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IDENTIFICATION OF THE REFERENCE BASE FOR HORIZONTAL DISPLACEMENTS BY "ALL-PAIRS METHOD"

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Abstract

In studies of movement essential element of the correctness of the obtained results is correct identification of the reference base. One method for such identification, referred to as "all-pairs method", is to examine the insignificance of the mutual displacement of potential reference points in all combinations of pairs. It is a well known exact method of identification, but its use in practice is limited to the study of vertical displacements. In this work we present the possibility of application of this method in the study of horizontal displacements. Numerical example illustrates the calculation procedure of the proposed method.

Keywords: displacement, identification procedure, reference base

1. Introduction

Both the construction process of various engineering structures (dams, tunnels, bridges, industrial chimneys, tall buildings, etc.) as well as their safe exploitation require the displacement monitoring of selected points of the object. These displacements are the result of deformation and/or change of position of an object in space, due to various factors. Regardless of the type of monitoring (automatic integrated systems or classic periodic control measurements) to determine the displacements of points is necessary to adopt a specific, clearly defined reference system. An important element of the reference system is the reference base, which is defined as a group of mutually fixed points. Already at the design stage of the control network to study displacements, the locations and the number of points of potential reference base are determined. Current mutual positions of these points are determined by control measurement in the network. Based on the results of two periodic measurements one can verify the mutual fixity of points of potential reference base and eventually detect such points that did not preserved their stability in relation to the others. The displacements of the controlled points are calculated with respect to only those points that preserved mutual fixity and form a so-called appropriate reference base. Identification of the appropriate reference base is an important step in the process of calculating the point displacements.

In the geodetic literature many different methods of identification of reference base for displacements can be found (Lazzarinii at al. 1977; Bryś and Przewłocki 1998; Laudyn 1980; Janusz 1962; Malarski 1987; Chrzanowski 1981). Only the most popular methods in the practice of determining displacements will be mentioned in this thesis, i.e. the methods of:

1. common confidence interval (Prószyński 1989);
2. checking the mutual fixity of the pairs of benchmarks (Prószyński and Kwaśniak 2006);
3. minimization of the first displacement vector norm of reference points (Chen at al. 1990);
4. successive adjustments, using the elementary reference system (Prószyński and Kwaśniak 2006);
5. exploratory transformations (Denli 2008; Caspary 1988; Prószyński 2010);
6. others.

These methods are based on different principles and are more or less accurate. Their specific characteristics can be found in the cited literature. The first two methods are most commonly applied in studies of vertical displacements, while the others more often in studies of horizontal displacements.

The subject of this work is the method of reference base identification on the basis of mutual displacements of pairs of points, hereinafter abbreviated by "all-pairs method". This method, considered as exact method, is often used for determining the vertical displacements. The aim of this work is to present the applicability of this method in studies of horizontal displacements.

2. Theoretical basis of the method

As mentioned above, a reference base for displacements may be constituted by a group of points, at which the mutual displacements of points can be considered as insignificant. Mutual displacement is defined as "a change in the relative position of two points, expressed by a change in the distance between these points" (Prószyński Kwaśniak 2006). The criterion for the insignificance of mutual displacement can be written as:

$$\left|\Delta d_{ij}\right| \le k_\alpha \sigma_{\Delta d_{ij}} \quad \text{or} \quad \frac{\left|\Delta d_{ij}\right|}{\sigma_{\Delta d_{ij}}} \le k_\alpha \qquad (1)$$

where: Δd_{ij} - mutual displacement of points i and j;

$\sigma_{\Delta d_{ij}}$ - standard error of mutual displacement;

k_α - factor of the transition from the standard error to maximal error (its value depends on the assumed confidence level α).

A reference base identification procedure discussed here requires verification of criterion (1) in all combinations of pairs of points in a potential reference base.

According to the given definition the mutual displacement of pair of points should be calculated as the distance change (difference) occurred between initial and actual measurement epochs. Appropriate distances should be calculated based on the coordinates of points obtained from preliminary adjustments of results of initial and actual measurements. It is also possible to calculate the standard error of the displacement.

If the values of point displacements are sufficiently small, one can use the approximate formula to calculate the mutual displacement with sufficient precision.

