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Reliability and risk improvement with components quantitative and qualitative redundancy of bulk cargo transporter

Keywords

reliability function, risk function, operation process, redundancy

Abstract

The joint general model of reliability of complex technical systems at variable operation conditions linking a semi-Markov modelling of the system operation processes with a multi-state approach to system reliability analysis and reliability improvement are applied in maritime transport to reliability and risk optimization of a bulk cargo transportation system

1. Introduction

Most real technical systems are very complex because of large numbers of components and subsystems and their operating complexity. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability parameters is very often met in real. A convenient tool for investigate this problems is a semi-Markov [2] modelling of the system operation process linked with a multi-state approach for the system reliability analysis [1], [5], [9]-[10]. Using this approach it is possible to find the complex system main reliability characteristics like the system reliability function, the system mean lifetimes in system reliability subsets and the system risk function [5]-[6], [8]. Having those characteristics it is possible to improve the system operation process to get their optimal values [8]. To this end the quantitative and qualitative redundancy [3] can be applied for maximizing the mean value of the system lifetime in the subset of the system reliability states not worse than the system critical reliability state.

2. System reliability at variable operation conditions

We suppose that the system during its operation process has v different operation states. Thus, we can define the system operation process $Z(t)$, $t \in <0,+\infty>$, as the process with discrete operation states from the set

$$Z = \{z_1, z_2, \ldots, z_v\}.$$

Further, we assume that $Z(t)$ is a semi-Markov process [2] with its conditional sojourn times θ_{bl} at the operation state z_b when its next operation state is z_l, $b, l = 1,2,...,v$, $b \neq l$. In this case the process $Z(t)$ may be described by:

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1xv}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{vxv}$, where $p_{bb}(t) = 0$ for $b = 1,2,..., v$,
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{vxv}$ of the process sojourn times θ_{bl}, $b \neq l$, in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,..., v$, $b \neq l$, and $H_{bb}(t) = 0$ for $b = 1,2,..., v$.

Under these assumptions, the sojourn times θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \quad b, l = 1,2,...,v, \ b \neq l. \quad (1)$$

The unconditional distribution functions of the sojourn times θ_b of the process $Z(t)$ at the operation states z_b, $b = 1,2,..., v$, are given by

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ b = 1,2,..., v.$$

The mean values $E[\theta_b]$ of the unconditional sojourn times θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,..., v, \quad (2)$$

where M_{bl} are defined by (1).
Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b), \ t \in <0,+\infty), \ b = 1,2,..., v,$$

are given by

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,..., v,$$

where the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases}$$

3. Reliability of multistate systems

We assume that the system is composed of n independent multistate components E_i, $i = 1,2,...,n$, and that the changes of the operation process $Z(t)$ states have an influence on the system components E_i reliability and on the system reliability structure as well. Consequently, we denote the component E_i lifetime in the reliability states subset $\{u, u+1,..., z\}$ by $T_i^{(b)}(u)$ and by

$[R_i\ (t,\cdot)]^{(b)} = [1, [R_i\ (t,1)]^{(b)}, [R_i\ (t,2)]^{(b)}, ...,$
$[R_i\ (t,z)]^{(b)}],$

where for $t \in <0,\infty)$, $b = 1,2,...,v$, $u = 1,2,..., z$,

$$[R_i\ (t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b),$$

its conditional reliability function while the system is at the operational state z_b, $b = 1,2,...,v$.
Similarly, we denote the system lifetime in the reliability states subset $\{u, u+1,..., z\}$ by $T^{(b)}(u)$ and by

$$[\boldsymbol{R}(t, \cdot)]^{(b)} = [1, \ [\boldsymbol{R}(t,1)]^{(b)} \ [\boldsymbol{R}(t,2)]^{(b)} ..., [\boldsymbol{R}(t,z)]^{(b)}$$
]

where

$$[\boldsymbol{R}(t,u)]^{(b)}, = P(T^{(b)}(u) > t | Z(t) = z_b),$$

is the conditional reliability function of the system while the system is at the operational state z_b, $b = 1,2,...,v$.
Thus, the reliability function $[R_i\ (t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the operation process *Z*(*t*) is at the operation state z_b. Similarly, the reliability function $[\boldsymbol{R}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the operation process *Z*(*t*) is at the operation state z_b.
In the case when the system operation time is large enough, the unconditional reliability function of the system is given by

$$\boldsymbol{R}(t, \cdot) = [1, \ \boldsymbol{R}(t,1), \ \boldsymbol{R}(t,2), \ ..., \ \boldsymbol{R}(t,z)\ ], \ t \geq 0,$$

where

$$\boldsymbol{R}(t,u) = P(T(u) > t) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)} \quad (3)$$

for $t \geq 0$, $u = 1,2,..., z$, and $T(u)$ is the unconditional lifetime of the system in the reliability state subset $\{u, u+1,..., z\}$.
The mean values of the system lifetimes in the reliability state subset $\{u, u+1,..., z\}$ are

$$\mu(u) = E[T(u)] \cong \sum_{b=1}^{v} p_b \mu_b(u), \ u = 1,2,..., z, \quad (4)$$

where

$$\mu_b(u) = \int_0^{\infty} [\boldsymbol{R}(t,u)]^{(b)}\, dt, \quad n_b \in \{1,2,...,n\}, \tag{5}$$

$$u = 1,2,...,\ z.$$

The mean values of the system lifetimes in the particular reliability states u, are [5]

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),\ u = 1,2,...,\ z-1,$$

$$\bar{\mu}(z) = \mu(z). \tag{6}$$

A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid R(0) = z) = P(T^{(b)}(r) \le t),$$

$$t \in (-\infty, \infty),$$

that the system is in the subset of reliability states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system or, in short, a risk [5].

