Pavel Záskalický
Technical University of Košice, Slovakia

HARMONIC ANALYSIS OF THE TWO-PHASE CONVERTERS FOR SMALL TWO-PHASE DRIVES

ANALIZA HARMONICZNA KONWERTERÓW DWUFAZOWYCH DLA MAŁYCH NAPĘDÓW DWUFAZOWYCH

Abstract: The paper deals with a harmonic analysis of two-phase converters output voltages. Output voltage each of transistors voltage-source inverters is controlled by pulse with modulation (PWM) controller. The mathematical models of the different converter connections are built on condition of assumption of idealized semiconductor devices. A complex Fourier series approach is used to predict terminal voltage waveforms. From complex Fourier series mathematical formulas a harmonic analysis for different inverters steady-state was made.

Streszczenie: Referat dotyczy analizy harmonicznej dwufazowych napięć wyjściowych konwerterów. Napięcie wyjściowe każdego z tranzystorów przetwornic napięcia jest kontrolowane przez impuls z modulacji (PWM) kontrolera. Matematyczne modele różnych połączeń przekształtników są zbudowane przy założeniu wzorcowych urządzeń półprzewodnikowych. Użyto kompleksowego podejścia szeregu Fouriera do symulacji przebiegów terminali. Wykonano analizę harmoniczną dla różnych falowników w stanie ustalonym przy użyciu formuł matematycznych szeregu Fouriera.

Keywords: *Fourier series, two-phase converter, mathematical model, harmonic analysis*
Słowa kluczowe: *szeregi Fouriera, konwerter 2-fazowy, model matematyczny, analiza harmoniczna*

1. Introduction

Electrical low-power drives (around 100W) which are supplied by a single-phase voltage, used in different industrial and domestic devices and presently increasingly use two-phase motors. They are deployed as drive of pumps in a washing machines and dishwashers, but also as the circulating pumps for central domestic heating.

The contribution deals with the various schemes of the two-phase inverters. Using the complex Fourier series is converter's output voltage mathematically expressed. On the base of output voltages formulas, the harmonic analysis was made.

For steady-state inverter's operation study we consider following idealized conditions:

- Power switch, that means the switch can handle unlimited current and blocks unlimited voltage.
- The voltage drop across the switch and leakage current through switch are zero.
- The switch is turned on and off with no rise and fall times.
- Sufficiently good size capacity of the input voltage capacitors divider, to can suppose inverter input DC voltage to by constant. [2], [3], [5]

This assumption helps us to analyze a power circuit and helps us to build a mathematical model of different concepts of two-phase converters.

2. Converter using a centre tapped two-phase inverter

The Fig.1 shows a scheme converter with centre tapped connected inverter.

An inverter consists of two transistor's branches, which form at the same time the output terminals of the convertor's. The load is connected between output terminals and the node on the capacitive voltage divider.

Power transistor each of branch can conduct only in the condition that the second branch transistor is blocked. Simultaneous transistors conducting creates a short circuit, simultaneous transistors blocking creates uncertain state. In the case of upper transistor conducting, the branch voltage is equal to the DC input voltage (U_e). In the case of lower transistor conducting, the branch voltage is equal zero.

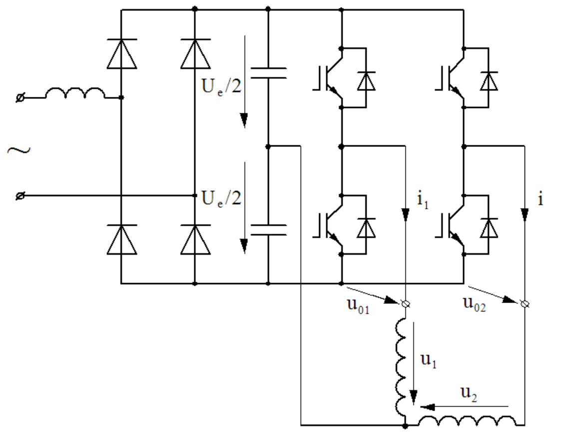

Fig. 1. Two-phase converter with centre tapped inverter connection

Branch voltages can by the only positive value of voltage. Either U_e or 0 value.

Assuming pulse-width modulation (PWM) of convertor's output voltage. PWM transform requested output voltage waveform to the series of impulsion of constant frequency and different width. Frequency of the output impulsion call modulated frequency.

Two parameters define the invertor control:

- *Coefficient of the modulation* m - equal to the ratio of the modulation and reference frequency.
- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the DC supply voltage.