With the components of horizontal displacements ΔX_i, ΔY_i, ΔX_j, ΔY_j of the pair of points and the side azimuth $\varphi_{ij}^{akt} \approx \varphi_{ij}^{init} = \varphi_{ij}$, the change in the length Δd_{ij} of the side can be calculated as:

$$\Delta d_{ij} = (\Delta X_j - \Delta X_i)\cos\varphi_{ij} + (\Delta Y_j - \Delta Y_i)\sin\varphi_{ij} \qquad (2)$$

This formula is a linearized form of the nonlinear formula for the distance difference. It represents a change of the distance between orthogonal projections of the points i and j (at the second measurement epoch) onto the line defined by the points i and j at the first measurement epoch (see Fig 1).

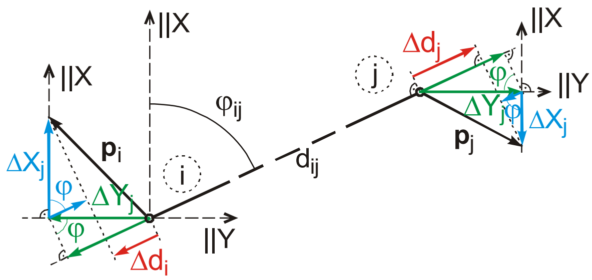

Fig. 1. Graphical illustration of conversion of point displacement components into mutual displacement

Author's numerical studies show that the maximum discrepancy value (Δd_{ij} calculated in an approximate and exact way) depends on the distance d_{ij} and the maximum length of the displacement vectors of points i, j. Figure 2 illustrates the variability of this discrepancy, assuming the maximum length of the displacement vectors $\|\mathbf{p}_i\| = \|\mathbf{p}_j\| = 0.10$m and $\|\mathbf{p}_i\| = \|\mathbf{p}_j\| = 0.05$m.

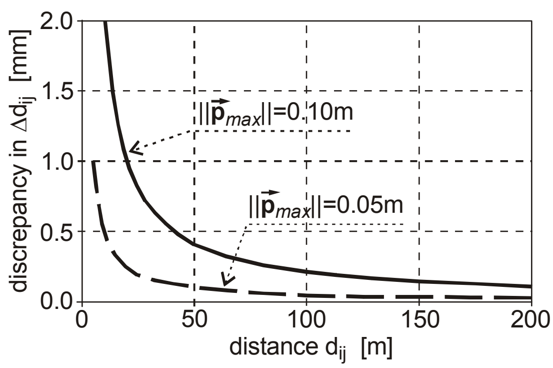

Fig. 2. Graphical illustration of discrepancy variability in Δd_{ij}

Looking at the graphs shown in Fig. 2 one can assume that in the majority of cases of the study of point displacements on the engineering objects application of the formula (2) will be sufficiently accurate. Therefore, further discussion will be carried out for the mutual displacements of pairs of points calculated in approximate way.

3. Calculation of the mutual displacements for all pairs of potential reference points

By restricting the analysis to the horizontal network (2D), from the preliminary adjustment of observation differences we obtain directly the displacement components of points $\mathbf{\Delta X}$ (as vector of unknowns) and their variance-covariance matrix $\mathbf{C_{\Delta X}}$. If the method of coordinate differences is used to calculate the point displacements, the vector of displacement components of points is calculated on the basis of coordinates obtained from preliminary adjustments of results of two periodic measurements:

$$\mathbf{\Delta X} = \mathbf{X}' - \mathbf{X} \tag{3}$$

where $\mathbf{X}'$ denotes the point coordinates obtained from adjustment of current measurement results.

The covariance matrix of the vector of displacement components of the points is calculated from the formula:

$$\mathbf{C_{\Delta X}} = \mathbf{C_X} + \mathbf{C_{X'}} \tag{4}$$

In order to obtain correct values of $\mathbf{\Delta X}$ and $\mathbf{C_{\Delta X}}$ from the method of coordinate differences, the use of identically defined reference system in both adjustments is necessary.

