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Comparative Analysis of Different Methods of Calculating Pressure Inside the Barrel in Post-Muzzle Period of a Shot

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Abstract. The article presents the results of calculations of the pressure of propellant gases inside the barrel in a post-muzzle period of a shot. Three methods of calculating the pressure in the barrel bore in the post-muzzle period of the shot were used in the work: the Brawin method, the method describing the outflow of gases from the barrel bore to the environment, and the method using the phenomenon of the transition of a pressure-averaging rarefaction wave from the barrel muzzle to the bottom of the combustion chamber. In the conducted tests, the obtained pressure courses were used to determine the impact of the use of various methods describing the post-muzzle period of a shot on the behaviour of the weapon automatics system operating on the recoil principle.

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A comparison of the pressure courses, obtained with the use of various methods, and the recoil velocities of the elements of the recoiling assembly, determined with the use of mathematical and physical models of recoil firearm operation, showed that in the case of recoil operated weapons, the differences between the methods are small - the differences between the recoil velocities determined with the use of various methods did not exceed 5%. Both, the pressure courses and the recoil velocities of the recoiling elements of the firearm operating on the recoil principle determined with the use of various methods of describing the pressure course in the post-muzzle period of the shot are at a similar level.
Keywords: firearm, internal ballistics, recoil, post-muzzle stage of a shot

1. INTRODUCTION

A post-muzzle stage of a shot is a part of a gun discharge process which takes place after the projectile leaves the barrel bore. Said stage lasts until propellant gas pressure drops to the level of atmospheric pressure. The post-muzzle stage of a shot is important especially in recoil-operated weapons, where the force, generated by propellant gas pressure, acts on a face of a bolt locked with a barrel, causing increase in recoil velocity of recoiling assembly (consisting of the said bolt and barrel) (Fig. 1). Similar research considering operation of gas-operated firearm in the post-muzzle stage of a shot was presented in [1]

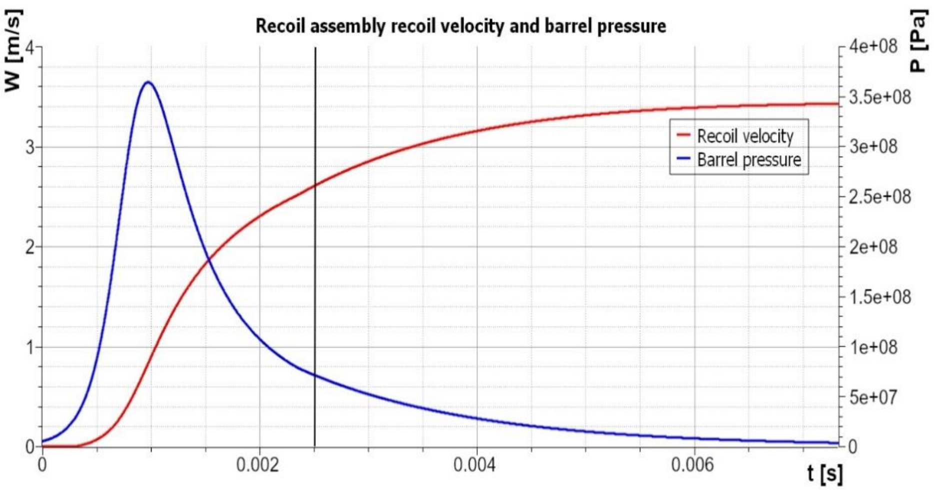

Fig. 1. Graph comparing the propellant gas pressure *P* inside the barrel and the recoil velocity of recoiling assembly *W*, black vertical line marks a moment of bullet leaving a barrel (based on [1])

The most common method of calculating post-muzzle pressure is based on Brawin's empirical formula, which describes an exponential drop of pressure inside the barrel [3]. Due to ease of use of Brawin's method, it is treated as a reference method in the presented work. Complete sets of equations for physical and mathematical models of recoil operated firearm operation and the sets of input parameters used in the paper were presented in [2].

Said models were based on a construction of caliber 12.7 mm recoil operated heavy machine gun, parameters for 12.7 × 99 mm M33 Ball NATO cartridge were used in calculations [2].

2. METHODS OF CALCULATING PRESSURE IN A POST-MUZZLE STAGE OF A SHOT

2.1. Brawin's formula

Propellant gases pressure evolution during the post-muzzle stage of a shot is described with Brawin's empirical formula (1) [3]:

$$p = p_w \cdot e^{-\frac{t_{op}}{b}} \tag{1}$$

where:

p – pressure in the barrel bore [Pa],
p_w – gas pressure at the moment of leaving the barrel by the projectile [Pa],
t_{op} – time in the post-muzzle stage of a shot [s],
b – constant for Brawin's formula [s].