Generally to control the inverter numeric control device is used. The turn on (α) and turn off (β) angles are calculated by the discredit of the reference sine-wave. That means the reference sine-wave is by a values discreet replaced. If the coefficient of modulation m is sufficiently great, the difference between real values and discrete values is negligible.

The phase's branches are control to create the output voltages as seen in equation:

$$u_{01} = \frac{U_e}{2} + r\frac{U_e}{2} sin\,\theta; \qquad u_{02} = \frac{U_e}{2} + r\frac{U_e}{2} cos\,\theta \tag{1}$$

Where: U_e - is a DC inverter's input voltage value.

Thus the requested phase voltages are given by:

$$u_1 = u_{01} - \frac{U_e}{2} = r\frac{U_e}{2} sin\,\theta; \qquad u_2 = u_{02} - \frac{U_e}{2} = r\frac{U_e}{2} cos\,\theta \tag{2}$$

To calculate a turn on (α) and turn off (β) angles we compare the DC impulse area with the requested voltage area for any of m intervals of the voltage period.

After the calculus we obtain for the turn-on and turn-off angles of the first transistors branch the following expressions:

$$\alpha_{01n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) + \frac{r}{2}\left[cos\,n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n-1)\right]$$

$$\beta_{01n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) + \frac{r}{2}\left[cos\,n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n+1)\right]$$

It will be similarly for the second transistor branch.

$$\alpha_{02n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) - \frac{r}{2}\left[sin\,n\frac{2\pi}{m} - sin\frac{\pi}{m}(2n-1)\right]$$

$$\beta_{02n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) - \frac{r}{2}\left[sin\,n\frac{2\pi}{m} - sin\frac{\pi}{m}(2n+1)\right]$$

Then we can write the output voltage of the first branch in the form of a complex Fourier series: [4], [1]

$$u_{01} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} \tag{3}$$

With Fourier series coefficients of the form

$$c_{01n} = \frac{1}{j2k\pi}\left(e^{-jk\alpha_{01n}} - e^{-jk\beta_{01n}}\right) \quad for \quad k \neq 0$$

$$c_{01n} = \frac{\beta_{01n} - \alpha_{01n}}{2\pi} \quad for \quad k = 0$$

Similarly for the output voltage of the second transistor branch:

$$u_{02} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} \tag{4}$$

With Fourier series coefficients of the form

$$c_{02n} = \frac{1}{j2k\pi}\left(e^{-jk\alpha_{02n}} - e^{-jk\beta_{02n}}\right) \quad for \quad k \neq 0$$

$$c_{02n} = \frac{\beta_{02n} - \alpha_{02n}}{2\pi} \quad for \quad k = 0$$

Fig.2 depicts the calculated branch voltage waveform calculated on the base of equation (3) and (4). Requested output frequency is $f = 50\,Hz$, modulation coefficient $m = 30$

voltage control coefficient $r = 1$ and DC inverter input voltage of $U_e = 325V$.

For the phase voltages, following formulas are valid:

$$u_1 = u_{01} - rU_e = 2U_e \sum_{k=1}^{\infty}\sum_{n=1}^{m} c_{01n} e^{jk\theta};$$
$$u_2 = u_{02} - \frac{U_e}{2} = 2U_e \sum_{k=1}^{\infty}\sum_{n=1}^{m} c_{02n} e^{jk\theta} \quad (5)$$

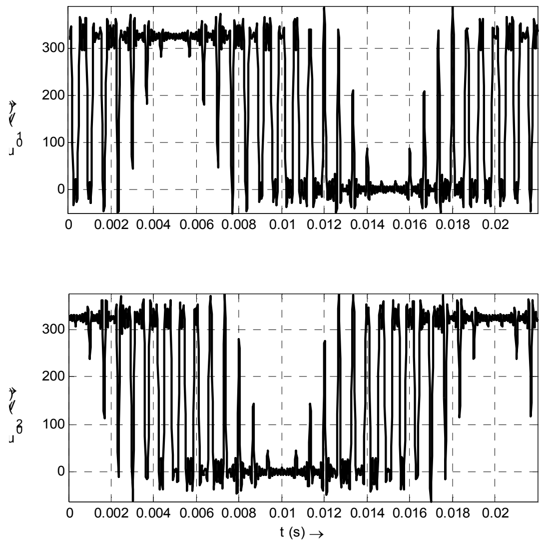

Fig. 2. Branch output voltage waveforms

In the Fig. 4 are shown the phase voltages waveforms. The voltages are bi-polar with amplitude equal to half of DC input voltage.