The next step in the identification procedure proposed here is to calculate (for all combinations of pairs of points in a potential reference base) the distance change vector (mutual displacements) $\mathbf{\Delta d}$. It may be written in matrix notation as:

$$\mathbf{\Delta d} = \mathbf{A} \cdot \mathbf{\Delta X} \tag{5}$$

where: $\mathbf{\Delta d}(w_p \times 1)$ is a distance change vector; $\mathbf{A}(w_p \times 2n_p)$ is a coefficient matrix; $\mathbf{\Delta X}(2n_p \times 1)$ is a horizontal displacement vector of components of potential reference points; n_p is a number of points of potential reference base; $w_p = C_{n_p}^2$ is a number of combinations of point pairs.

On the basis of covariance matrix $\mathbf{C_{\Delta X}}$ of the displacement components, the covariance matrix $\mathbf{C_{\Delta d}}$ of the distance change vector can be also calculated as:

$$\mathbf{C_{\Delta d}} = \mathbf{A} \mathbf{C_{\Delta X}} \mathbf{A}^{\mathrm{T}} \tag{6}$$

Using diagonal elements of the matrix $\mathbf{C_{\Delta d}}$ and the vector $\mathbf{\Delta d}$, the criterion of mutual fixity (1) can be verified for any pair of points belonging to the potential reference base.

4. Determination of the appropriate reference base for displacements

Determination of the reference base for displacements begins with identification of all pairs of the potential reference base points, for which the mutual displacements meet criterion (1). This verification leads to creating the point subsets (B_i) of the potential reference base, for which the aggregated criterion of mutual fixity is met:

$$\bigwedge_{i,j \in B} \left|\Delta d_{ij}\right| \leq k_\alpha \sigma_{\Delta d_{ij}} \tag{3}$$

The found subsets of points have to be verified using the aggregated criterion for components of point displacements:

$$\bigwedge_{i \in B} |\Delta X_i| \leq k_\alpha \sigma_{\Delta X_i} \;\wedge\; |\Delta Y_i| \leq k_\alpha \sigma_{\Delta Y_i} \tag{4}$$

The values of displacement components should be equal to those obtained from free network adjustment (differential or two separate adjustments) at the reference system defined on the points of the subset *B*. Here it is possible to use known transformation formulas presented in the literature, to transform the displacement vector and its covariance matrix from one reference system to another. (Caspary 1998; Prószyński 1990; Prószyński and Kwaśniak 2006).

The author proposes a different approach to calculate the displacement components of points belonging to the subset *B*. The following data will be used: the previously calculated distance change vector $\mathbf{\Delta d}$ (see (5)), its covariance matrix $\mathbf{C}_{\Delta \mathbf{d}}$ (see (6)) and the coefficient matrix **A** created for those distances.

5. Verification of found reference bases for displacements

Points belonging to a subset *B*, together with the distances calculated between them, can be treated as a linear network (Fig. 3) while the values in $\mathbf{\Delta d}_B$ refer to differences between results of two measurements in such network.

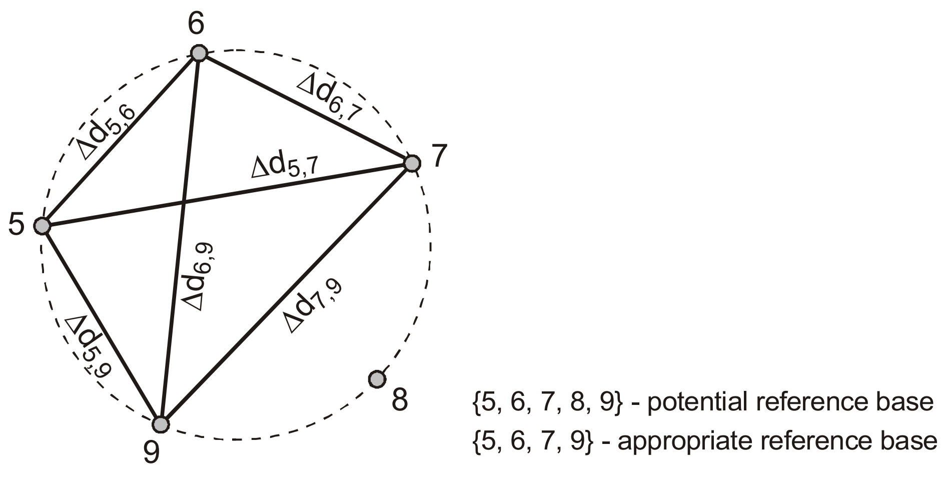

Fig. 3. Illustration of the subset *B* as a linear network in terms of differential

