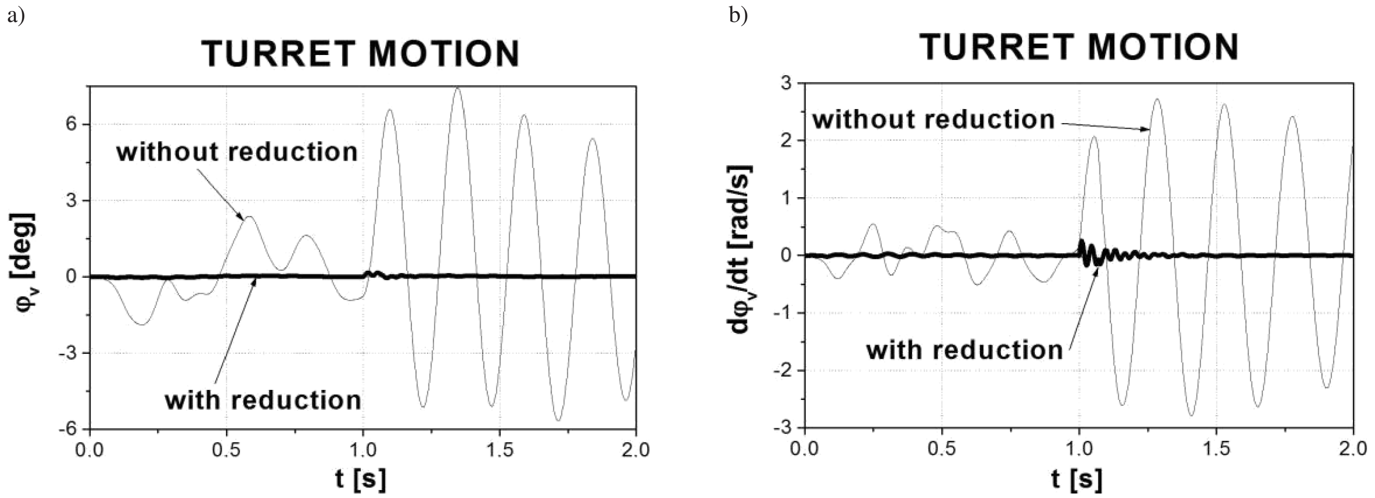

Fig. 12. a) Angular relocation of the turret in the inclination motion, b) angular velocity of the turret in the inclination motion

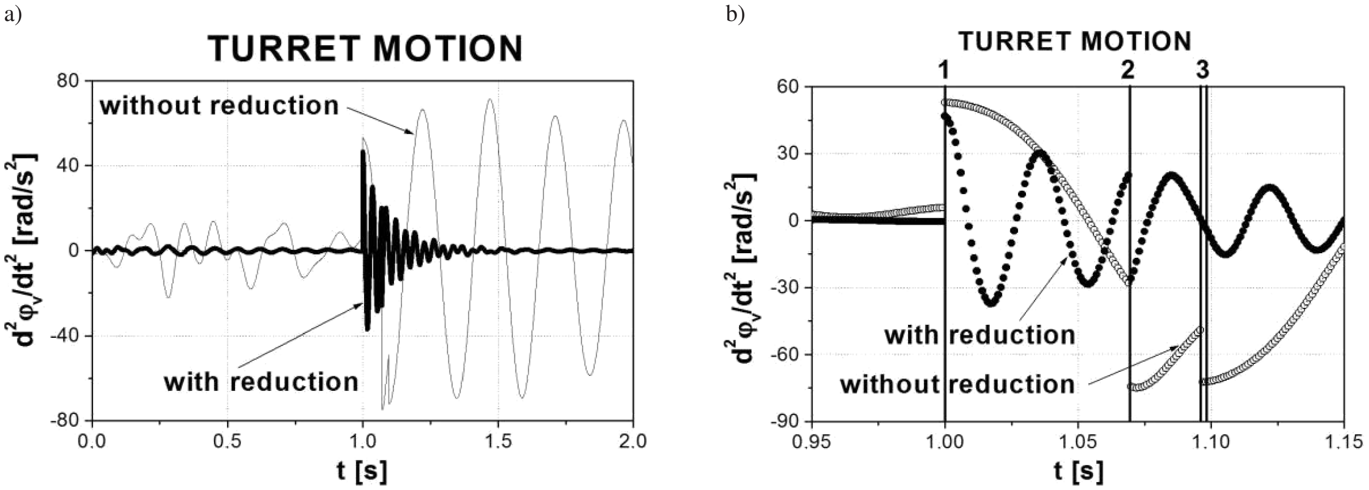

Fig. 13. a) Angular turret acceleration in the inclination motion, b) angular turret acceleration in the inclination motion at the take-off of missile 1

