THE METHOD OF HIGH ACCURACY CALCULATION OF ROBOT TRAJECTORY FOR THE COMPLEX CURVES

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Abstract:
The geometric model accuracy is crucial for product design. More complex surfaces are represented by the approximation methods. On the contrary, the approximation methods reduce the design quality. A new alternative calculation method is proposed. The new method can calculate both conical sections and more complex curves. The researcher is able to get an analytical solution and not a sequence of points with the destruction of the object semantics. The new method is based on permutation and other symmetries and should have an origin in the internal properties of the space. The classical method consists of finding transformation parameters for symmetrical conic profiles, however a new procedure for parameters of linear transformations determination was acquired by another method. The main steps of the new method are theoretically presented in the paper. Since a double result is obtained in most stages, the new calculation method is easy to verify. Geometric modeling in the AutoCAD environment is shown briefly. The new calculation method can be used for most complex curves and linear transformations. Theoretical and practical researches are required additionally.

Key words: *analytical method, linear transformations, planar complex curves, symmetries*

INTRODUCTION

The mathematical calculations have always been the base of any engineering creations [1]. It takes a long time to develop exact mathematical methods and a human life is too short to get a solution. The first approximate calculation methods were suggested by the ancient Greeks. Newton proposed a canonical form for the detection of a complex curve on plane. He found canonical formulas for solving the problem:

$$Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F = 0 \qquad (1)$$

$$Ax^3 + 3Bx^2y + 3Cxy^2 + 3Ex^2 + 6Fxy + 3Gy^2 + 3Hx + 3Jy + I = 0 \qquad (2)$$

etc. But an analytical result was obtained single for the form (1) of the second stage in next times only by Euler. Thus, Newton designed of the advanced approximation methods [2] for form (2).

Artificial intelligence methods have been used to solve this problem. The equivalence of the concepts of automorphism in geometry and universal in linguistics was showed by an analysis of the texts of the works of De Saussure and Bachmann. A hypothesis was put forward about space built as a natural language text.

To solve the characteristic equations, it is necessary to accept the basis of these hypotheses. A classic solution is used for their calculations. This solution follows from two postulates. Then the type of the basic equation does not change and so it is possible to obtain the solution in an orthogonal way. The transformation matrix then has a great influence on the change of the basic equation. However, everything changes the symmetry of space and therefore it is necessary to use a non-orthogonal basis of the solution to solve complicated processes of nonlinearity.

The next stages of the study were related to the analysis of symmetries in geometry. Symmetry were used for theoretical constructions by Euclid, but only mirror symmetry is used in based researches us rule. Let us look at how Newton's problem is solved now.

REVIEW OF THE LITERATURE

The approximation theory is currently in the development stage. The authors [3, 4, 5, 6, 7] describe and develop the

theory of approximation, methods of approximation, algorithms for data analysis, discrete approximation, from which the authors of this article draw the basic theoretical background. Many authors are currently working on this issue. Authors [8] solve the Apostol-Euler-Dunkl polynomials with applications to series involving zeros of Bessel functions, and author in [9] describe the growth of polynomials outside of a compact set-The Bernstein-Walsh inequality revisited, and further, the authors [10] deal with the approximation and Entropy Numbers of Embeddings Between Approximation Spaces.

Many outputs of approximation techniques are widely used in mechanics, economics, mechatronics, robotics [11, 12], instrumentation, technology [13, 14, 15], assembly [16, 17], informatics [18, 19] etc. The general disadvantage of all methods is that they have miscalculations. Obviously, the exact computations accuracy bases on the ideal conditions required for the project.

From the point of view of the trajectory of the robot, we recognize two basic tasks: the first is the task of the trajectory of the mobile robot and the second is the trajectory of the robot effector.

Many authors deal with calculations, planning and simulation of robot effector trajectories [20, 21, 22, 23, 24]. In this case, the product precession depends on the tolerances of the manufacturing. Even though the scientific scan for a systematic problem-solving in the theory was finished a century ago, some production conditions dictate more strict claims nowadays. The task to rise the industrial engineering precision was set about fifty years ago. The main objects of this research are the Jordan curves. The issue of Jordan curves was dealt with by the authors in [25], where the main goal was to calculate the Jordan trajectory of robot movement. The solutions to the characteristic equation for industrial robot's elliptic trajectories are designed in the article [26].