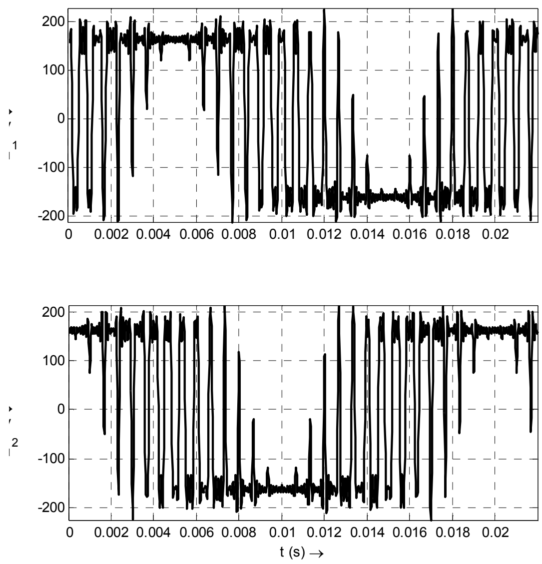

Fig. 4. Phase voltages output waveforms

Based on the complex Fourier series formulas of the phase supply voltages, harmonic analysis of the waveforms can be made.

Phasor of each of k voltage harmonics is given by a sum of complex Fourier series coefficients and DC input voltage. For any of harmonic following formulas are walid

$$U_1^k = 2U_e \sum_{n=1}^{m} c_{01}^k$$
$$U_2^k = 2U_e \sum_{n=1}^{m} c_{02}^k \quad (6)$$

With anplitude and phase

$$A^k = \left|U^k\right|; \quad P^k = arg\left(U^k\right) \quad (7)$$

For practical application we consider only significant harmonics. Therefore we neglect the harmonics with amplitude less then $10V$.

On the Fig.5 are shown the amplitudes and phases of the significant harmonics of the voltage u_1 for centre tapped inverter.

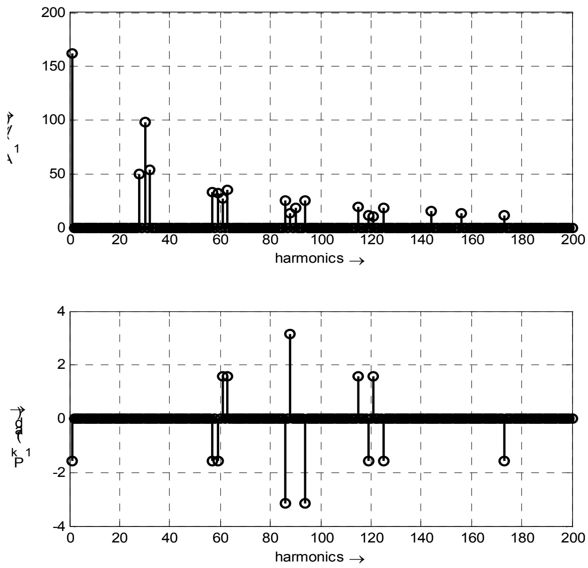

Fig. 5. Harmonic analysis phase u_1 :amplitude - top and phase – bottom

Fig.5 depicts, that all harmonics are in the vicinity of an integer multiple of the modulation frequency.

3. Converter using a half-bridge two-phase inverter

Fig.6 shows a connection of a two phase converter with half-bridge inverter.

The inverter consists of three transistor branches. The first one is common branch for both others. The load is connected between output terminals second and threed branch and the node on the first transistor branch. The advantage compared to the centre tapped connection is that there is not neccesary capacitive divider. Maximum output voltage is of $\sqrt{2}$ higher than in the case of centre tapped connection. The disadvantage is to control of three transistor branches, what increase realization fee as well as complexity of inverter control (control of 6 transistors).