Module and direction angles of the linear velocity vector of the missile and linear missile velocity in the motion along the launcher guide.

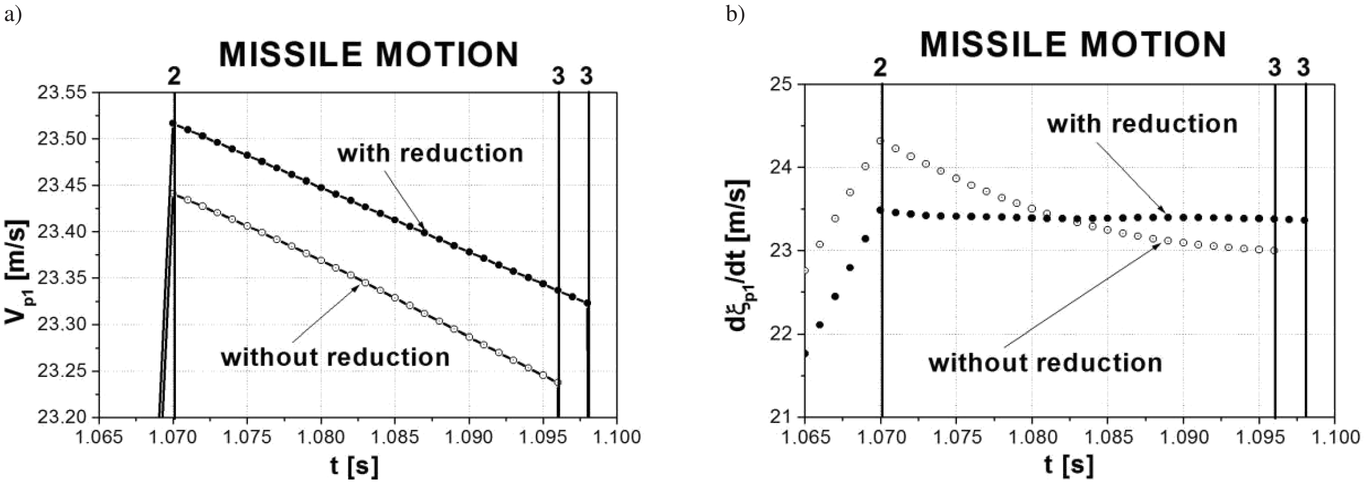

Fig. 14. a) Module of the missile linear velocity vector, b) linear velocity of the missile in the motion along the launcher guide