Other authors follow the trajectory of the mobile robot itself. Authors in the article [27] presents a novel method to generate a trajectory for a mobile robot moving in an uncertain environment where only a few way-points are available to reach a nearby target state. The proposed method interpolates way-points by the combination of a straight line and a fifth-order Bezier curve along which the curvature varies continuously.

In this paper [28] authors are proposed an algorithm that calculates the suboptimal movement between two positions, which automatically generates a cloud of safe via points around the workpiece and then by exploiting such points finds the suboptimal safe path between the two positions that minimizes movement time.

Mathematical linguistics and the theory of symmetries create the theoretical base of this study. Authors of the mentioned study obtained the first results at the beginning of the 90's of the 20th century. They repeated the classical method of study in various ways.

The correct answer to the equation $\mathbf{P}\vec{u} = \lambda\vec{u}$,
where:
$\mathbf{P}$ – transformation matrix,
$\vec{u}$ – vector,

λ – scalars, is necessary for acquiring results in many areas of science, such as physics, mechatronics, optimal management, cryptography, etc.

The classical method consists of finding transformation parameters for symmetrical conic profiles [29]. A new procedure for parameters of linear transformations determination was acquired by another method [30, 31]. The Cartesian product in the Euclidean plane $\mathbf{R}\times\mathbf{R}$ for reflections $\mathbf{R}\times\mathbf{R}\rightarrow\mathbf{R}\times\mathbf{R}$ is the main object of the research. The specific equation can be resolved by the application of new mathematical methods by means of projective transformations in spaces with a large number of dimensions [32]. The resolution is extraordinarily difficult for engineering calculations [33, 34, 35, 36, 37]. The goal of the authors was to find a method that results in formulas without radical dependencies. Radical dependencies usually do not allow further analytical calculations. The new method should have an origin in the internal properties of the space. Lower level headings remain unnumbered; they are formatted as run-in headings.

METHODOLOGY AND RESULTSOF RESEARCH
We focused our research approach on the creation of an algorithm of complex curves linear transformation, as well as on computer modeling with experimental research.

Algorithmus of complex curves linear transformation
Let there be a trajectory by Ω – planar differentiable curve in the Euclidean plane $\mathbf{R}\times\mathbf{R}$. Curve is located in a Cartesian coordinate. Parametrical system defined by equations:

$$\begin{cases} x = k_x f_x(t) \\ y = k_y f_y(t)' \end{cases} \qquad (3)$$

where:
$x, y, t, k_x, k_y, \in \mathbf{R}$.

A parametric system of equations $\begin{cases} x = k_x t \\ y = k_y t \end{cases}$ cannot be used. The equations define a line, thus, functions $f_x(t) \neq t$ and $f_y(t) \neq t$ at the same time. A system of parametric equations $\begin{cases} x = t \\ y = k_y f_y(t) \end{cases}$ or $\begin{cases} x = k_x f_x(t) \\ y = t \end{cases}$ can be composed to solve any canonical equation from a Newton systematic. Let us do a linear transformation of (3) by the matrix $\mathbf{P} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$,
where:
$a_{ij} \in \mathbf{R}$. The parameters of the transformed figure must be obtained.

The calculation method for centrally symmetric conical sections was described in articles [26, 35, 36] earlier, therefore we will not dwell on it. But for a complete understanding, we recommend that the reader refer to these materials, since the angles α and β are taken from them.

The method is based on such a fundamental property of a plane (space) as symmetry. Symmetry has many definitions as it is used in various sciences. The Dieudonne definition is used in this study since it is the most modern postulate in analytic geometry. Dieudonne considered three types of symmetries on the plane: the unitary matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the mirror matrixes $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, permutations

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The unitary matrix does not change the Euclidean plane. Mirror symmetries determine the direction of the numerical axes.