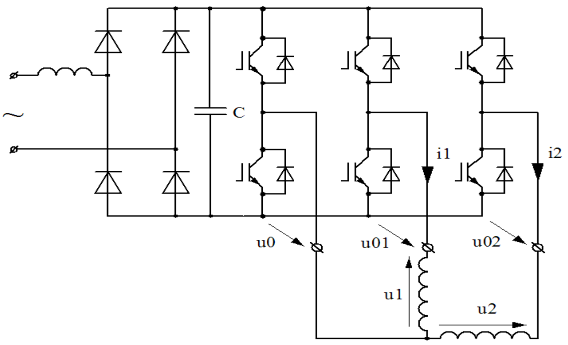

Fig. 6. Two-phase converter with half-bridge inverter connection

In this case required output voltage of the branches has a form

$$u_0 = \frac{U_e}{2} + r\frac{U_e}{2}\sin\theta;$$
$$u_{01} = \frac{U_e}{2} + r\frac{U_e}{2}\cos\theta; u_{02} = \frac{U_e}{2} - r\frac{U_e}{2}\cos\theta \qquad (8)$$

For positive cosine and sine function up derived formulas are valid. For the negative cosine function following equations was developed

$$\alpha_{02n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) + \frac{r}{2}\left[\sin n\frac{2\pi}{m} - \sin\frac{\pi}{m}(2n-1)\right]$$
$$\beta_{02n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) + \frac{r}{2}\left[\sin n\frac{2\pi}{m} - \sin\frac{\pi}{m}(2n+1)\right]$$

The branch voltages can be expressed in a form of complex Fourier series

$$u_0 = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{0n} e^{jk\theta};$$
$$u_{01} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{01n} e^{jk\theta}; \qquad (9)$$
$$u_{02} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{02n} e^{jk\theta}$$

The phase voltages a given by a difference of branch voltages

$$u_1 = u_0 - u_{01} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}\left(c_{0n} - c_{01n}\right) e^{jk\theta};$$
$$u_2 = u_0 - u_{02} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}\left(c_{0n} - c_{02n}\right) e^{jk\theta}; \qquad (10)$$

In the Fig. 7 are shown the phase voltages waveforms, which were calculated on the base on equations (10).
On the Fig.8 are shown the amplitudes and phases of the significant harmonics of the voltage u_1 for half-bridge connected inverter.

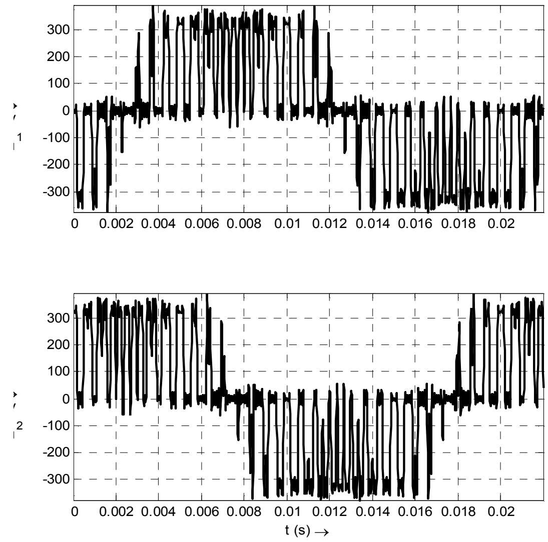

Fig. 7. Phase voltages output waveforms

From equations (10) for harmonics following formulas are valid

$$U_1^k = 2U_e \sum_{n=1}^{m}\left(c_{0n}^k - c_{01n}^k\right)$$
$$U_2^k = 2U_e \sum_{n=1}^{m}\left(c_{0n}^k - c_{02n}^k\right) \qquad (11)$$

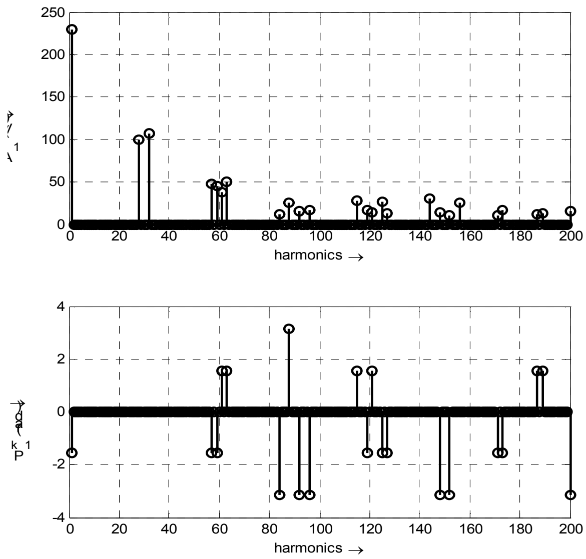

Fig. 8. Harmonic analysis phase u_1 :amplitude - top and phase – bottom

Compared with the Fig.5 we can conclude that the harmonic component equal to the resonant frequency disappeared. On the other hand harmonic $m \pm 2$ became stronger.