The constant value b is calculated by formula (2) [3]:

$$b = \frac{(\beta - 0.5) \cdot \omega}{s \cdot (p_w - p_{pw})} \cdot v_w \tag{2}$$

where:

β – coefficient of the post-muzzle gas effect [-],
ω – mass of propellant [kg],
s – barrel bore cross-sectional surface area [m^2],
p_{pw} – gas pressure at the end of the post-muzzle stage (atmospheric pressure) [Pa],
v_w – muzzle velocity of the projectile [m/s].

A value of the coefficient of the post-muzzle gas effect β, present in relationship (2), is expressed by empirical formula (3) [3]:

$$\beta = 1.5 + \frac{6.45}{\left(\frac{p_m}{p_w} \cdot \frac{l_w}{d}\right) \cdot 0.23} \tag{3}$$

where:

p_m – maximum gas pressure inside the barrel [Pa],

l_w – overall projectile travel inside the barrel [m],

d – calibre [m].

2.2. Outflow of gases from the barrel

In order to determine validity of application of the relationship describing outflow of propellant gases from the barrel to the environment, an analysis of the relationship between flow velocity of the gas-powder mixture and wave velocity in the gas-powder mixture after the projectile exit was carried out. The flow velocity of the mixture was determined by carrying out calculations with an internal ballistics' thermodynamic model for a fictionally elongated barrel and using relationship (4):

$$U_w = v\frac{l_w}{l} \quad (4)$$

where:

U_w – velocity of the gas-powder mixture [m/s],

v – current bullet velocity [m/s],

l – current projectile travel (for a fictionally elongated barrel) [m].

Velocity of the wave in the gas-powder mixture was determined using equation (5) [4]:

$$c = \frac{\sqrt{kRT\psi}}{1-(1-\psi)\frac{\rho}{\rho_p}-\alpha\rho\psi} \quad (5)$$

where:

c – velocity of a wave in the gas-powder mixture [m/s],

k – propellant gas adiabatic exponent [-],

R – gas constant [J/(kg·K)],

T – gas temperature [K],

ρ – density of the gas-powder mixture [kg/m^3],

ρ_p – propellant density [kg/m^3],

ψ – relative volume of the burnt propellant [-],

α – propellant gases co-volume [m^3/kg].

Density of the gas-powder mixture was determined from formula (6):

$$\rho = \frac{\omega}{W_0 + ls} \quad (6)$$

where:

W_0 – volume of the combustion chamber [m^3],

l – projectile's travel in the barrel bore [m].

Comparison of flow velocity of gas-powder mixture and wave motion velocity in gas-powder mixture in fictionally elongated part of the barrel is shown in Fig. 2.

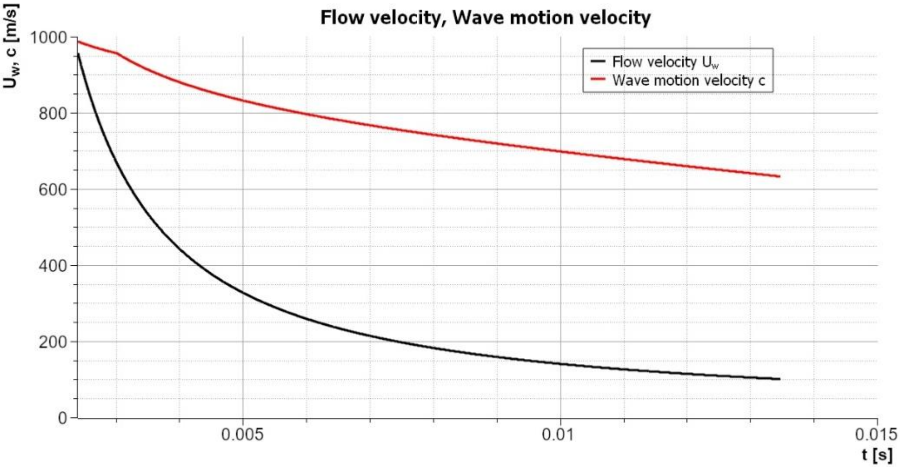

Fig. 2. Graph comparing flow velocity of the gas-powder mixture U_w and velocity of the wave motion c in the fictional part of the barrel, which is an extension of a standard-length barrel

Change of the slope in the first part of the curve of the wave motion velocity c, visible in Fig. 2, is caused by completion of remained propellant burning (ψ < 1 at the moment of projectile exit from the barrel of the standard length). Calculations for fictionally elongated barrel were performed for the same amount of time it takes the pressure to drop to the level of atmospheric pressure in Brawin's method, which is treated as a reference. Considering that fact, the fictional elongation of barrel corresponds to 15 meters of bullet travel.