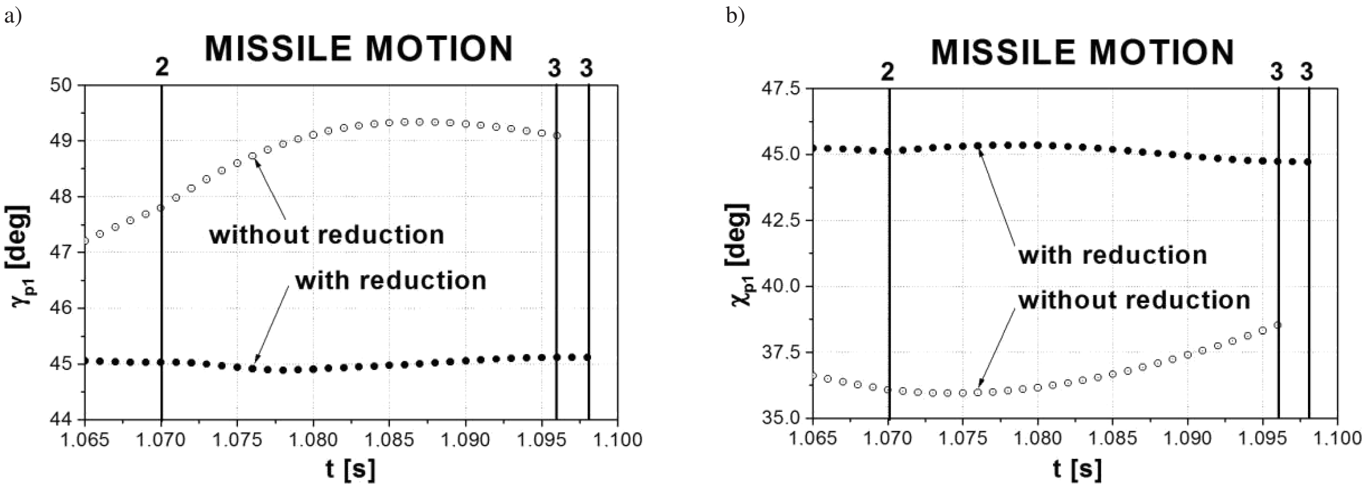

Fig. 15. a) Direction angle γ_{p1} of the missile linear velocity vector, b) direction angle χ_{p1} of the missile linear velocity vector

Missile angular velocity and angular acceleration vector components.

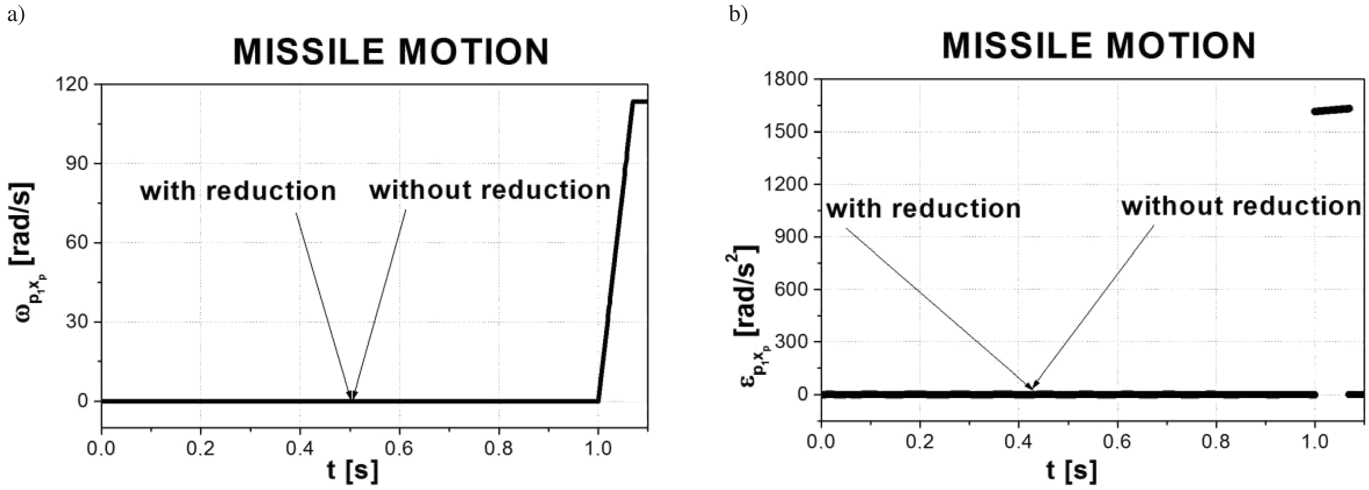

Fig. 16. a) Component $\omega_{p_1x_p}$ of missile angular velocity vector, b) component $\varepsilon_{p_1x_p}$ of missile angular acceleration vector