Under this definition, from (3), we have

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,r),\ t \in (-\infty, \infty). \tag{7}$$

and if τ is the moment when the risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{8}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

4. System reliability improvement by components quantitative and qualitative redundancy

Considering the equation (3), it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (6) for the mean values of the system unconditional lifetimes in the reliability state subsets. To improve reliability and the except values of the system unconditional lifetime in reliability state subsets we can use quantitative and qualitative redundancy. The quantitative redundancy we obtain by assuming that every single system component has hot reservation. The qualitative redundancy we obtain by using components with better reliability i.e. If the system' components have the multistate exponential reliability function the failure rate of every component will be decreased by a factor $\rho(u)$, $\{u, u+1, ..., z\}$ where $0 \le \rho(u) \le 1$.

Definition 1. A multistate system is called series if its lifetime $T(u)$ in the reliability state subset $\{u, u + 1,...,z\}$ is given by

$$T(u) = \min_{1 \le i \le n}\{T_i(u)\},\ u = 1,2,...,z.$$

It is easy to motivate that the reliability function of the multistate series system composed of component with multistate exponential reliability function

$$R_i(t,\cdot) = [1,\ R_i(t,1),\ \ldots,\ R_i(t,z)]$$

where

$$R_i(t,u) = \exp[-\lambda_i(u)t] \ \text{for}\ t \in <0,\infty),\ u = 1,2,...,z,$$

is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)]$$

with the coordinates

$$\boldsymbol{R}(t,u) = \prod_{i=1}^{n} R_i(t,u) = \prod_{i=1}^{n} \exp[-\lambda_i(u)t], \tag{9}$$

for $t \in <0,\infty)$, $u = 1,2,...,z$.

Definition 2. A multistate series system is called a system with a hot single reserve of its components if its lifetime $T_h(u)$ in the state subset $\{u,u+1,...,z\}$ is given by

$$T_h(u) = \min_{1 \le i \le n}\{\max_{1 \le j \le 2}\{T_{ij}(u)\}\},\ u = 1,2,...,z,$$

where $T_{i1}(u)$ are the lifetimes of the system basic components in the basic system and $T_{i2}(u)$ are the lifetimes of their reserve components.

The scheme of a series system with hot single reserve of particular component is given in *Figure 1*.

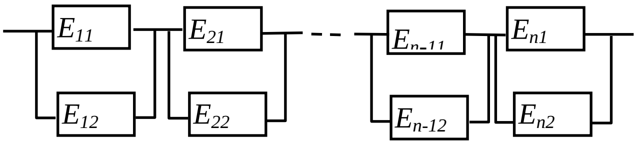

Figure 1. The scheme of a series system with hot reserve of its components

The reliability function of the non-homogeneous multistate series system with a hot reserve of its components is given by a vector

$$\boldsymbol{R_h}(t,\cdot) = [1, \boldsymbol{R_h}(t,1),..., \boldsymbol{R_h}(t,z)],$$

with the coordinates

$$\boldsymbol{R_h}(t,u) = \prod_{i=1}^{n}[1-[F_i(t,u)]^2] = \prod_{i=1}^{n}[1-[1-\exp[-\lambda_i(u)t]]^2] = \prod_{i=1}^{n}[2\exp[-\lambda_i(u)t]-\exp[-2\lambda_i(u)t]] = \prod_{i=1}^{n}\exp[-\lambda_i(u)t][2-\exp[-\lambda_i(u)t]], \quad (10)$$

for $t \in <0,\infty)$, $u = 1,2,...,z$.

Definition 3. A multistate exponential system is called multistate system with qualitative redundancy of its component if the failure rate of its component $\lambda_i(u)$, $i = 1,2,...,n$, $u = 1,2,...,z$, is reduced by the factor $\rho(u)$, $0 \le \rho(u) \le 1$, $\{u, u+1,..., z\}$.
Thus, if the multistate reliability function of the component E_i is given by the vector

$$R_{\rho i}(t,\cdot) = [1, R_{\rho i}(t,1), \ldots, R_{\rho i}(t,z)]$$

with the coordinates

$$R_{\rho i}(t,u) = \exp[-\lambda_i(u)\rho(u)t] \quad (11)$$

for $t \in <0,\infty)$, $u = 1,2,...,z$,

then the reliability function of the non-homogenous multistate series system with qualitative redundancy of its component in the reliability state subset $\{u, u + 1,...,z\}$ is given by the vector

$$\boldsymbol{R}_\rho(t,\cdot) = [1, \boldsymbol{R}_\rho(t,1),..., \boldsymbol{R}_\rho(t,z)]$$

where

$$\boldsymbol{R}_\rho(t,u) = \prod_{i=1}^{n} R_{\rho i}(t,u) = \prod_{i=1}^{n}\exp[-\lambda_i(u)\rho(u)t] \quad (12)$$

for $t \in <0,\infty)$, $u = 1,2,...,z$.

From linear equation (4), we can see that the mean value of the system unconditional lifetime $\mu(u)$, $u = 1,2,..., z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation states and the mean values $\mu_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,..., z$, of the system conditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z$, given by (5).
Therefore, the system lifetime improvement approach based on the quantitative and qualitative component redundancy can be proposed. Namely, we may look for the values of factor $\rho(r)$, $r \in \{1,..., z\}$ that ensure the same level of the system reliability or the mean values of the system conditional lifetimes at the critical state r, $r \in \{1,..., z\}$.
Finally, after finding the reliability function and the mean values of the system conditional lifetimes of the system with hot single component's reservation and of the system with qualitative redundancy of its component at the critical state r, comparing obtained results for both systems, we get the values of factor $\rho(r)$, for a fixed $r \in \{1,2,..., z\}$, from the equations

$$\boldsymbol{R_h}(t,r) = \boldsymbol{R}_\rho(t,r), \quad (13)$$

or from equation

$$\mu_h(r) = \mu_\rho(r), \quad (14)$$

where

$$\mu_h(r) \cong \sum_{b=1}^{\nu} p_b \mu_{hb}(r), \quad \mu_\rho(r) \cong \sum_{b=1}^{\nu} p_b \mu_{\rho b}(r),$$

$r \in \{1,2,..., z\}$.