Although the plots, shown in Fig. 2, refer to a fictive length of the barrel, they prove that the flow of gas-propellant mixture, approaching the muzzle after the projectile exit, is subsonic flow. Due to this, rarefaction wave starts to travel down the barrel bore from the moment of bullet exit, causing acceleration of propellant gas flow in the muzzle cross section of the barrel to the sound velocity. Due to that fact, it is possible to use dependencies describing outflow of propellant gases from the barrel to the environment.

2.2.1. Propellant burning after leaving a barrel by a projectile

Equation of energy balance in the barrel, taking into account combustion of propellant charge and outflow of gases from the barrel bore to the environment (7) [5]:

$$\frac{dRT}{dt} = \frac{\frac{d\psi}{dt}(\theta q_s - RT) - \theta RT\frac{d\gamma}{dt}}{\psi - \gamma} \tag{7}$$

where:

t – time [s],

θ – function of the adiabatic exponent of propellant gas ($\theta = k - 1$) [K],

q_s – heat of propellant combustion [J/kg],

γ – relative volume of the propellant gas that flowed from the barrel to the environment [-].

Equation of state of propellant gases in the barrel (8) [5]:

$$p = \frac{\omega(\psi - \gamma)RT}{W_0 + sl - \frac{\omega}{\rho_p}(1 - \psi) - \alpha\omega(\psi - \gamma)} \tag{8}$$

Equation of relative volume rate of propellant charge combustion (9) [5]:

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\sqrt{1 + 4\frac{\lambda_1}{\chi_1}\psi} \cdot u_1 p \tag{9}$$

where:

S_1 – propellant grain initial surface area [m^2],

Λ_1 – propellant grain initial volume [m^3],

λ_1, χ_1 – propellant grain shape coefficients [-],

u_1 – coefficient of linear burning rate [m/(s·Pa)].

Equation of relative mass velocity of propellant gas outflow from the barrel to the environment (10) [5]:

$$\frac{d\gamma}{dt} = \frac{\xi_w}{\omega}\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{\frac{2k}{k+1}} \cdot \frac{p}{\sqrt{RT}} \tag{10}$$

where:

ξ_w – loss coefficient [-].

2.2.2. Outflow of gases from the barrel after complete propellant burn

Equation of energy balance in the barrel after complete combustion of the propellant charge (ψ = 1), taking into account outflow of propellant gases from the barrel bore to the environment (11) [5]:

$$\frac{dRT}{dt} = \frac{-\theta RT \frac{d\gamma}{dt}}{1-\gamma} \tag{11}$$

Equation of state of propellant gases in the barrel (12) [5]:

$$p = \frac{\omega(1-\gamma)RT}{W_0 + sl - \alpha\omega(1-\gamma)} \tag{12}$$

Equation of relative mass velocity of propellant gas outflow from the barrel to the environment stays the same as in the stage of propellant burning (10).

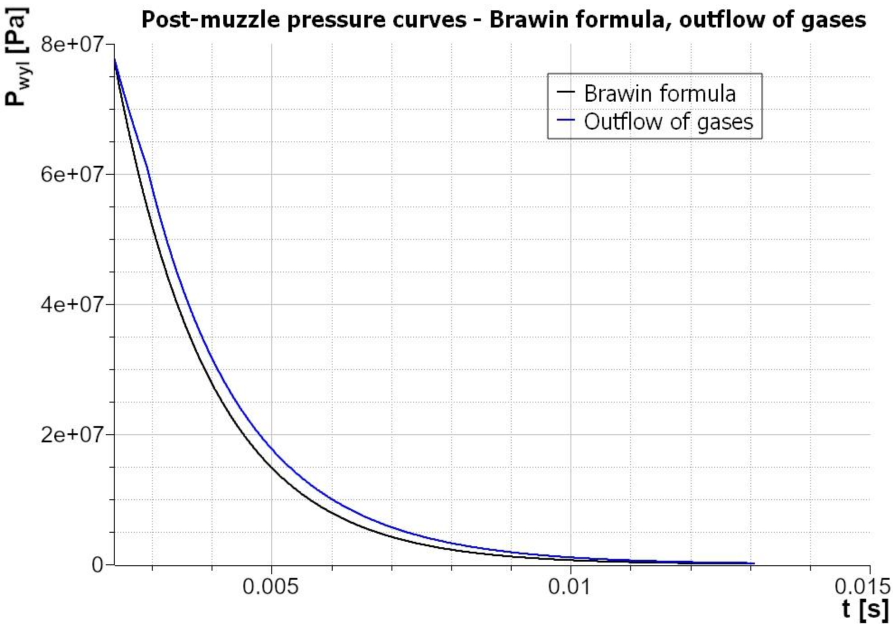

Fig. 3. Comparison of the pressure curves P_{wyl} in post-muzzle period determined using Brawin's formula and equations of outflow of gases from the barrel bore

Comparison of pressure curves obtained from both methods, visible in the graph (Fig. 3), shows significant deviation of the curves in the initial part of the post-muzzle period which is caused by the afterburning of the propellant charge.