The first angle α is the corner of rotation of the quadratic form by classic theory and the second angle β is the corner by permutation symmetry. A non-orthogonal basis can be considered based on these angles.

The basis was used to define a direct method for linear transformations of central symmetrical conic sections by two angles. Further research with scale and transformations of rotation proved the method to be non-applicable for an exact transformation, but to coincide with classical results in the neighborhood. The transformations in question do not have a non-orthogonal basis. Own angles $\alpha_1 \ne \alpha_2$ for calculations of compression along the axis, if the angle is not zero, results are similar as to results obtained by a classical method. A non-orthogonal basis coinciding with the orthogonal basis does exist, no other non-orthogonal bases are present.

The biggest distinction to be found was for the process of singular transformations, where by applying a classical method each curve had to be transformed individually, the new method in contrast offered eight groups of transformations (independent of shape of the curve) [26].

Four transformations $\begin{pmatrix} m & n \\ m & -n \end{pmatrix}$, $\begin{pmatrix} -m & n \\ m & n \end{pmatrix}$, $\begin{pmatrix} m & kn \\ -n & km \end{pmatrix}$, $\begin{pmatrix} km & n \\ -kn & m \end{pmatrix}$ were found for the deviation of the own angle, where $\beta \in \{-\pi/2, 0, \pi/2\}$. In this case two non-orthogonal bases can be found by a classical method, each coinciding with an orthogonal basis. Exact parameters are generated the new method. Digital experiments made it possible to obtain a chain of transformations for finding the analytical formula of the transformed curve.

1. The transformation matrix is split into a product of two matrices:

$\mathbf{P_1} = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$

and $\mathbf{P_2} = \begin{pmatrix} 1 & a_{12}/a_{22} \\ a_{21}/a_{11} & 1 \end{pmatrix}$.

2. The transformation $\begin{pmatrix} 1 & a_{12}/a_{22} \\ a_{21}/a_{11} & 1 \end{pmatrix} \begin{pmatrix} f_x(t) \\ f_y(t) \end{pmatrix}$ parameters (3) are discovered as a centrally symmetric conic section, where α is the angle of first ort (own angle); β is the angle of second ort (angle of permutation symmetry); k'_x and k'_y are both scalars of the characteristic equation.

3. New curve description system: $\begin{cases} x = a_{11}k_x f_x(t) \\ y = a_{22}k_y f_y(t) \end{cases}$ is considered from (3).

4. An inverse transformation [37] $\mathbf{T}$ depending on β is performed. If the value of β will be negative, the transformation will result in $\mathbf{T} = \begin{pmatrix} cos\,\beta & -sin\,\beta \\ cos\,\alpha & sin\,\alpha \end{pmatrix}$, if the value will be positive, $\mathbf{T} = \begin{pmatrix} -cos\,\beta & sin\,\beta \\ cos\,\alpha & sin\,\alpha \end{pmatrix}$, in this manner a transition from a non-orthogonal basis to an orthogonal basis can be achieved.

5. The curve scalars b will be multiplied by coefficients k'_x and k'_y.

6. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ will be transformed into a parametric trajectory equations system in the form of $\begin{cases} x = k_x f_x(t) \\ y = k_y t \end{cases}$ from (3) and if the system has an alternative description into transformation $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. This step of the transformation chain needs to be investigated in more detail. Since it is necessary to take into account the characteristic equation separation. Additional research is needed. However, the algorithm should be correct for many calculations.

7. By rotating the curve by its own angle α, the analytical formula of the transformed complex curve is found.

The last research was building by parabola with equations system $\begin{cases} x = t \\ y = t^2 \end{cases}$ by step 6. Matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ must execute if signature angles α and β are negative and matrix must execute $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ if signature angles α and β are positive. Experiments show that the trajectory obeys the laws of quantum physics than classical analytical geometry additionally.