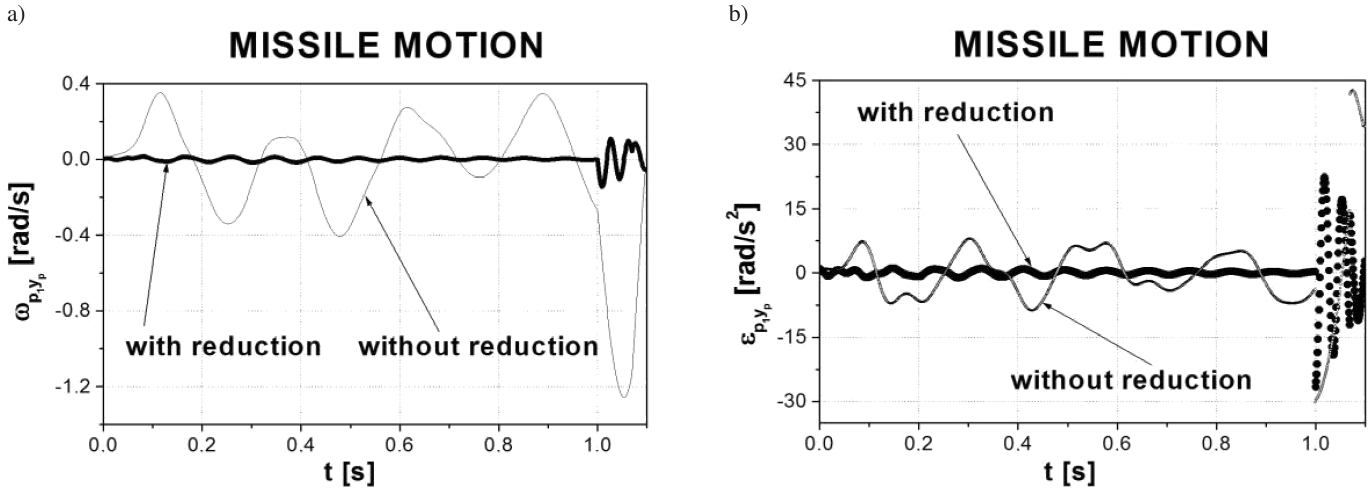

Fig. 17. a) Componenet $\omega_{p_1y_p}$ of missile angular velocity vector, b) component $\varepsilon_{p_1y_p}$ of missile angular acceleration vector

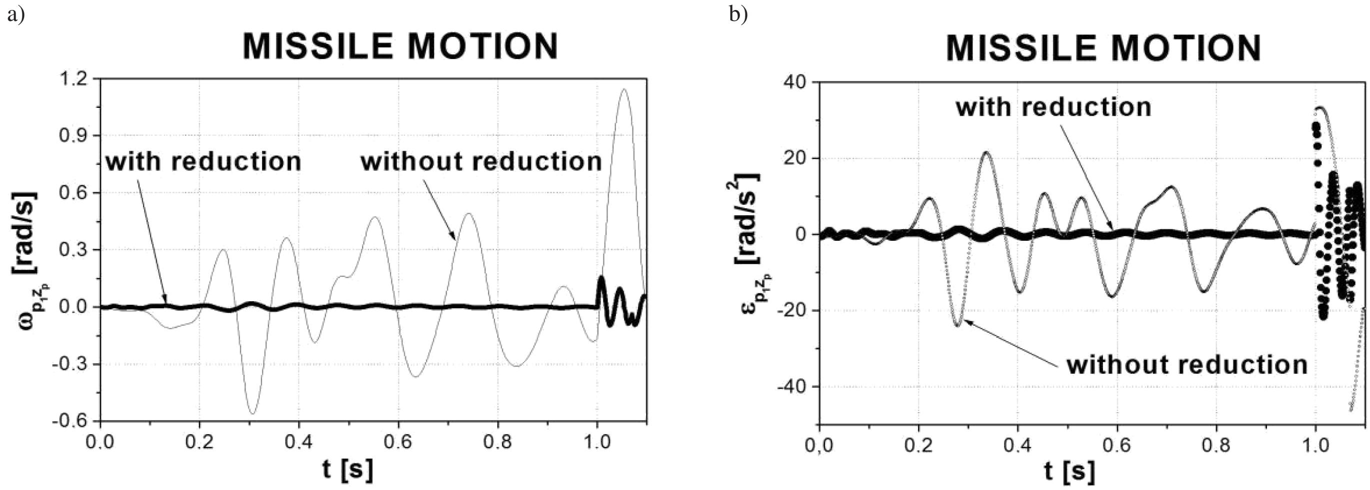

Fig. 18. a) Component $\omega_{p_1z_p}$ of missile angular velocity vector, b) component $\varepsilon_{p_1z_p}$ of missile angular acceleration vector