5. The bulk cargo transportation system reliability and risk functions

The considered bulk cargo terminal placed at the Baltic seaside is designated for storage and reloading of bulk cargo such as different kinds of fertilizers i.e.: ammonium sulphate, but its main area of activity is to load bulk cargo on board the ships for export.
There are two independent transportation systems:

1. The system of reloading rail wagons.
2. The system of loading vessels.

Cargo is brought to the terminal by trains consisting of self discharging wagons which are discharged to a hopper and then by means of conveyors are transported into the one of four storage tanks (silos). Loading of fertilizers from storage tanks on board the ship is done by means of special reloading system which consists of several belt conveyors and one bucket conveyor which allows the transfer of bulk cargo in a vertical direction. Researched system is a system of belt conveyors, called later on the transport system.

In the conveyor reloading system we distinguish three bulk cargo transportation subsystems, the belt conveyors S_1, S_2 and S_3 .

The conveyor loading system is composed of six bulk cargo transportation subsystems, the dosage conveyor S_4, the horizontal conveyor S_5, the horizontal conveyor S_6, the sloping conveyors S_7, the dosage conveyor with buffer S_8, the loading system S_9.

The bulk cargo transportation subsystems are built, respectively:

- the subsystem S_1 composed of 1 rubber belt, 2 drums, set of 121 bow rollers, set of 23 belt supporting rollers,
- the subsystem S_2 composed of 1 rubber belt, 2 drums, set of 44 bow rollers, set of 14 belt supporting rollers,
- the subsystem S_3 composed of 1 rubber belt, 2 drums, set of 185 bow rollers, set of 60 belt supporting rollers,
- the subsystem S_4 composed of three identical belt conveyors, each composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_5 composed of 1 rubber belt, 2 drums, set of 125 bow rollers, set of 45 belt supporting rollers,
- the subsystem S_6 composed of 1 rubber belt, 2 drums, set of 65 bow rollers, set of 20 belt supporting rollers,
- the subsystem S_7 composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem S_8 composed of 1 rubber belt, 2 drums, set of 162 bow rollers, set of 53 belt supporting rollers,
- the subsystem S_9 composed of 3 rubber belts, 6 drums, set of 64 bow rollers, set of 20 belt supporting rollers.

The scheme of the bulk cargo transportation system is presented in *Figure 2*.

Taking into account the operation process of the considered system we distinguish the following as its three operation states:

- an operation state z_1 – the loading of fertilizers from rail wagons on board the ship is done by using S_1, S_2, S_3, S_6, S_7, S_8 and S_9 subsystems.
- an operation state z_2 – the discharging rail wagons to storage tanks or hall when subsystems S_1, S_2 and S_3, are used,
- an operation state z_3 – the loading of fertilizers from storage tanks or hall on board the ship is done by using S_4, S_5, S_6, S_7, S_8 and S_9, subsystems.

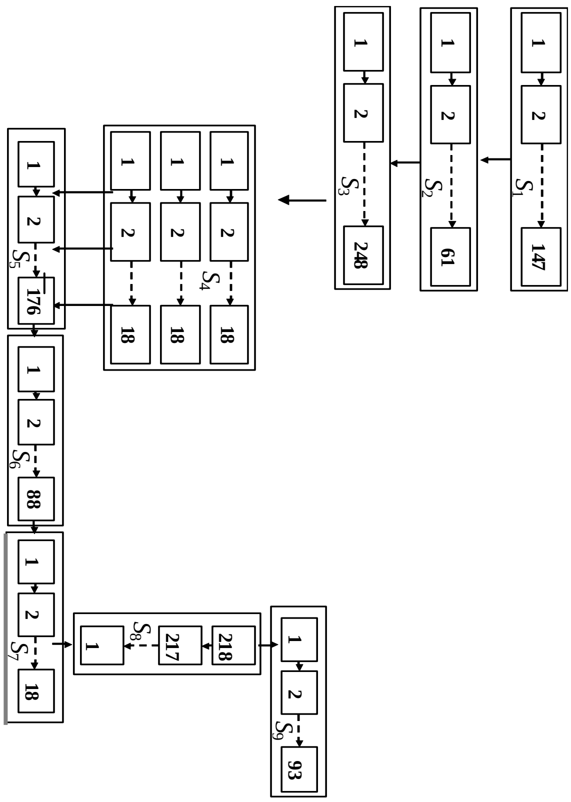

Figure 2. The scheme of port bulk cargo transportation system

The limit values of the bulk cargo transportation systems operation process transient probabilities $p_b(t)$ at the operation states z_b, $b = 1,2,3,$ determined in [4], on the bases of the data coming from experts are

$$p_1 = 0.2376, \quad p_2 = 0.6679, \quad p_3 = 0.0945. \qquad (15)$$

Further, assuming that the system is in the reliability state subset {*u*,*u*+1,…,z} if all its subsystems are in this subset of reliability states, we conclude that the bulk cargo transportation system is a series system [5] of subsystems S_1, S_2, S_3, S_6, S_7, S_8 and S_9 with a scheme presented in *Figure 2*.