4. Converter using a full-bridge two-phase inverter

The Fig.9 shows a scheme of two-phase converter with full-bridge connected inverter.
The inverter of the converter consists of four transistor branches. Two transistor branches supplies each of phases. Advantage of this converter compare to the precedents two connections is a higher maximal output voltage. Maximum output voltage is equal to DC input voltage. Disadvantage is four transistor branches (8 transistors), which increase realization fees.

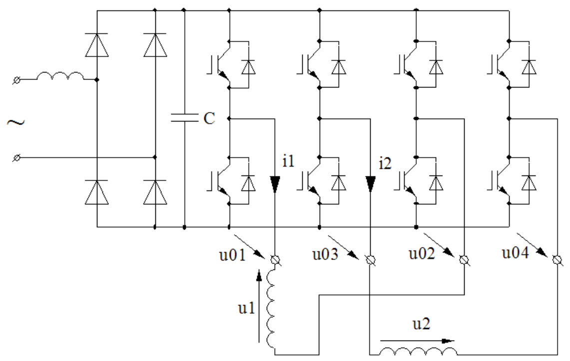

Fig. 9. Two-phase converter with full-bridge inverter connection

Required output voltage of the branches has a form:

$$u_{01} = \frac{U_e}{2} + r\frac{U_e}{2} sin\,\theta; u_{02} = \frac{U_e}{2} - r\frac{U_e}{2} sin\,\theta$$
$$u_{03} = \frac{U_e}{2} + r\frac{U_e}{2} cos\,\theta; u_{04} = \frac{U_e}{2} - r\frac{U_e}{2} cos\,\theta \qquad (12)$$

For the negative sine function for turn-on and turn-off angles the following equations were developed

$$\alpha_{02n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) - \frac{r}{2}\left[cos\, n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n-1)\right]$$
$$\beta_{02n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) - \frac{r}{2}\left[cos\, n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n+1)\right]$$

The branch voltages, we can express in a form of complex Fourier series

$$u_{01} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{01n} e^{jk\theta}; u_{02} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{02n} e^{jk\theta}$$
$$u_{03} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{03n} e^{jk\theta}; u_{04} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} c_{04n} e^{jk\theta} \qquad (13)$$

The phase voltages a given by a difference of branch voltages

$$u_1 = u_{01} - u_{02} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}\left(c_{01n} - c_{02n}\right) e^{jk\theta};$$
$$u_2 = u_{03} - u_{04} = U_e \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}\left(c_{03n} - c_{04n}\right) e^{jk\theta}; \qquad (14)$$

In the Fig. 10 are shown the phase voltages waveforms, which were calculated on the base on equations (14).