6. Conclusions

The study of the results of the dynamics analysis of the missile take-off from a launcher positioned on a motor vehicle brings to mind the following conclusions and observations:

- **The application of the active vibroisolation system effectively shapes the characteristics of the initial missile flight parameters.**
  - At the start of the take-off engine and the moment the missile leaves the guide the missile linear velocity vector keeps the desired direction. During the missile motion along the guide the vector direction is virtually invariable, and its module in the case of active vibroisolation reaches higher value.
  - Angular velocity component $\omega_{p_1x_1}$ reaches a more favourable value for the single-channel missile control system. The application of the hybrid vibroisolation lowers the fluctuation level of the angular velocity components $\omega_{p_1y_1}$ and $\omega_{p_1z_1}$ within the whole range of the assembly performance observation. In the course of missile motion along the guide and at the moment of its launching the values of these components are also lower.
  - The application of the active vibroisolation lowers the fluctuation level of the angular acceleration components $\varepsilon_{p_1y_1}$ and $\varepsilon_{p_1z_1}$ within the entire observation range of the assembly performance. In the course of missile motion along the guide the occurring value oscillation of these components at the time of launching is definitely smaller.
- **The application of the active vibroisolation system effectively shapes the turret motion.**
  - The missile constitutes a kinematic pair with the guide, which is an integral element of the turret. Both the linear and angular vibrations of the turret affect the variability course of such kinematic values as the vector of location and missile mass centre velocity, the vector of velocity and missile angular acceleration.
  - The application of active vibroisolation lowers the fluctuation level of relocation, velocity and linear acceleration of the launcher turret in the vertical direction.
  - The application of active vibroisolation lowers the fluctuation level of relocation, velocity and angular acceleration of the launcher turret in the inclination motion.
  - The application of the active vibroisolation lowers the fluctuation level of relocation, velocity and angular acceleration of the launcher turret in the tilting motion.
- **The interpretation of the characteristic time moments occurring during the missile motion along the launcher guide.**
  - Physical phenomena resulting from mechanical reaction occur when the take-off engine starts and finishes work and when the missile leaves the launcher.
  - The moment the engine starts working is the reason for changing the characteristics of the acceleration variability course $\ddot{y}_v$, $\ddot{\vartheta}_v$, $\ddot{\varphi}_v$, $\ddot{\varepsilon}_{p_1x_p}$, $\ddot{\varepsilon}_{p_1y_p}$, $\ddot{\varepsilon}_{p_1z_p}$. The acceleration characteristics has the properties of the non-continuous function. The non-continuity of the acceleration curve results from the mathematical interpretation of the angular point of the first derivative. The pitch of acceleration value depends on the physical nature of the processes taking place at that moment. Applying a force of constant value in time to the missile leads to the pitch of the acceleration value. The rocket thrust makes the missile move along the guide determining the weight distribution.
  - Because of the existing coupling between the missile motion and the turret performance the generated disturbance is the cause of a particular course of variability of physical values characterizing the motion of both objects.
  - The moment the take-off engine stops work leads to the change of the acceleration variability course $\ddot{y}_v$, $\ddot{\vartheta}_v$, $\ddot{\varphi}_v$, $\ddot{\varepsilon}_{p_1x_p}$, $\ddot{\varepsilon}_{p_1y_p}$, $\ddot{\varepsilon}_{p_1z_p}$. The missile is not loaded by thrust. The acceleration value pitch is the result of the thrust reduction to 0. The discussed system is not only a system with a variable weight distribution but also a variable mass. After finishing work by the take-off engine the missile moves along the guide using the acquired kinetic energy.
  - The moment the missile leaves the launcher the turret-missile system mass makes a single and discrete change and amounts to the newly-formed objects. These objects no longer make a kinematic pair. In a single moment of time point 3 joins two different structures. On the one hand, it describes the end of the missile movement along the guide and on the other, it determines the initial conditions of the objects motion after the process of natural degeneration.

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