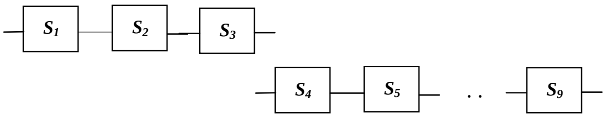

Figure 3. The scheme of port bulk cargo transportation system reliability structure

Additionally, we assume that the subsystems S_i, $i = 1,2,3,...,9$, are composed of four-state components, with the exponential reliability functions with the parameters $\lambda_i^{(\upsilon)}(u)$, $i = 1,2,...,i^{(\upsilon)}$, $u = 1,2,3$, $\upsilon = 1,2,3$, presented in *Tables 1-3.*

Table 1. Bulk cargo transportation subsystem S_1, S_2, S_3, component parameters $\lambda_i^{(k)}(u)$, $u = 1,2,3$

S_1	$\lambda_i^{(1)}(u)$	S_2	$\lambda_i^{(2)}(u)$	S_3	$\lambda_i^{(3)}(u)$
$i = 1$		$i = 1$		$i = 1$	
$u = 1$	0,124	$u = 1$	0,124	$u = 1$	0,124
$u = 2$	0,167	$u = 2$	0,167	$u = 2$	0,167
$u = 3$	0,250	$u = 3$	0,250	$u = 3$	0,250
$i = 2,3$		$i = 2,3$		$i = 2,3$	
$u = 1$	0,049	$u = 1$	0,049	$u = 1$	0,049
$u = 2$	0,055	$u = 2$	0,055	$u = 2$	0,055
$u = 3$	0,061	$u = 3$	0,061	$u = 3$	0,061
$i = 4,...,124$		$i = 4,...,47$		$i = 4,...,188$	
$u = 1$	0,097	$u = 1$	0,097	$u = 1$	0,097
$u = 2$	0,124	$u = 2$	0,124	$u = 2$	0,124
$u = 3$	0,164	$u = 3$	0,164	$u = 3$	0,164
$i = 125,...,147$		$i = 48,...,61$		$i = 189,...,248$	
$u = 1$	0,051	$u = 1$	0,051	$u = 1$	0,051
$u = 2$	0,056	$u = 2$	0,056	$u = 2$	0,056
$u = 3$	0,062	$u = 3$	0,062	$u = 3$	0,062

Table 2. Bulk cargo transportation subsystem S_4, S_5, S_6 component parameters $\lambda_i^{(k)}(u)$, $\lambda_{ij}^{(k)}(u)$, $u = 1,2,3$

S_4	$\lambda_{ij}^{(4)}(u)$ $j = 1,2,3$	S_5	$\lambda_i^{(5)}(u)$	S_6	$\lambda_i^{(6)}(u)$
$i = 1$		$i = 1$		$i = 1$	
$u = 1$	0,124	$u = 1$	0,124	$u = 1$	0,124
$u = 2$	0,167	$u = 2$	0,167	$u = 2$	0,167
$u = 3$	0,250	$u = 3$	0,250	$u = 3$	0,250
$i = 2,3$		$i = 2,3$		$i = 2,3$	
$u = 1$	0,049	$u = 1$	0,049	$u = 1$	0,049
$u = 2$	0,055	$u = 2$	0,055	$u = 2$	0,055
$u = 3$	0,061	$u = 3$	0,061	$u = 3$	0,061
$i = 4,...,15$		$i = 4,...,128$		$i = 4,...,67$	
$u = 1$	0,189	$u = 1$	0,097	$u = 1$	0,097
$u = 2$	0,195	$u = 2$	0,124	$u = 2$	0,124
$u = 3$	0,202	$u = 3$	0,164	$u = 3$	0,164
$i = 16,...,18$		$i = 129,...,173$		$i = 68,...,88$	
$u = 1$	0,087	$u = 1$	0,051	$u = 1$	0,051
$u = 2$	0,113	$u = 2$	0,056	$u = 2$	0,056
$u = 3$	0,160	$u = 3$	0,062	$u = 3$	0,062

Table 3. Bulk cargo transportation subsystem S_7, S_8, and S_9, component parameters $\lambda_i^{(k)}(u)$, $u = 1,2,3$

S_7	$\lambda_i^{(7)}(u)$	S_8	$\lambda_i^{(8)}(u)$	S_9	$\lambda_i^{(9)}(u)$
$i = 1$		$i = 1$		$i = 1,2,3$	
$u = 1$	0,124	$u = 1$	0,124	$u = 1$	0,124
$u = 2$	0,167	$u = 2$	0,167	$u = 2$	0,167
$u = 3$	0,250	$u = 3$	0,250	$u = 3$	0,250
$i = 2,3$		$i = 1,2,3$		$i = 4,..,9$	
$u = 1$	0,049	$u = 1$	0,049	$u = 1$	0,049
$u = 2$	0,055	$u = 2$	0,055	$u = 2$	0,055
$u = 3$	0,061	$u = 3$	0,061	$u = 3$	0,061
$i = 4,...,15$		$i = 4,...,165$		$i = 10,...,73$	
$u = 1$	0,189	$u = 1$	0,097	$u = 1$	0,077
$u = 2$	0,195	$u = 2$	0,124	$u = 2$	0,098
$u = 3$	0,202	$u = 3$	0,164	$u = 3$	0,124
$i = 16,...,18$		$i = 167,...218$		$i = 74,...,93$	
$u = 1$	0,087	$u = 1$	0,051	$u = 1$	0,039
$u = 2$	0,113	$u = 2$	0,056	$u = 2$	0,048
$u = 3$	0,160	$u = 3$	0,062	$u = 3$	0,055

Under the assumption that the changes of the bulk cargo transportation system operation states have an influence on the subsystem S_i, $i = 1,2,3,...,9$, reliability and on the whole reliability structures as well [8], on the basis of expert opinions and statistical data the bulk cargo transportation system reliability structures and their components reliability functions at different operation states can be determined.