Computer modeling and natural experiments

The method of geometric modeling was used to verify the found patterns. The experiments were carried out in the AutoLisp language in AutoCAD 2007 environment. The requested transformation parameter: let us have curve by parametrical equations system (3). The curve is drawing by the system of equations:

$$\begin{cases} x = a_{11}k_x f_x(t) + a_{12}k_y f_y(t) \\ y = a_{21}k_x f_x(t) + a_{22}k_y f_y(t) \end{cases} \quad (4)$$

as a sequence of thickened straight-line segments with black color. The parameters of the curve are obtaining the by the new method Another curve is drawing based on the calculation results as a sequence of thin segments of straight lines in red. A hypothesis is putting forward if the second curve is completely inside the first.

The experiments were carried out for next curves from (4):

astroid by system $\begin{cases} x = cos^3 t \\ y = sin^3 t \end{cases}$

where:

$t \in [0, 2\pi]$; cardiode by system $\begin{cases} x = 2\,cos\,t\,(1 + cos\,t) \\ y = 2\,sin\,t\,(1 + cos\,t) \end{cases}$

where:

$t \in [0, 2\pi]$; parabola by system $\begin{cases} x = t \\ y = t^2, \end{cases}$

where:

$t \in [-2, 2]$; circle by system $\begin{cases} x = cos\,t \\ y = sin\,t \end{cases}$

where:

$t \in [0, 2\pi]$; strofoid by system $\begin{cases} x = a\frac{tan^2 t - 1}{tan^2 t + 1} \\ y = a\,tan\,t\,\frac{tan^2 t - 1}{tan^2 t + 1} \end{cases}$

where:

$t \in [0, \pi]$; hypotrohoid by system $\begin{cases} x = 2\,cos\,t + 5\,cos(\frac{2}{3}t) \\ y = 2\,sin\,t - 5\,cos(\frac{2}{3}t) \end{cases}$

where:

$t \in [0, 6\pi]$.

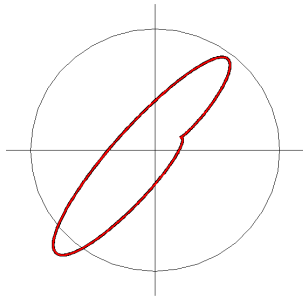The formulas were taken from books [38, 39]. The results are shown in Fig. 1-6.
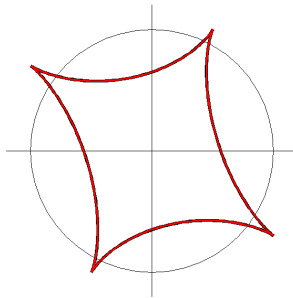

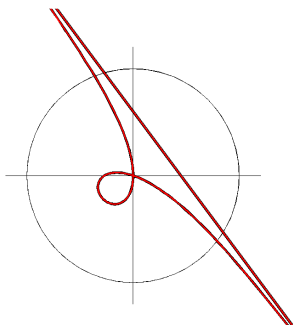**Fig. 1 Hypocycloid with four cusps**

Fig. 2 Cardioid

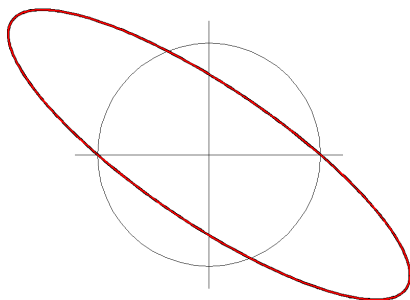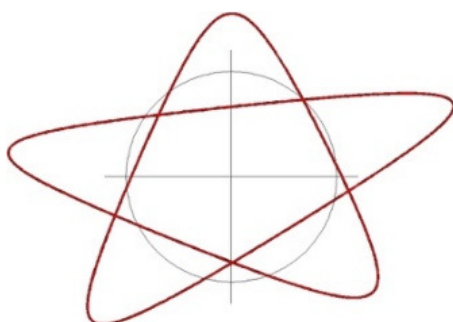**Fig. 3 Strofoid**

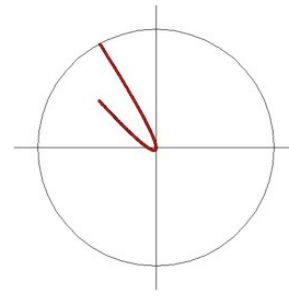**Fig. 4 Circle**

Fig. 5 Hypotrochoid

Fig. 6 Parabola

The characteristic equation $\mathbf{P}\vec{u} = \lambda\vec{u}'$ is divided into four types $\vec{u}' \in \left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} -x \\ -y \end{pmatrix}, \begin{pmatrix} y \\ x \end{pmatrix}, \begin{pmatrix} -y \\ -x \end{pmatrix} \right\}$.