For the considered subset B a system of equations is created that have to be met by the sought displacement components of the points $\Delta\mathbf{X}_B$:

$$\mathbf{A}_B \cdot \Delta\mathbf{X}_B = \Delta\mathbf{d}_B \tag{5}$$

where: $\mathbf{A}_B(w_B \times 2n_B)$ is a matrix of coefficients, formed on the basis of the matrix $\mathbf{A}$; $\Delta\mathbf{X}_B(2n_B \times 1)$ is a vector of sought components of horizontal displacements of the points; $\Delta\mathbf{d}_B(w_B \times 1)$ is a distance change vector between the points of the subset B; n_B is a number of points in the subset B; $w_B = C^2_{n_B}$ is a number of all combinations of pairs of points in the subset B.

Table 1 presents the relevant indicators for the equation system (9) corresponding to different number of points in the subset B.

Table 1. Different values of parameters related to the subset B and to equation system (9)

Number of points in the subset B	Number of equations	Number of unknowns	Number of independent equations
n_B	$w_B = C^2_{n_B}$	$2n_B$	$u_B = 2n_B - 3$
3	3	6	3
4	6	8	5
5	10	10	7
6	15	12	9
7	21	14	11
8	28	16	13
9	36	18	15
10	45	20	18

Because of $w_B > u_B$ (for $n_B > 3$) the solution of the equation system (9) may be obtained by using a special case of general inverse of matrix $\mathbf{A}_B$.

$$\Delta\mathbf{X}_B = \mathbf{A}^+_B \Delta\mathbf{d}_B \tag{10}$$

where $\mathbf{A}^+_B$ is Moore-Penrose inverse, which satisfies condition $\Delta\mathbf{X}^{\mathrm{T}}_B \Delta\mathbf{X}_B = \min$.

The author of this work proposes another way to solve the equation system (9), which requires simpler calculations and creates a task of smaller size. Note that (9) is the coherent equation system, because the distance changes $\Delta\mathbf{d}$ caused by the point movements were calculated previously (see (5)) based on the known displacement components vector $\Delta\mathbf{X}$. So, the equation system (9) may be reduced by removing the redundant equations.

The simplest rule to choose the independent equations is shown in Fig. 4. Placing the points of the subset B on the circle, the independent equations may correspond to the figure contour and all diagonals linking arbitrarily selected point with other points.

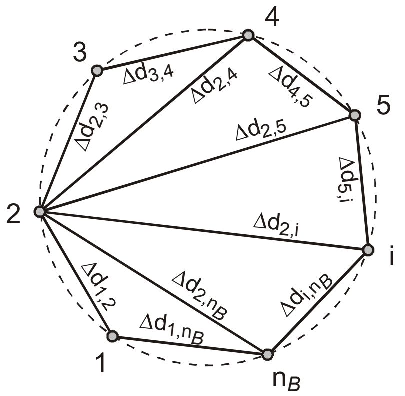

Fig. 4. Illustration of rule of choosing the independent equations

Then, on the basis of (9) the reduced equation system is obtained:

$$\mathbf{A}_{r,B} \cdot \mathbf{\Delta X}_B = \Delta\mathbf{d}_{r,B} \tag{6}$$

where *r* is the number of independent equations.

Solving the equation system (11) in accordance with the rules of conditional procedure of the least squares estimation (LSE) the following results can be obtained:

$$\mathbf{\Delta X}_B = \mathbf{A}^{\mathrm{T}}_{r,B}(\mathbf{A}_{r,B}\mathbf{A}^{\mathrm{T}}_{r,B})^{-1}\Delta\mathbf{d}_{r,B} \tag{12}$$

Resulting displacement components vector of the points of the subset *B* satisfies condition $\mathbf{\Delta X}^{\mathrm{T}}_B\mathbf{\Delta X}_B = \min$ similarly as solution (10) or as the result of free network adjustment using the parameter procedure of LSE.