The calculations, carried out with the thermodynamic model of internal ballistics, show that in the analyzed case about 93.5% of the propellant charge was burnt when the projectile exited the barrel.

2.3. Propagation of rarefaction wave from the muzzle to the bottom of the barrel bore

Third method of mathematical description of the post-muzzle period of a shot is based on calculations for a fictitious elongation of the barrel. It is based on the fact that the pressure acting on the breech for the elongated barrel changes in the same way as for the real barrel until the rarefaction wave (formed when the projectile leaves the barrel muzzle) traveling through the barrel bore reaches the bottom of the combustion chamber. Rarefaction wave is treated as some 'information' about opening the barrel bore – when the front of the wave reaches the bottom of the combustion chamber, pressure in the barrel bore starts to drop rapidly. A concept of rarefaction wave propagation in the barrel bore was also used in the Rarefaction Wave Gun (RAVEN) project, where the phenomenon was used to decrease recoil of large caliber guns by opening the barrel bore in the breech area, allowing to vent propellant gases from the barrel. Venting gases from the barrel was synchronized with rarefaction wave position, not allowing to reach a base of the projectile by the wave – rarefaction wave reached the barrel muzzle after the projectile left the barrel bore, thus preventing pressure to drop while the projectile was still accelerating inside the barrel [6].

Determination of the characteristic time of wave processes, justifying the inclusion of said processes in modelling, begins with comparison of the characteristic times of wave motion and the motion of the projectile (Fig. 4).

A curve of the characteristic time of wave transport process t_f in relation to the duration of the firing process was determined by referring a wave velocity to a sum of lengths of projectile travel in the barrel and the combustion chamber (13):

$$t_f = \frac{l_k + l}{c} \tag{13}$$

where:

t_f – characteristic time of wave motion [s].

l_k – length of the combustion chamber [m].

The rarefaction wave velocity was calculated by using formulas (5) and (6). Based on the graph showing the relation between the characteristic times of projectile motion and wave transport processes (Fig. 4), it can be concluded that it is reasonable to take wave processes into account in the modelled phenomenon due to the same order of magnitude of both times.

Drop of a value of the characteristic time of projectile motion t_p , visible in the first part of the graph, is caused by a pirostatic period, in which propellant burns in a constant volume.

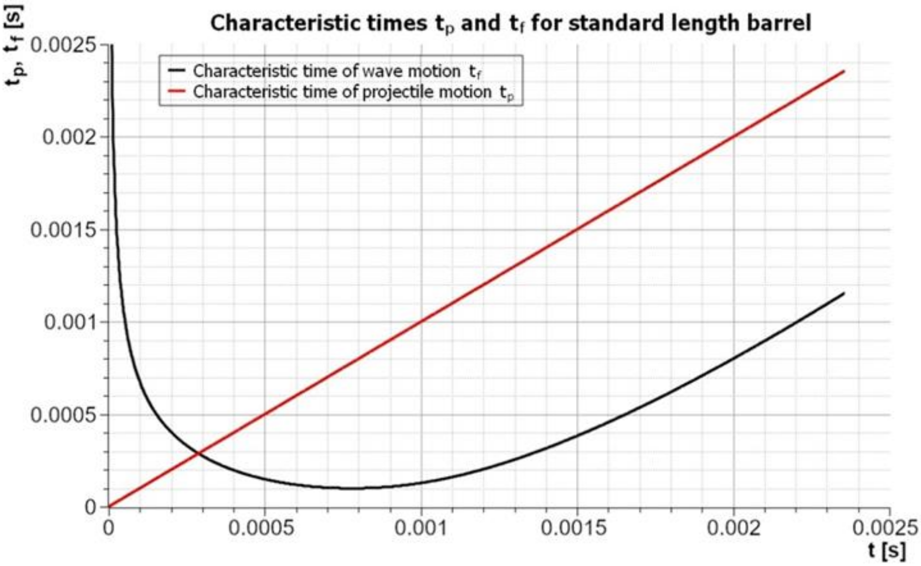

Fig. 4. Comparison of the characteristic times of wave motion t_f and the projectile motion t_p for a standard-length barrel (the length used in the actual layout)