Division was not taken into account in previous studies. Obviously, this division has a significant impact on step 6 of the transformation chain. Therefore, this step is the most difficult at this stage of the researches.

Kinematic pairs with simple paths are used to design most robot designs now. Newton proposed a complex classification of curves specifically for the design of robots with complex trajectories. Designing a robot with a complex trajectory is possible using approximate methods now, and designing precise robots is not possible.

RESULTS OF RESEARCH

The results of theoretical and modeling research can be used in the field of robotics, mainly due to its accuracy of trajectory calculation and rendering, but also by reverse validation of motion control of monitored robot effectors. The proposed model is applicable for simulation of a two-wheel self-balancing mobile robotic system or other unstable robotic vehicles with parallel wheel alignment in nature. Robotic devices that belong to this category of robotic systems re-quire some degree autonomous behavior, described in literature as a mobile service robot. A convenient addition to the precise regulation and control of mobile service robots is the implementation of inertia navigation, which provides reverse validation and control, for example for a tilting system implemented on a two-wheel self-balancing mobile robotic system.

Table 1
The experiments results

Curve	Matrix	Angle $\beta°$	Angle $\alpha°$	Scalar k_x k_y
Astroid (Fig. 1)	$\begin{pmatrix} 0.5 & -1 \\ 1 & 0.7 \end{pmatrix}$	15.13	75.87	0.759, 2.270
Cardioid (Fig. 2)	$\begin{pmatrix} 0.2 & 0.2 \\ 0.1 & 0.3 \end{pmatrix}$	47.38	42.61	1.587, 0.420
Strofoid (Fig. 3)	$\begin{pmatrix} 0.3 & -0.3 \\ 0.2 & 0.4 \end{pmatrix}$	-62.66	-27.34	1.267, 1.184
Circle (Fig. 4)	$\begin{pmatrix} 1.5 & -1 \\ -0.5 & 1.2 \end{pmatrix}$	-52.02	-37.98	1.614, 0.447
Hypotrochoid (Fig. 5)	$\begin{pmatrix} 0.3 & -0.1 \\ 0.2 & 0.2 \end{pmatrix}$	-56.31	-33.69	1.167, 1.0837
Parabola (Fig. 6)	$\begin{pmatrix} 0.01 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$	-1	-89	1.000, 0.493

SUMMARY

Engineering computations define the material life of the person. The precision of the calculation has an impact on the comfort of the entity as well as on the way of living. The

basic method for gaining exact engineering calculations is the inverse matrix method. However, this method contains one defect: it is applicable only to conical sections. Other sections are impossible to apply with the inverse matrix method. For this reason, Newton proposed a new curves classification. Even though this proposal solved the defect, the calculation methods were not found.

The disadvantage of the transformation types acquired sooner is that they also viewed only the orthogonal basis. The current findings enabled to find an algorithm for a non-orthogonal basis calculation. The latest ways for the calculation were examined on robot trajectories in laboratory conditions [40, 41, 42]. The authors of this paper expect that the described method will not only increase the accuracy of calculations but will also help to reveal the practical solutions and meanings for many technical sciences[43, 44, 45, 46, 47, 48]. The finding of the analytical solution for plane differentiable curves did not solve all issues. The records have to be based on theoretical works. Some linear transformations break the characteristic equation into four parts. Two main tasks must be solved: to determine the boundaries of the linear transformations zones in the characteristic equation and to formulate a parametric equations system in conditions similar to system (1).

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