At the operation state z_1, at loading of fertilizers from rail wagons on board the ship, system is composed of seven non-homogenous series subsystems S_1, S_2, S_3, S_6, S_7, S_8, and S_9 forming a series structure. The conditional reliability function of the system while it is at the operation state z_1 is given by

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}, [\boldsymbol{R}(t,3)]^{(1)}],$$

where

$$[\boldsymbol{R}(t,u)]^{(1)} = [\boldsymbol{R}_{147}(t,u)]^{(1)} [\boldsymbol{R}_{61}(t,u)]^{(1)} [\boldsymbol{R}_{248}(t,u)]^{(1)} [\boldsymbol{R}_{88}(t,u)]^{(1)} [\boldsymbol{R}_{18}(t,u)]^{(1)} [\boldsymbol{R}_{218}(t,u)]^{(1)} [\boldsymbol{R}_{93}(t,u)]^{(1)}$$

for $t \in <0,\infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}(t,1)]^{(1)} = \exp[-74.426t] \quad (16)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \exp[-93.472t], \quad (17)$$

$$[\boldsymbol{R}(t, 3)]^{(1)} = \exp[-150.206t], \quad (18)$$

The expected values of the conditional lifetimes in the reliability state subsets calculated from the above result given by (16)-(18), according to (5), at the operation state z_1 are:

$$\mu_1(1) \cong 0.013, \ \mu_1(2) \cong 0.011, \\ \mu_1(3) \cong 0.007 \text{ years}, \quad (19)$$

and further, using (6), it follows that the conditional lifetimes in the particular reliability states at the operation state z_1 are:

$$\bar{\mu}_1(1) \cong 0.002, \ \bar{\mu}_1(2) \cong 0.004, \ \bar{\mu}_1(3) \cong 0.007 \text{ years}.$$

At the operation state z_2, i.e. at the discharging rail wagons to storage tanks or hall state the system is built of three subsystems S_1, S_2 and S_3 forming a series structure [5]. The conditional reliability function of the bulk cargo transportation system at the operation state z_2 is given by

$$[\boldsymbol{R}(t, \cdot)]^{(2)} = [1, [\boldsymbol{R}(t, 1)]^{(2)}, [\boldsymbol{R}(t, 2)]^{(2)}, [\boldsymbol{R}(t, 3)]^{(2)}],$$

where

$$[\boldsymbol{R}(t,u)]^{(2)} = [\boldsymbol{R}_{147}(t, u)]^{(2)} [\boldsymbol{R}_{61}(t, u)]^{(2)} [\boldsymbol{R}_{248}(t, u)]^{(2)}$$

for $t \in <0, \infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}(t,1)]^{(2)} = \exp[-39.563t], \quad (20)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \exp[-49.663t], \quad (21)$$

$$[\boldsymbol{R}(t,3)]^{(2)} = \exp[-64.280t]. \quad (22)$$

The expected values of the conditional lifetimes in the reliability state subsets calculated from the above result given by (20)-(22), according to (5), at the operation state z_2 are:

$$\mu_2(1) \cong 0.025, \ \mu_2(2) \cong 0.020, \ \mu_2(3) \cong 0.016, \quad (23)$$

and further, using (6), it follows that the conditional lifetimes in the particular reliability states at the operation state z_2 are:

$$\bar{\mu}_2(1) \cong 0.005, \ \bar{\mu}_2(2) \cong 0.004, \ \bar{\mu}_2(3) \cong 0.016.$$

At the operation state z_3, i.e. at the loading of fertilizers from storage tanks or hall on board, the bulk cargo transportation system is built of six subsystems one series-parallel subsystem S_4 and five series subsystems S_5, S_6, S_7, S_8, S_9 forming a series structure [5]. The conditional reliability function of the system while it is at the operation state z_3 is given by

$$[\boldsymbol{R}(t, \cdot)]^{(3)} = [1, [\boldsymbol{R}(t, 1)]^{(3)}, [\boldsymbol{R}(t, 2)]^{(3)}, \\ [\boldsymbol{R}(t, 3)]^{(3)}],$$

where

$$[\boldsymbol{R}(t, u)]^{(3)} = [\boldsymbol{R}_{3,18}(t, u)]^{(3)} \cdot [\boldsymbol{R}_{173}(t, u)]^{(3)} \\ \cdot [\boldsymbol{R}_{88}(t, u)]^{(3)} \cdot [\boldsymbol{R}_{18}(t, u)]^{(3)} \cdot [\boldsymbol{R}_{218}(t, u)]^{(3)} \\ \cdot [\boldsymbol{R}_{93}(t, u)]^{(3)}$$

for $t \in <0, \infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}(t, 1)]^{(3)} = \exp[-57.758t] - 3\exp[-55.007t] \\ + 3\exp[-52.256t], \quad (24)$$

$$[\boldsymbol{R}(t, 2)]^{(3)} = \exp[-70.974t] - 3\exp[-68.018t] \\ + 3\exp[-65.062t], \quad (25)$$

$$[\boldsymbol{R}(t, 3)]^{(3)} = \exp[-89.416t] - 3\exp[-86.140t] \\ + 3\exp[-82.864t]. \quad (26)$$

The expected values of the conditional lifetimes in the reliability state subsets calculated from the above result given by (24)-(26), according to (5), at the operation state z_3 are:

$$\mu_3(1) \cong 0.020, \ \mu_3(2) \cong 0.016, \ \mu_3(3) \cong 0.013, \quad (27)$$

and further, using (6), it follows that the conditional lifetimes in the particular reliability states at the operational state z_3 are:

$$\bar{\mu}_3(1) \cong 0.004, \ \bar{\mu}_3(2) \cong 0.003, \ \bar{\mu}_3(3) \cong 0.013 \text{ years}.$$

In the case when the system operation time is large enough, according to (3), the unconditional reliability function of the bulk cargo transportation system is given by the vector

$$\boldsymbol{R}(t, \cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)], \ t \geq 0,$$

where, according to (3) and after considering the values of p_b, $b = 1,2,3$, given by (15), its co-ordinates are as follows:

$$\boldsymbol{R}(t,u) = p_1 \cdot [\boldsymbol{R}(t,u)]^{(1)} + p_2 \cdot [\boldsymbol{R}(t,u)]^{(2)} + p_3 \cdot [\boldsymbol{R}(t,u)]^{(3)} \quad (28)$$

for $t \geq 0$, $u = 1,2,3$, where $[\boldsymbol{R}(t,u)]^{(1)}$ and $[\boldsymbol{R}(t,u)]^{(2)}$ and $[\boldsymbol{R}(t,u)]^{(3)}$ are respectively given by (16)-(18) and (20)-(22) and (24)-(26), i.e.

$$\boldsymbol{R}(t,1) = 0.6679\exp[-39.563t] + 0.0945\exp[-74.426t] + 0.2376[\exp[-57.758t] - 3\exp[-55.007t] + 3\exp[-52.256t]], \quad (29)$$

$$\boldsymbol{R}(t,2) = 0.6679\exp[-93.472t] + 0.0945\exp[-49.663t] + 0.2376[\exp[-70.974t] - 3\exp[-68.018t] + 3\exp[-65.062t]], \quad (30)$$

$$\boldsymbol{R}(t,3) = 0.6679\exp[-150.206t] + 0.0945\exp[-64.280t] + 0.0945[\exp[-89.416t] - 3\exp[-86.140t] + 3\exp[-82.864t]], \quad (31)$$

for $t \geq 0$.