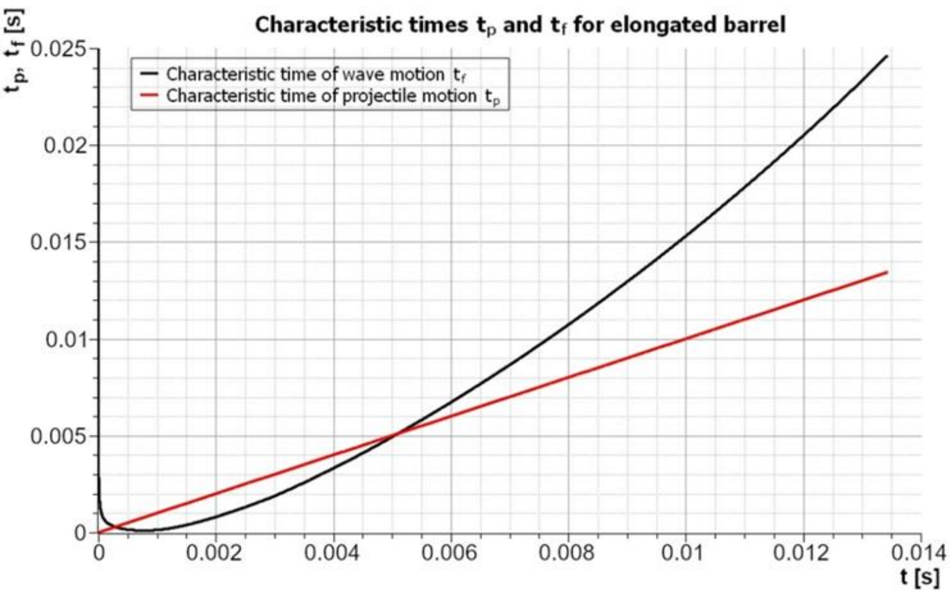

Fig. 5. Comparison of the characteristic times of wave motion t_f and the bullet motion t_p for an extended barrel

When comparing the times t_p and t_f for an extended barrel (analysis was carried out for duration of the phenomenon equal to duration of the pressure applied to the breech assembly determined using the Brawin's formula) (Fig. 5), in the entire duration of the phenomenon, both characteristic times are characterized by the same order of magnitude. The increasing value of the time t_f , in case of an elongated barrel, is caused by an increase in volume, and thus, a decrease in density of the medium (gas-powder mixture) in which wave transport processes take place.

Taking into account wave transport processes, in operation of the modelled system, was carried out by fictitious extension of the barrel bore until the rarefaction wave moving deeper into the barrel reaches the bottom of the combustion chamber (Fig. 6). A position of the rarefaction wave front was determined by relating the distance travelled by it to a sum of length of the barrel bore and the combustion chamber. The path of the wave front was determined by integrating a wave velocity over time, being the difference between the flow velocity and the velocity of sound in the gas-powder mixture (due to opposite directions of the mixture and the wave front).

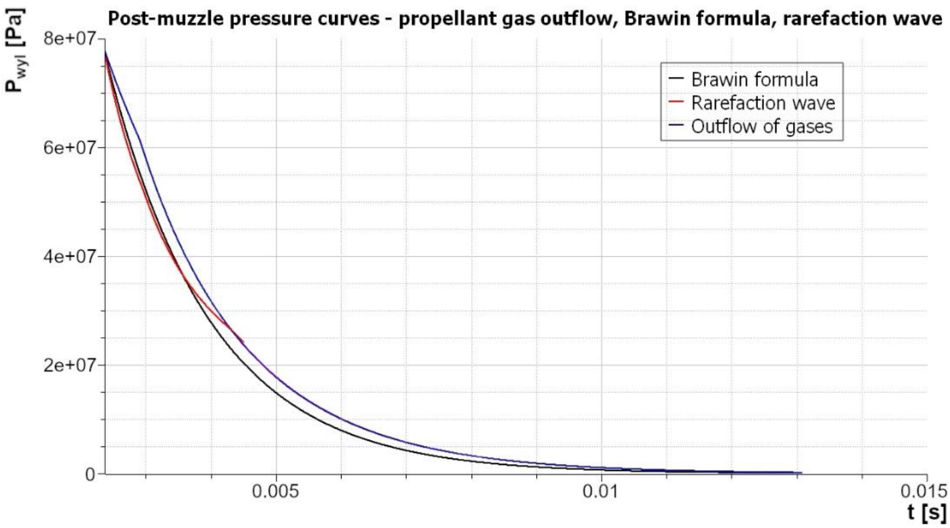

Fig. 6. Comparison of the pressure curves P_{wyl} in the post-muzzle period, determined using: the Brawin's formula, the relationships describing the outflow of gases from the barrel tube to the environment and a fictitious extension of the barrel bore during the transition of the rarefaction wave to the bottom of the combustion chamber

Comparison of calculation results for the elongated barrel, the Brawin's formula and the equations of gas outflow to the environment shows that in the analyzed case, almost immediately when the rarefaction wave reaches bottom of the combustion chamber, the pressure curve for the long barrel intersects with the curve determined from the gas flow equations - curves intersect occurs at the time t = 0.0043 s, while the rarefaction wave reaches the bottom of the chamber at t = 0.0044 s. Therefore, in order to determine a pressure curve until the end of the post-muzzle period, it was decided to use a simplification in form of a direct transition from the curve for elongated barrel to the curve for the outflow of gases from the moment of their intersection (Fig. 7).