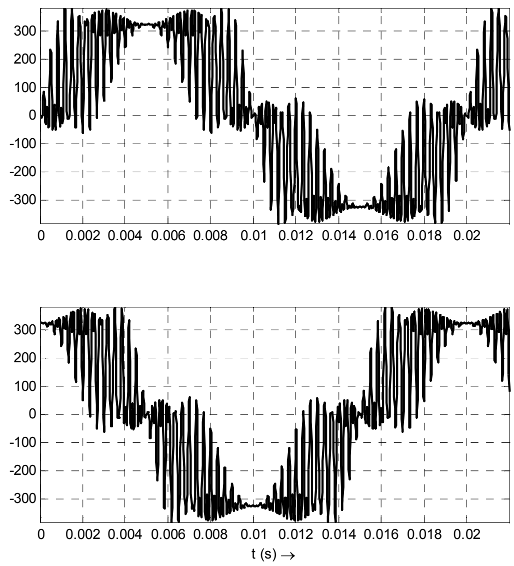

Fig. 10. Phase voltages output waveforms

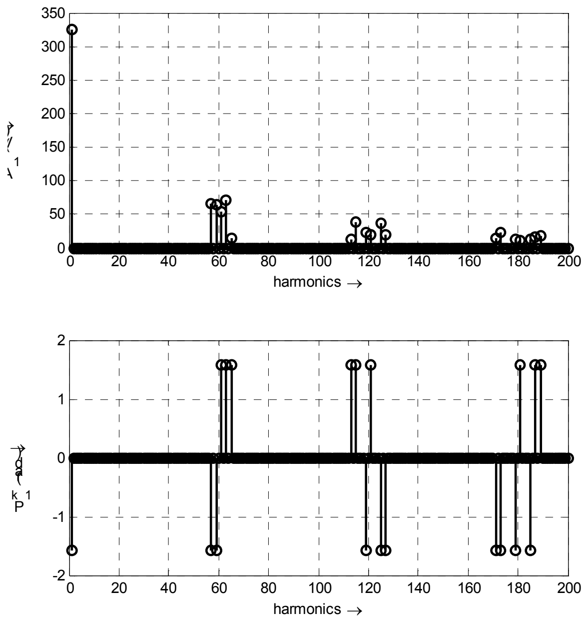

Fig. 11. Harmonic analysis phase u_1 :amplitude - top and phase – bottom

On the Fig.11 are shown the amplitudes and phases of the significant harmonics of the voltage u_1 for full-bridge connected inverter.

The harmonics in the Fig.11 were calculated on the base of formulas

$$U_1^k = 2U_e \sum_{n=1}^{m} \left(c_{01n}^k - c_{02n}^k \right)$$
$$U_2^k = 2U_e \sum_{n=1}^{m} \left(c_{03n}^k - c_{04n}^k \right) \quad (15)$$

As seen in the Fig.11, harmonic around odd multiples of the resonant frequency disappeared.

Conclusion

In a presented contribution analyze of the two-phase converters was provided. The developed equations of each connection are valid for any coefficient of modulation, frequency and voltage control coefficient. Inverter in full-bridge connection provides the upper most output voltage. That is extremely favorable, when motor operate in region of high speed and call for higher supply current. Voltage output waveform for full-bride connection has a lowest number of harmonic components.

Acknowledgment

The financial support of the Slovak Research and Development Agency under the contract N^0: APVV-0138-10 and National Scientific Agency under the contract N^0: VEGA-1/0121/15 is acknowledged.

Bibliography

[1]. Záskalický P., Dobrucký, B.: *Complex Fourier–series mathematical model of a three-phase inverter with Improved PWM output voltage control;* Elektronika ir Elektrotechnika, 2012m, Nr.7 (123), pp.65-68, Kaunas, KTU, Lithuanie.

[2]. Ferková Ž.,: *Two-Phase Asynchronous Motor - Simulation in ANSYS Maxwell and MATLAB-Simulink;* EDPE 2013 : 17th International Conference on Electrical Drives and Power Electronics : 6th joint Croatian-Slovak Conference : October 2-4, 2013, Dubrovník, Croatia

[3]. Záskalická M., Záskalický P., Beňová M., Mahmud A.R., Dobrucký B.: *Analysis of complex time function of converter output quantities using complex Fourier transform/series;* Communications-Scientific letters of the University of Žilina, pp. 23-30, vol.12, No.1 2010, Žilina.

[4]. Ferková Ž., Kaňuch J.: *Two-Phase Asynchronous Motor - Simulation and Measurement;* Maszyny Elektryczne - Zeszyty Problemowe. Vol. 100, no. 4 (2013), p. 25-30, Katowice, Poland

[5]. Dobrucký B., Záskalický P., Kaščák S., Praženica M.: *Comparison of modified sinusoidal and space vector modulation for symmetrical two-phase induction motor under field oriented control*; IREMOS Int'l Review of Modelling and Simulation, 2012, vol. 5, no. 6, pp. 2445-2451.

[6]. Kaňuch J., Višnyi P.: *Control of two-phase induction motor using a conventional three-phase bridge inverter*; Maszyny Elektryczne - Zeszyty Problemowe, Katowice, Poland, Vol. 100, no. 4 (2013), p. 171-174.

[7]. Kaščák S., Záskalický P., Dobrucký B., Praženica M.: *Two-Phase Space Vector Modulation of FOC Controlled ASM Fed by 2-Phase VSI Inverter;* EPE-PEMC 2012 ECCE Europe", Novi Sad (RS), 4-6 Sept. 2012, pp. DS2c: 131-135.

[8]. Záskalický P., Dobrucký B., Praženica M.: *Modelling of Converter with PWM Output for Two-phase Motor Applications*; Elektronika ir Elektrotechnika, vol.20, Nr.1 2014, pp.25-28, Kaunas, KTU, Lithuanie.

Author

Prof. Ing. Pavel Záskalický, PhD., is with Department of Electrical Engineering and Mechatronic, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Letná 9, 04001 Košice, Slovakia.
(Email: pavel.zaskalicky@tuke.sk
tel.:+421 55 602 2272