Having previously calculated covariance matrix $\mathbf{C}_{\Delta\mathbf{d}}$, from which the matrix $\mathbf{C}_{\Delta\mathbf{d}B}$ is created, it can also be calculated:

$$\mathbf{C}_{\mathbf{\Delta X}_B} = \mathbf{A}^{\mathrm{T}}_{r,B}(\mathbf{A}_{r,B}\mathbf{A}^{\mathrm{T}}_{r,B})^{-1}\mathbf{C}_{\Delta\mathbf{d}_{r,B}}(\mathbf{A}_{r,B}\mathbf{A}^{\mathrm{T}}_{r,B})^{-1}\mathbf{A}_{r,B} \tag{7}$$

Verification of the found reference base (subset *B*) comes down to check the significance of point displacements, using the criterion (8) or the more exact criterion given in (Prószyński and Kwaśniak 2006, pp. 47-49). Thus, verifying in this manner each of the subsets *B* one can eventually choose the appropriate reference base to determine displacements of controlled points.

6. Numerical example

The method of identification of the reference base for horizontal displacements presented above will be illustrated on the example of a 6-point linear network (Fig. 5). Points No. 5-9 constitute the potential reference base.

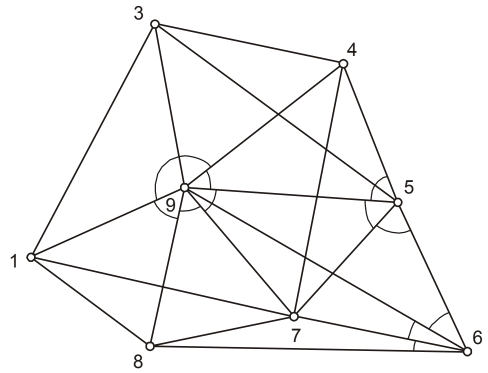

Fig. 5. An example of a 6-point horizontal network

Table 2 presents the results of two periodic measurements performed in the network.

Table 2. The results of the two periodic measurements

Linear observations [m]					Angular observations [g]				
c	l	p	d	d'	c	l	p	β	β'
7	1	0	271.7062	271.7044	9	1	3	119.6487	119.6486
7	5	0	155.7243	155.7237	9	3	4	69.8366	69.8336
7	6	0	178.4659	178.4633	9	4	5	45.8021	45.8039
7	8	0	148.0717	148.0729	9	5	7	49.3107	49.3111
7	9	0	177.3419	177.3412	9	7	8	56.3676	56.3690
8	1	0	149.9983	150.0006	9	8	1	59.0337	59.0347
8	9	0	167.7055	167.7085	6	8	7	11.5719	11.5733
8	7	0	148.0732	148.0752	6	7	9	21.1879	21.1877
8	6	0	320.0413	320.0416	6	9	5	38.4484	38.4502
1	3	0	266.1756	266.1607	5	6	7	74.9047	74.9036
1	9	0	167.7057	167.6948	5	7	9	58.6636	58.6631
1	7	0	271.7081	271.7032	5	9	4	70.3981	70.3992
3	5	0	304.0138	304.0066					
3	9	0	161.9417	161.9366					
3	4	0	194.1629	194.1558					
4	5	0	150.4167	150.4229					
4	7	0	259.8561	259.8654					
4	9	0	204.0224	204.0271					
6	9	0	336.1555	336.1522					
6	5	0	165.5280	165.5316					
standard error $\sigma_{d□} = 0.001+10^{-6}d$ [m]					standard error $\sigma_{\beta□} = 3^{cc}$				

Due to the identity of network geometry in both measurements, the method of observation differences can be used for calculation. The point displacements and their standard errors obtained from the preliminary adjustment are shown in Table 3. The so-called elementary reference system was defined on point No. 9 (fixed point) and point No. 6 (fixed azimuth 9-6).

Table 3. Displacement components of the points and their standard errors [mm]

No.	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$
1	7.56	1.19	6.78	1.02
3	-5.84	1.27	3.15	1.04
4	9.53	1.00	-0.65	1.06
5	3.04	0.78	1.13	0.89
6	0.82	0.52	-1.40	0.88
7	1.79	0.68	0.94	0.76
8	-3.16	1.22	-1.11	0.93
9	0.00	0.00	0.00	0.00

A covariance matrix of the point displacements of the potential reference base (5÷9) is presented in Table 4.