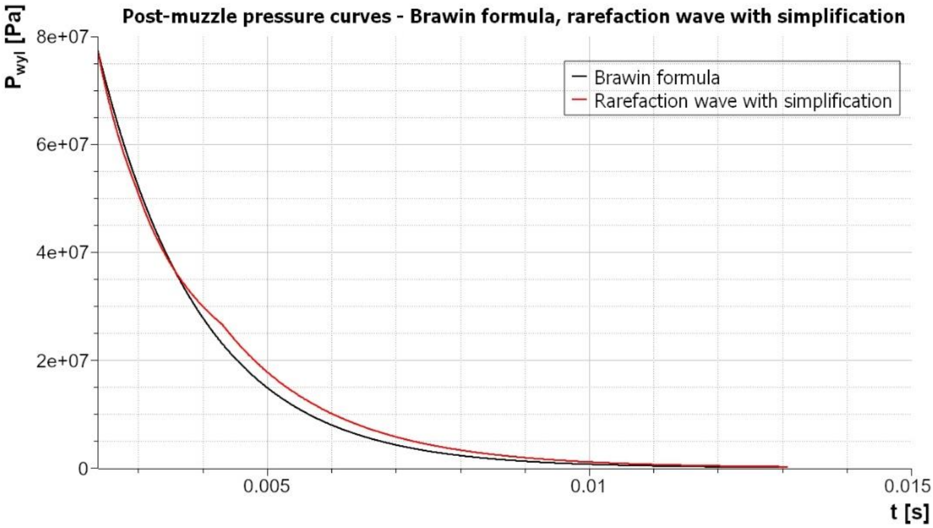

Fig. 7. Comparison of the pressure curves P_{wyl} in a post-muzzle period, determined using the Brawin's formula and a fictitious elongation of the barrel during the transition of the rarefaction wave, which turns into the outflow of gases to the environment

3. COMPARISON OF METHODS

The diagram (Fig. 8) shows a comparison of pressure courses in the barrel bore during post-muzzle period for the three methods used. On the basis of the summary, it is possible to notice similar course of all curves in the entire post-muzzle period. The lowest pressure values in almost entire run were obtained using the Brawin's formula. In the initial part of the post-muzzle period, a value of the pressure obtained for the elongated barrel is lower by about 3%, at the time t = 0.00355 s it intersects with the curve obtained for the Brawin's formula and its slope decreases. The highest-pressure values in the entire period were obtained for a method based on the outflow of gases from the barrel to the environment. The list of selected parameters for the entire pressure courses and for the post-muzzle period is presented in Table 1.

Calculations were made using a mathematical model of a short recoil operated firearm with accelerator described in [2].

The models describe operation of a firearm during one half of a single shot cycle divided into four characteristic stages (Fig. 9).

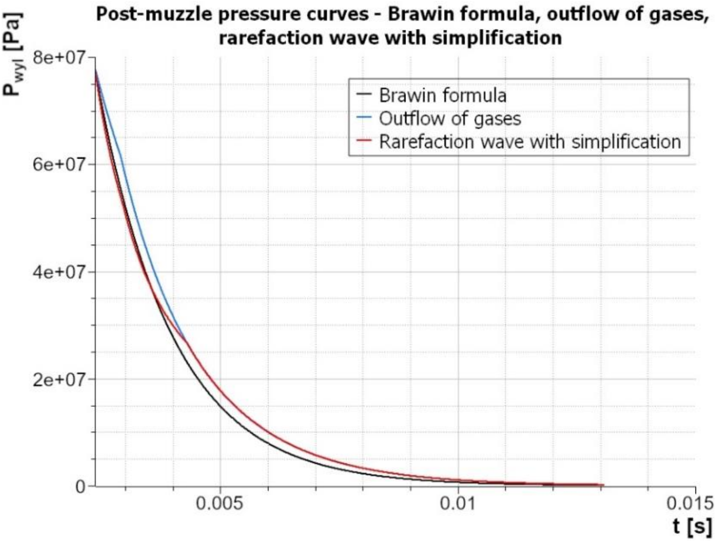

Fig. 8. Comparison of the pressure curves P_{wyl} in the post-muzzle period obtained with the use of three methods