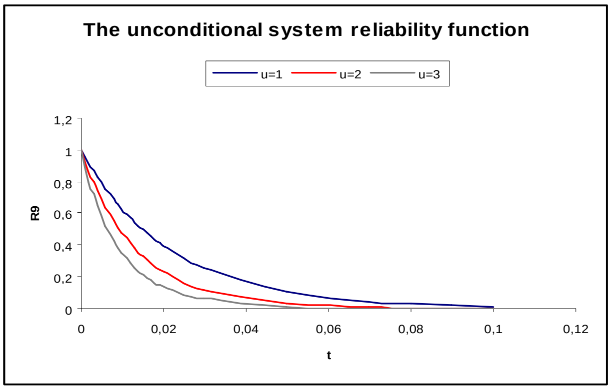

Figure 4. The graph of the port bulk cargo transportation system unconditional reliability function $\boldsymbol{R}(t,u)$, $u = 1,2,3$

The mean values of the system unconditional lifetimes in the reliability state subsets, according to (4)-(5) respectively are:

$$\mu(1) \cong 0.016, \ \mu(2) \cong 0.013, \ \mu(3) \cong 0.009, \quad (32)$$

The mean values of the system lifetimes in the particular reliability states, by (6), are

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.003,$$
$$\bar{\mu}(2) = \mu(2) - \mu(3) = 0.004,$$
$$\bar{\mu}(3) = \mu(3) = 0.009.$$

If the critical reliability state is $r = 2$, then the system risk function, according to (7) and (30), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2) \ \text{for} \ t \geq 0. \quad (33)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, from (8), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.000627 \text{ years}. \quad (34)$$

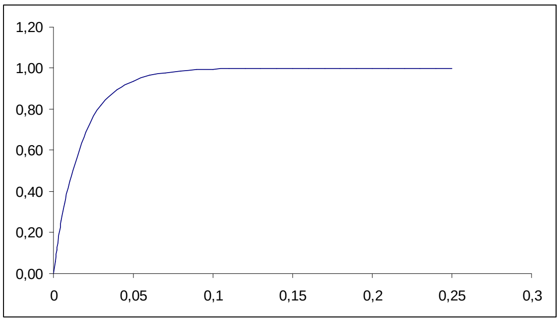

Figure 5. The graph of the port bulk cargo transportation system risk function

5. Reliability of improved bulk cargo transportation system in operation process

Now we assume the quantitative and qualitative redundancy of system's components. In the first case considering the expression (3), we get the unconditional reliability function of the system (29)-(30).

The conditional reliability function of the system with hot single reservation of components while it is at the operation state z_1, after considering the expression (10) is given by

$$[\boldsymbol{R}_h(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}_h(t,1)]^{(1)}, [\boldsymbol{R}_h(t,2)]^{(1)}, [\boldsymbol{R}_h(t,3)]^{(1)}],$$

with the coordinates

$$[\boldsymbol{R}_h(t,u)]^{(1)} = [\boldsymbol{R}_{h147}(t,u)]^{(1)} \ [\boldsymbol{R}_{h61}(t,u)]^{(1)} \ [\boldsymbol{R}_{h248}(t,u)]^{(1)} \ [\boldsymbol{R}_{h88}(t,u)]^{(1)} \ [\boldsymbol{R}_{h18}(t,u)]^{(1)} \ [\boldsymbol{R}_{h218}(t,u)]^{(1)}$$

$$[\boldsymbol{R}_{h93}(t,u)]^{(1)}$$

for $t \in <0,\infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}_h(t,1)]^{(1)} = \exp[-74.874t](2-\exp[-0.124t])^9 (2-\exp[-0.049t])^{18} (2-\exp[-0.097t])^{577} (2-\exp[-0.189t])^{12} (2-\exp[-0{,}077t])^{64} (2-\exp[-0.051t])^{170} (2-\exp[-0.087t])^3 (2-\exp[-0.039t])^{20}, \quad (35)$$

at the critical state $r = 2$, we get

$$[\boldsymbol{R}_h(t,2)]^{(1)} = \exp[-93.472t](2-\exp[-0.167t])^9 (2-\exp[-0.055t])^{18} (2-\exp[-0.124t])^{577} (2-\exp[-0.195t])^{12} (2-\exp[-0{,}098t])^{64} (2-\exp[-0.056t])^{170} (2-\exp[-0.113t])^3 (2-\exp[-0.048t])^{20}, \quad (36)$$

$$[\boldsymbol{R}_h(t,3)]^{(1)} = \exp[-120.456t](2-\exp[-0.25t])^9 (2-\exp[-0.061t])^{18} (2-\exp[-0.164t])^{577} (2-\exp[-0.202t])^{12} (2-\exp[-0{,}124t])^{64} (2-\exp[-0.062t])^{170} (2-\exp[-0.160t])^3 (2-\exp[-0.055t])^{20}, \quad (37)$$

for $t \in <0,\infty)$,

The expected values of the conditional lifetimes at the reliability critical state calculated from the above result given by according to (5), at the operation state z_1 are:

$$\mu_{h1}(1) = 0.3442,\ \mu_{h1}(2) \cong 0.2745,\ \mu_{h1}(3) = 0.2109. \quad (38)$$