Table 4. Covariance matrix of the point displacements of the potential reference base [mm^2]

	ΔX_5	ΔY_5	ΔX_6	ΔY_6	ΔX_7	ΔY_7	ΔX_8	ΔY_8	ΔX_9	ΔY_9
ΔX_5	0.6047	-0.1374	-0.0250	0.0427	0.2643	0.0369	0.0862	0.1665	0.0000	0.0000
ΔY_5	-0.1374	0.7910	-0.2269	0.3871	-0.1478	0.2990	0.0099	0.1682	0.0000	0.0000
ΔX_6	-0.0250	-0.2269	0.2663	-0.4543	0.0737	-0.2082	-0.0063	-0.1749	0.0000	0.0000
ΔY_6	0.0427	0.3871	-0.4543	0.7750	-0.1257	0.3551	0.0108	0.2983	0.0000	0.0000
ΔX_7	0.2643	-0.1478	0.0737	-0.1257	0.4640	-0.0094	0.1963	0.1447	0.0000	0.0000
ΔY_7	0.0369	0.2990	-0.2082	0.3551	-0.0094	0.5796	0.0708	0.4347	0.0000	0.0000
ΔX_8	0.0862	0.0099	-0.0063	0.0108	0.1963	0.0708	1.4799	0.1417	0.0000	0.0000
ΔY_8	0.1665	0.1682	-0.1749	0.2983	0.1447	0.4347	0.1417	0.8557	0.0000	0.0000
ΔX_9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ΔY_9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

In order to identify the appropriate reference base for displacements, we calculate the distance changes between the points of potential reference base for all combinations of pairs according to (5).

$$\mathbf{A} = \begin{bmatrix} 0.9062 & -0.4229 & -0.9062 & 0.4229 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7385 & 0.6743 & 0 & 0 & -0.7385 & -0.6743 & 0 & 0 & 0 & 0 \\ 0.5017 & 0.8650 & 0 & 0 & 0 & 0 & -0.5017 & -0.8650 & 0 & 0 \\ -0.0905 & 0.9959 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0905 & -0.9959 \\ 0 & 0 & -0.1961 & 0.9806 & 0.1961 & -0.9806 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0156 & 0.9999 & 0 & 0 & 0.0156 & -0.9999 & 0 & 0 \\ 0 & 0 & -0.5057 & 0.8627 & 0 & 0 & 0 & 0 & 0.5057 & -0.8627 \\ 0 & 0 & 0 & 0 & 0.2026 & 0.9793 & -0.2026 & -0.9793 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.7612 & 0.6485 & 0 & 0 & 0.7612 & -0.6485 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.9839 & -0.1789 & 0.9839 & 0.1789 \end{bmatrix}$$

$$\Delta\mathbf{d}^{\mathrm{T}} = \begin{bmatrix} 0.944 & 1.055 & 5.052 & 0.849 & -2.110 & -0.353 & -1.626 & 3.014 & -0.749 & 3.310 \end{bmatrix}$$

$$
\mathbf{C}_{\Delta\mathbf{d}} = \begin{bmatrix}
1.2102 & 0.1170 & -0.1106 & -0.1479 & 0.4678 & 0.2568 & 0.7121 & -0.0964 & 0.0208 & -0.1480 \\
0.1170 & 0.6099 & 0.5636 & 0.3258 & 0.2647 & 0.3090 & 0.1692 & -0.0155 & 0.0827 & 0.1497 \\
-0.1106 & 0.5636 & 1.3124 & 0.4632 & 0.1905 & 0.6631 & 0.1076 & 0.5590 & 0.0795 & 0.9043 \\
-0.1479 & 0.3258 & 0.4632 & 0.8142 & 0.0958 & 0.2328 & 0.4424 & 0.1040 & 0.3212 & -0.0293 \\
0.4678 & 0.2647 & 0.1905 & 0.0958 & 0.6919 & 0.5512 & 0.5519 & -0.1116 & -0.0889 & 0.0316 \\
0.2568 & 0.3090 & 0.6631 & 0.2328 & 0.5512 & 1.0388 & 0.5608 & 0.4346 & 0.1556 & 0.2048 \\
0.7121 & 0.1692 & 0.1076 & 0.4424 & 0.5519 & 0.5608 & 1.0413 & 0.0324 & 0.3779 & -0.0741 \\
-0.0964 & -0.0155 & 0.5590 & 0.1040 & -0.1116 & 0.4346 & 0.0324 & 0.5733 & 0.1551 & 0.3978 \\
0.0208 & 0.0827 & 0.0795 & 0.3212 & -0.0889 & 0.1556 & 0.3779 & 0.1551 & 0.5219 & 0.0711 \\
-0.1480 & 0.1497 & 0.9043 & -0.0293 & 0.0316 & 0.2048 & -0.0741 & 0.3978 & 0.0711 & 1.5098
\end{bmatrix}
$$