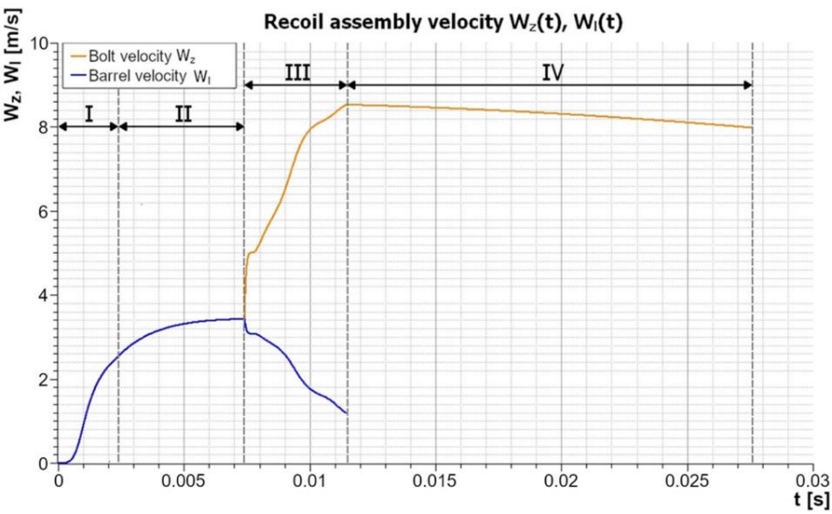

Fig. 9. Recoil velocity of the recoil assembly consisting of bolt (W_z) and barrel (W_l) with the characteristic's stages of operation shown: I – stage of bullet moving inside the barrel, II – post-muzzle stage, III – stage of bolt acceleration, IV – stage of bolt movement by forces of inertia (based on [2])

Tab. 1. Selected parameters for the entire pressure courses and for post-muzzle period obtained with different methods

Brawin's formula		Outflow of gases		Rarefaction wave	
t_k [s]	0.0115	t_k [s]	0.0112	t_k [s]	0.0113
p_k [MPa]	0.2578	p_k [MPa]	0.5583	p_k [MPa]	0.5108
max W_z [m/s]	8.5310	max W_z [m/s]	8.9520	max W_z [m/s]	8.7610
min W_l [m/s]	1.1820	min W_l [m/s]	1.2390	min W_l [m/s]	1.2170
W_r [m/s]	3.4270	W_r [m/s]	3.5500	W_r [m/s]	3.4840
t_r [s]	0.0074	t_r [s]	0.0073	t_r [s]	0.0074
t_o [s]	0.0276	t_o [s]	0.0265	t_o [s]	0.0270

The t_k parameter, presented in Table 1, refers to the accelerator operation's termination time determined with the use of post-muzzle period calculation methods discussed in the paper. The p_k parameter corresponds to a theoretical value of pressure in the barrel bore at t_k. In calculations of the automatic operation, a simplification was used in form of pressure acting on the bottom of the combustion chamber only until the accelerator operation was finished. Simplification was adopted due to relatively small protrusion of the cartridge case from the cartridge chamber during the accelerator operation (at the end of acceleration process, cartridge case protrudes from the chamber to a distance of about 22 mm, while length of the cartridge case in the analyzed system is 99 mm, the length of the chamber is ~93.8 mm). In addition, the table also shows: recoil time of the bolt assembly (t_o), maximum recoil velocity of the bolt (W_z), minimum recoil velocity of the barrel (W_l) (velocity of the barrel after acceleration of the bolt - at the moment of its stopping), recoil assembly recoil velocity at the moment of separation of the bolt and barrel (W_r) and corresponding time span. The table also includes the values of pressure impulse in the post-muzzle period I_p expressed in MPa·s.

Based on the analysis of the parameters, listed in Table 1, there is a relatively small difference between values of the parameters determined by the use of various methods describing the post-muzzle period. Despite maximum difference between the pressure impulses reaching 14.3% (between the outflow of gases and the Brawin's formula), differences between the maximum bolt recoil velocities, barrel recoil velocities, and the velocities at the moment of separation of the parts do not exceed 5% when compared to the velocities determined using the Brawin's formula. For the times t_r and t_k, the differences do not exceed 0.3 ms and 0.1 ms, respectively, the recoil time t_o of the bolt is in a range of 26.5 ms - 27.6 ms (the difference is 1 ms). Analysis of the pressures p_k, corresponding to the moments t_k of the end of acceleration of the bolt, allows us for validation of the introduced simplification concerning taking into account the pressure only until the end of the accelerator operation.