The conditional reliability function of the system with hot single reservation of components while it is at the operation state z_2 is given by

$$[\boldsymbol{R}_h(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}_h(t,1)]^{(2)}, [\boldsymbol{R}_h(t,2)]^{(2)}, [\boldsymbol{R}_h(t,3)]^{(2)}],$$

with the coordinates

$$[\boldsymbol{R}_h(t,u)]^{(2)} = [\boldsymbol{R}_{h147}(t,u)]^{(2)} [\boldsymbol{R}_{h61}(t,u)]^{(2)} [\boldsymbol{R}_{h248}(t,u)]^{(2)}$$

for $t \in <0,\infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}_h(t,1)]^{(2)} = \exp[-39.563t](2-\exp[-0.124t])^3 (2-\exp[-0.049t])^6 (2-\exp[-0.097t])^{350} (2-\exp[-0.051t])^{97}, \quad (39)$$

at the critical state $r = 2$, we get

$$[\boldsymbol{R}_h(t,2)]^{(2)} = \exp[-49.663t](2-\exp[-0.167t])^3 (2-\exp[-0.055t])^6 (2-\exp[-0.124t])^{350} (2-\exp[-0.056t])^{97}, \quad (40)$$

$$[\boldsymbol{R}_h(t,3)]^{(2)} = \exp[-64.530t](2-\exp[-0.250t])^3 (2-\exp[-0.061t])^6 (2-\exp[-0.164t])^{350} (2-\exp[-0.062t])^{97} \quad (41)$$

for $t \in <0,\infty)$.

The expected values of the conditional lifetimes at the reliability critical state calculated from the above result given by according to (5), at the operation state z_2 are:

$$\mu_{h2}(1) \cong 0.4798,\ \mu_{h2}(2) \cong 0.3789,\ \mu_{h2}(3) \cong 0.2885 \quad (42)$$

The conditional reliability function of the system with hot single reservation of components while it is at the operation state z_3 is given by

$$[\boldsymbol{R}_h(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}_h(t,1)]^{(3)}, [\boldsymbol{R}_h(t,2)]^{(3)}, [\boldsymbol{R}_h(t,3)]^{(3)}],$$

with the coordinates

$$[\boldsymbol{R}_h(t,u)]^{(3)} = [\boldsymbol{R}_{h3,18}(t,u)]^{(3)} [\boldsymbol{R}_{h173}(t,u)]^{(3)} [\boldsymbol{R}_{h88}(t,u)]^{(3)} [\boldsymbol{R}_{h18}(t,u)]^{(3)} [\boldsymbol{R}_{h218}(t,u)]^{(3)} [\boldsymbol{R}_{h93}(t,u)]^{(3)}$$

for $t \in <0,\infty)$, $u = 1,2,3$, i.e.

$$[\boldsymbol{R}_h(t,1)]^{(3)} = \exp[-53.704t] (2-\exp[-0.124t])^8 (2-\exp[-0.087t])^6 (2-\exp[-0.049t])^{16} (2-\exp[-0.189t])^{24} (2-\exp[-0.097t])^{352} (2-\exp[-0.051t])^{118}$$

$$(2-\exp[-0.077t])^{64}(2-\exp[-0.039t])^{20}$$
$$[3-3\exp[-2{,}751t](2-\exp[-0.124t])$$
$$(2-\exp[-0.049t])^{2}\,(2-\exp[-0.189t])^{12}$$
$$(2-\exp[-0.087t])^{3}+\exp[-5{,}502t]$$
$$(2-\exp[-0.124t])^{2}(2-\exp[-0.049t])^{4}$$
$$(2-\exp[-0.189t])^{24}(2-\exp[-0.087t])^{6}], \quad (43)$$

at the critical state $r = 2$, we get

$$[\boldsymbol{R}_h(t,\,2)]^{(3)} = \exp[-65.062t]$$
$$(2-\exp[-0.167t])^{8}(2-\exp[-0.055t])^{16}$$
$$(2-\exp[-0.195t])^{24}(2-\exp[-0.113t])^{6}$$
$$(2-\exp[-0.124t])^{352}(2-\exp[-0.056t])^{118}$$
$$(2-\exp[-0.098t])^{64}(2-\exp[-0.048t])^{20}$$
$$[3-3\exp[-2{,}956t](2-\exp[-0.167t])$$
$$(2-\exp[-0.055t])^{2}\,(2-\exp[-0.195t])^{12}$$
$$(2-\exp[-0.113t])^{3}+\exp[-5{,}912t]$$
$$(2-\exp[-0.167t])^{2}(2-\exp[-0.055t])^{4}$$
$$(2-\exp[-0.195t])^{24}(2-\exp[-0.113t])^{6}], \quad (44)$$

$$[\boldsymbol{R}_h(t,\,3)]^{(3)} = \exp[-82.864t]$$
$$(2-\exp[-0.25t])^{8}(2-\exp[-0.061t])^{16}$$
$$(2-\exp[-0.202t])^{24}(2-\exp[-0.16t])^{6}$$
$$(2-\exp[-0.164t])^{352}(2-\exp[-0.062t])^{118}$$
$$(2-\exp[-0.124t])^{64}(2-\exp[-0.055t])^{20}$$
$$[3-3\exp[-3{,}276t](2-\exp[-0.250t])$$
$$(2-\exp[-0.061t])^{2}\,(2-\exp[-0.202t])^{12}$$
$$(2-\exp[-0.16t])^{3}+\exp[-6{,}552t]$$
$$(2-\exp[-0.25t])^{2}(2-\exp[-0.061t])^{4}$$
$$(2-\exp[-0.202t])^{24}(2-\exp[-0.16t])^{6}] \quad (45)$$

for $t \in <0,\infty)$.