Then the criterion of mutual fixity for each pair of points in the potential reference base is checked. The result of this checking is presented in the Table 5 and in Fig. 6, where the lines between the points represent the fulfillment of criterion (1).

Table 5. Result of checking of the criterion (1)

i	j	$\Delta d/\sigma_{\Delta d}$	Result
5	6	0.86	fulfilled
5	7	1.35	fulfilled
5	8	4.41	not fulfilled
5	9	0.94	fulfilled
6	7	-2.54	not fulfilled
6	8	-0.35	fulfilled
6	9	-1.59	fulfilled
7	8	3.98	not fulfilled
7	9	-1.04	fulfilled
8	9	2.69	not fulfilled

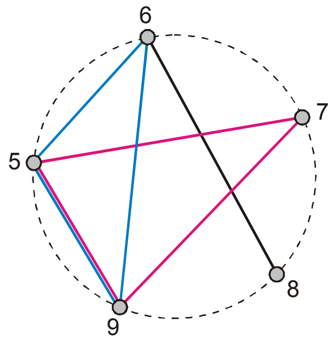

Fig. 6. Graphical illustration of checking of criterion (1)

The results show the possible existence of two three-point reference bases: B_1 = {5, 6, 9}, B_2 = {5, 7, 9}, which satisfy the aggregated criterion (7). As stated in section 4 of this work, the obtained point subsets B_1 and B_2 must be verified by re-calculating the displacements of all points in free network adjustments with reference system specified on the points of these subsets. Instead of realizing the full algorithm for calculating the displacements formulas (12) and (13) are used.

<u>Verification of the subset B_1 = {5, 6, 9}</u>

$$
\mathbf{A}_{r,B} = \begin{bmatrix}
0.9062 & -0.4229 & -0.9062 & 0.4229 & 0.0000 & 0.0000 \\
-0.0905 & 0.9959 & 0.0000 & 0.0000 & 0.0905 & -0.9959 \\
0.0000 & 0.0000 & -0.5057 & 0.8627 & 0.5057 & -0.8627
\end{bmatrix}
$$

$$
\Delta\mathbf{d}_{r,B} = \begin{bmatrix} 0.944 \\ 0.849 \\ -1.626 \end{bmatrix} \text{ [mm]} \qquad
\mathbf{C}_{\Delta\mathbf{d}_{r,B}} = \begin{bmatrix}
1.2102 & -0.1479 & 0.7021 \\
-0.1479 & 0.8142 & 0.4424 \\
0.7121 & 0.4424 & 1.0413
\end{bmatrix} \text{ [mm}^2\text{]}
$$

$$
\Delta\mathbf{X}_B^{\mathrm{T}} = \begin{bmatrix} 1.714 & 1.256 & -0.567 & -1.400 & -1.148 & 0.144 \end{bmatrix} \text{ [mm]}
$$

$$\mathbf{C}_{\Delta\mathbf{X}_B} = \begin{bmatrix} 0.2676 & -0.0356 & -0.2408 & 0.0687 & -0.0268 & -0.0331 \\ -0.0356 & 0.2363 & 0.0614 & -0.0865 & -0.0258 & -0.1498 \\ -0.2408 & 0.0614 & 0.2775 & -0.1689 & -0.0367 & 0.1075 \\ 0.0687 & -0.0865 & -0.1689 & 0.2094 & 0.1002 & -0.1229 \\ -0.0268 & -0.0258 & -0.0367 & 0.1002 & 0.0634 & -0.0744 \\ -0.0331 & -0.1498 & 0.1075 & -0.1229 & -0.0744 & 0.2727 \end{bmatrix}$$