At the end of its operation, the pressure in the barrel bore is at a relatively low level, in the range of 0.2578 MPa - 0.5583 MPa (atmospheric pressure is about 0.1 MPa). Compiled curves of the pressures and the recoil velocities of the barrel and breech from the moment the projectile exits the barrel are shown in Fig. 10.

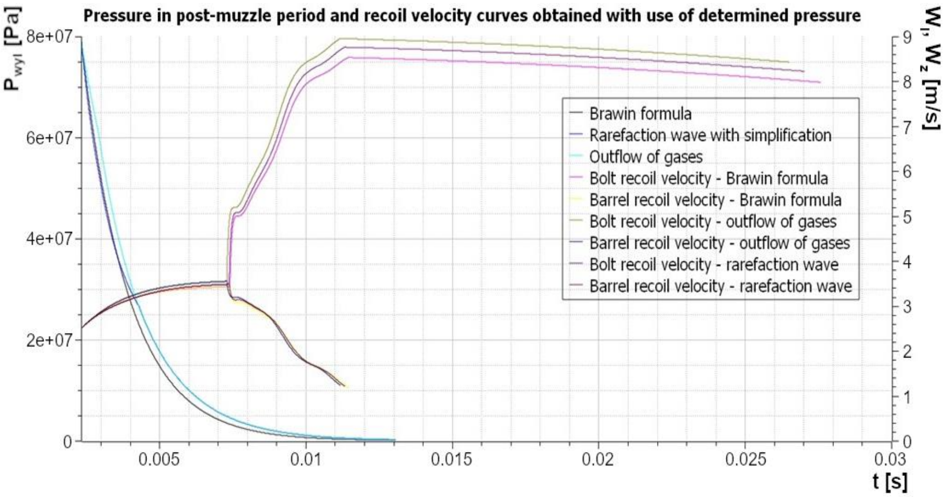

Fig. 10. Comparison of the pressure curves P_{wyl} and curves of the recoil velocities W_z and W_l of the recoiling assembly elements, determined by taking into account the obtained values of pressure in the post-muzzle stage of a shot

4. CONCLUSIONS

1. Taking into account the phenomena of wave motion are justified in the case of modelling the post-muzzle period of a gunshot. Taking into account passage of the rarefaction wave through the barrel bore and the combustion chamber combined with the outflow of gases from the barrel to the environment gives the results on the level of other analyzed methods.
2. Due to a length of the barrel of the analyzed system, afterburning of the powder charge (in outflow of the gases from the barrel method) results in obtaining the highest value of the post-muzzle pressure impulse in relation to other methods - higher by 14.3% compared to the values of pressure obtained using the Brawin's formula. Despite the differences in value of the pressure impulse, relatively large mass of the elements of the recoiling assembly and presence of the return springs result in a slight increase in the recoil velocity of the elements - the greatest difference is 4.9% (gas flow from the barrel to the environment in relation to the Brawin's formula).

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Analiza porównawcza metod obliczania ciśnienia w przewodzie lufy w powylotowym okresie strzału

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Streszczenie. W artykule przedstawiono wyniki obliczeń ciśnienia gazów prochowych w przewodzie lufy w powylotowym okresie strzału. W pracy wykorzystano trzy metody opisu przebiegu ciśnienia w przewodzie lufy w powylotowym okresie strzału: metodę Brawina, metodę wykorzystującą zależności na wypływ gazów z przewodu lufy do otoczenia oraz metodę wykorzystującą zjawisko przejścia uśredniającej ciśnienie fali rozrzedzenia od wylotu lufy do dna komory spalania. W prowadzonych badaniach otrzymane przebiegi ciśnienia zostały wykorzystane do określenia wpływu wykorzystania różnych metod opisu powylotowego okresu strzału na zachowanie układu automatyki broni działającej na zasadzie odrzutu. Porównanie przebiegów ciśnienia otrzymanych z wykorzystaniem różnych metod oraz prędkości odrzutu elementów zespołu odrzucanego wyznaczonych z wykorzystaniem modelu matematycznego i fizycznego wykazało, że w przypadku broni działającej na zasadzie odrzutu różnice między poszczególnymi metodami są niewielkie - różnice między prędkościami odrzutu wyznaczonymi z użyciem różnych metod nie przekraczały 5%. Zarówno przebiegi ciśnienia jak i prędkości odrzutu elementów zespołu odrzucanego broni działającej na zasadzie odrzutu wyznaczone z wykorzystaniem różnych metod opisu przebiegu ciśnienia w powylotowym okresie strzału znajdują się na zbliżonym poziomie.
Słowa kluczowe: balistyka wewnętrzna, broń palna, powylotowy okres strzału, odrzut