The expected values of the conditional lifetimes at the reliability critical state calculated from the above result given by according to (5), at the operation state z_3 are:

$$\mu_{h3}(1) \cong 0.4227,\ \mu_{h3}(2) \cong 0{,}3393,$$
$$\mu_{h3}(3) \cong 0{,}2623. \quad (46)$$

In the case when the system operation time is large enough, the unconditional reliability function of the bulk cargo transportation system with hot single reservation of its component at the critical state $r = 2$ according to (3) and after considering the values of p_b, $b = 1,2,3,$ given by (15), is respectively given by

$$\boldsymbol{R}_h(t,2) = 0.6679\cdot[\boldsymbol{R}_h(t,2)]^{(1)}$$
$$+0.0945\cdot[\boldsymbol{R}_h(t,2)]^{(2)}$$
$$+0.2376\cdot[\boldsymbol{R}_h(t,2)]^{(3)} \quad (47)$$

The mean value of the system unconditional lifetimes in the critical reliability state, according to (4)-(5) respectively is:

$$\mu(2) \cong 0.6679\cdot 0.2745 + 0.0945\cdot 0.3789$$
$$+ 0.2376\cdot 0.3393 = 0.2998 \text{ years} \quad (48)$$

If the critical reliability state is r = 2, then the system risk function, according to (7) , is given by

$$\boldsymbol{r}(t) = 1-\boldsymbol{R}_h(t,2) \ \text{ for } t \geq 0. \quad (49)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, from (10), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.07382 \text{ years}. \quad (50)$$

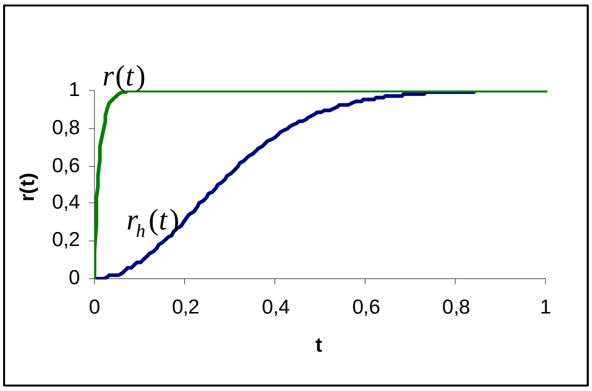

Figure 6. The graphs of the port bulk cargo transportation system risk function and the risk function of this system with hot single reservation of its components

In the second case when the system operation time is large enough, the unconditional reliability function of the bulk cargo transportation system with qualitative redundancy of its component is given by the vector

$$\boldsymbol{R}_{\rho}(t,\cdot)=[1,\ \boldsymbol{R}_{\rho}(t,1),\ \boldsymbol{R}_{\rho}(t,2),\ \boldsymbol{R}_{\rho}(t,3)\ ],\ t\geq 0,$$

where, according to (3) and after considering the values of p_b, $b=1,2,3,$ given by (15), its co-ordinates are as follows:

$$\boldsymbol{R}_{\rho}(t,u)=0.6679\cdot[\boldsymbol{R}_{\rho}(t,u)]^{(1)} +0.0945\cdot[\boldsymbol{R}_{\rho}(t,u)]^{(2)} +0.2376\cdot[\boldsymbol{R}_{\rho}(t,u)]^{(3)} \qquad (51)$$

for $t\geq 0,\ u=1,2,3,$ where $[\boldsymbol{R}_{\rho}(t,u)]^{(1)}$, $[\boldsymbol{R}_{\rho}(t,u)]^{(2)}$ and $[\boldsymbol{R}_{\rho}(t,u)]^{(3)}$ are respectively given by (35)-(37) and (39)-(41) and (43)-(45), i.e.

$$\boldsymbol{R}(t,1)=0.6679\exp[-39.563\rho(1)t] +0.0945\exp[-74.426\rho(1)t] +0.2376(\exp[-57.758\rho(1)t] -3\exp[-55.007\rho(1)t] +3\exp[-52.256\rho(1)t]), \qquad (52)$$

$$\boldsymbol{R}(t,2)=0.6679\exp[-93.472\rho(2)t] +0.0945\exp[-49.663\rho(2)t] +0.2376(\exp[-70.974\rho(2)t] -3\exp[-68.018\rho(2)t] +3\exp[-65.062\rho(2)t]), \qquad (53)$$

$$\boldsymbol{R}(t,3)=0.6679\exp[-150.206\rho(3)t] +0.0945\exp[-64.280\rho(3)t] +0.0945(\exp[-89.416\rho(3)t] -3\exp[-86.140\rho(3)t] +3\exp[-82.864\rho(3)t]). \qquad (54)$$

If the critical reliability state is r = 2, then the system risk function, according to (8) , is given by

$$\boldsymbol{r}(t)=1-\ \boldsymbol{R}(t,2)\quad \text{for } t\geq 0. \qquad (55)$$

The mean value of the system unconditional lifetimes in the critical reliability state is r = 2, according to (4)-(5), respectively are:

$$\mu(2)\cong\frac{0,013}{\rho(2)}. \qquad (56)$$

Now comparing the mean value of the system unconditional lifetimes in the critical reliability state (48) and (56), according to the equation (14), we determine the value of the factor $\rho(2)$, i.e. the factor of components failure rates reduction,

$$0.2998=\frac{0.013}{\rho(2)}.$$

Hence

$$\rho(2)=0.434. \qquad (57)$$

5. Conclusion

The joint model of reliability of complex technical systems at variable operation conditions linking a semi-Markov modelling of the system operation processes with a multi-state approach to system reliability analysis and system reliability improvement was constructed. Next, the final results obtained from this joint model and a linear programming were used to build the model of complex technical systems reliability optimization. These tools can be useful in reliability evaluation and optimization of a very wide class of real technical systems operating in varying conditions that have an influence on changing their reliability structures and their components reliability characteristics. These tools practical application to reliability and risk evaluation and optimization of a technical system of a bulk cargo transportation system operating at variable operation conditions and the results achieved are interesting for reliability practitioners from maritime transport industry and from other industrial sectors as well.

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