Table 6. Checking of the criterion (8) for B_1 (k_α = 2.5)

Component	$\Delta\mathbf{X}_B$ [mm]	$\Delta\mathbf{X}/\sigma_{\Delta\mathbf{X}}$	Result
ΔX_5	1.71	3.31	not fulfilled
ΔY_5	1.26	2.58	not fulfilled
ΔX_6	-0.57	-1.08	fulfilled
ΔY_6	-1.40	-3.06	not fulfilled
ΔX_9	-1.15	-4.56	not fulfilled
ΔY_9	0.14	0.28	fulfilled

Table 6 shows that the points 5, 6 and 9 cannot be a reference base for displacements, because the subset B_1 does not satisfy the criterion (8).

<u>Verification of the subset B_2 = {5, 7, 9}</u>

$$\mathbf{A}_{r,B} = \begin{bmatrix} 0.7385 & 0.6743 & -0.7385 & -0.6743 & 0.0000 & 0.0000 \\ -0.0905 & 0.9959 & 0.0000 & 0.0000 & 0.0905 & -0.9959 \\ 0.0000 & 0.0000 & -0.7612 & 0.6485 & 0.7612 & -0.6485 \end{bmatrix}$$

$$\Delta\mathbf{d}_{r,B} = \begin{bmatrix} 1.055 \\ 0.849 \\ -0.749 \end{bmatrix} \text{ [mm]} \qquad \mathbf{C}_{\Delta\mathbf{d}_{r,B}} = \begin{bmatrix} 0.6099 & 0.3258 & 0.0827 \\ 0.3258 & 0.8142 & 0.3212 \\ 0.0827 & 0.3212 & 0.5219 \end{bmatrix} \text{ [mm}^2\text{]}$$

$$\Delta\mathbf{X}_B^{\mathrm{T}} = \begin{bmatrix} 0.257 & 0.783 & 0.140 & -0.654 & -0.397 & -0.129 \end{bmatrix} \text{ [mm]}$$

$$\mathbf{C}_{\Delta\mathbf{X}_B} = \begin{bmatrix} 0.0851 & 0.0390 & -0.0848 & -0.0722 & -0.0003 & 0.0332 \\ 0.0390 & 0.2484 & -0.0294 & -0.0754 & -0.0097 & -0.1730 \\ -0.0848 & -0.0294 & 0.1564 & 0.0093 & -0.0716 & 0.0201 \\ -0.0722 & -0.0754 & 0.0093 & 0.1210 & 0.0630 & -0.0456 \\ -0.0003 & -0.0097 & -0.0716 & 0.0630 & 0.0719 & -0.0533 \\ 0.0332 & -0.1730 & 0.0201 & -0.0456 & -0.0533 & 0.2185 \end{bmatrix}$$

The results shown in Table 7 indicate that the subset B_2 can be a reference base for displacements, because it satisfy the criterion (8).

Table 7. Checking of the criterion (8) for B_2 (k_α = 2.5)

Component	$\Delta\mathbf{X}_B$ [mm]	$\Delta\mathbf{X}/\sigma_{\Delta\mathbf{X}}$	Result
ΔX_5	0.26	0.88	fulfilled
ΔY_5	0.78	1.57	fulfilled
ΔX_6	0.14	0.35	fulfilled
ΔY_6	-0.65	-1.88	fulfilled
ΔX_9	-0.40	-1.48	fulfilled
ΔY_9	-0.13	-0.28	fulfilled

7. Conclusions

The "all-pairs method" of a reference base identification for horizontal displacements has the following properties:

1. The identification procedure is very simple both in concept and in numerical realization.
2. The method may be applied to reference base identification in both methods of displacement calculation, i.e. the method of observation differences and the method of coordinate differences.
3. The method allows to detect of all alternative reference bases in the control network.
4. The reference base determined use of the "all-pairs method" meets the two aggregating criteria at the same time:
   - for mutual displacements;
   - for relative displacements.
5. Presented version of the method may be applied to reference base identification in networks of 1D, 2D and 3D.

The results of effectiveness analyses of the proposed procedure in relation to other popular methods of a reference base identification for displacements will be presented in separate publication.

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