Paper

Call Blocking Probabilities of Multirate Elastic and Adaptive Traffic under the Threshold and Bandwidth Reservation Policies

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Abstract—This paper proposes multirate teletraffic loss models of a link that accommodates different service-classes of elastic and adaptive calls. Calls follow a Poisson process, can tolerate bandwidth compression and have an exponentially distributed service time. When bandwidth compression occurs, the service time of new and in-service elastic calls increases. Adaptive calls do not alter their service time. All calls compete for the available link bandwidth under the combination of the Threshold (TH) and the Bandwidth Reservation (BR) policies. The TH policy can provide different QoS among service-classes by limiting the number of calls of a service-class up to a predefined threshold, which can be different for each service-class. The BR policy reserves part of the available link bandwidth to benefit calls of high bandwidth requirements. The analysis of the proposed models is based on approximate but recursive formulas, whereby authors determine call blocking probabilities and link utilization. The accuracy of the proposed formulas is verified through simulation and found to be very satisfactory.

Keywords*—*adaptive traffic policy, Call Blocking Probabilities, Multirate Loss Model, threshold and bandwidth reservation policies.

1. Introduction

Multirate elastic traffic refers to in-service calls of different service-classes which have the ability to compress/expand their bandwidth and simultaneously increase/decrease their service time, during their lifetime in a system. A variation of elastic traffic is the so-called adaptive traffic. The service time of adaptive in-service calls is not affected by their bandwidth compression/expansion. Assuming that the system behaves as a loss system (i.e. calls are not allowed to wait in order to be serviced) and that the call arrival process is Poisson then the calculation of various performance measures such as Call Blocking Probabilities (CBP), and system's utilization can be based on the classical Erlang Multirate Loss Model (EMLM) [1], [2]. The latter has led to numerous loss models proposed for the call-level analysis of wired (e.g. [3]–[19]), wireless (e.g. [20]–[32]) and optical networks (e.g. [33]–[37]).

In the EMLM, a link accommodates calls of different service-classes. New calls compete for the available link bandwidth according to the Complete Sharing (CS) policy (i.e., calls compete for all bandwidth resources) and have fixed bandwidth requirements and generally distributed service time [1]. The term "fixed" means that in-service calls do not compress their bandwidth during their lifetime in the system. A new call is blocked and lost if its required bandwidth is not available. The steady state probabilities in the EMLM have a Product Form Solution (PFS), which leads to an accurate CBP calculation [1], [2]. In [5], the EMLM is extended to include the case of elastic traffic. The authors name this model Elastic EMLM (E-EMLM). In the E-EMLM, instead of rejecting immediately a blocked call, the link may accept this call by compressing its bandwidth and the bandwidth of all in-service calls. Bandwidth compression is permitted down to a minimum bandwidth, which can be different for each service-class. Elastic calls increase their service time so that the product *bandwidth by service time* remains constant. When a call with compressed bandwidth leaves the system, then the remaining calls expand their bandwidth in proportion to their initial bandwidth requirement. Call blocking occurs when the value of the minimum bandwidth requirement is still higher than the available bandwidth. The model of [5] has been extended in [9] in order to include the case of adaptive traffic. The authors name the model of [9], Elastic-Adaptive EMLM (EA-EMLM).

In this paper, authors initially consider a link that accommodates Poisson arriving calls of elastic service-classes and modify the admission mechanism to include the Threshold (TH) and the Bandwidth Reservation (BR) policies. The proposed model is named E-EMLM/TH-BR. In addition, authors propose the EA-EMLM/TH-BR whereby a link accommodates elastic and adaptive service-classes. In the TH policy, the number n_k of in-service calls k of service-class should not exceed a pre-defined threshold, after the acceptance of a new service-class k call. Otherwise, the call is blocked and lost. The TH policy is significant since it analyzes a multirate access tree network which accommodates different service-classes [38] and may differentiate service-classes in terms of CBP or revenue rates by a proper threshold selection (see e.g. [39], [40]). The BR policy is used to reserve bandwidth to benefit calls of high bandwidth requirements and is mainly used when CBP equalization is required among calls of different service-classes. The fact that the BR policy has been extensively applied in the literature (e.g. [6], [8], [18], [28], [42]–[47]) evinces its importance in call admission control.

To model the proposed E-EMLM/TH-BR and EA-EMLM/TH-BR, the Markov chain method is used. However, due to the existence of the compression/expansion mechanism and the BR policy, the reversibility of the Markov chains is destroyed, and the steady state probabilities in the proposed models cannot be determined via a PFS. Therefore, authors resort to approximate Markov chains which provide recursive formulas for the efficient determination of the link occupancy distribution and, consequently, CBP and link utilization. The accuracy of the proposed formulas is verified through simulation and found to be very satisfactory. On the other hand, the comparison of the proposed models with existing models shows the necessity of the new models, as well as their consistency over changes of their parameters (e.g. compression factor and offered traffic-load). The term "consistency" is referred to the anticipated behavior of the proposed models over changes, such as the increase of offered traffic-load or the increase of the compression factor.

This paper is organized as follows. In Section 2, the E-EMLM/TH is presented and formulas for the calculation of the various performance measures are proved. In Section 3, the E-EMLM/TH is extended to include the BR policy. In Section 4, the E-EMLM/TH-BR is extended to include the case of adaptive traffic. In Section 5, authors provide numerical results whereby the E-EMLM/TH and the E-EMLM/TH-BR are compared to existing models and evaluated via simulation. The paper is concluded in Section 6.

2. The Elastic EMLM/TH (E-EMLM/TH)

2.1. The System Model

Consider a link of capacity C bandwidth units (b.u.) that accommodates K elastic service-classes. Calls of service-class k $(k = 1,\dots,K)$ follow a Poisson process with arrival rate λ_k and request b_k b.u. Bandwidth compression is introduced in the system by allowing the occupied link bandwidth to virtually exceed C up to T b.u., i.e. $j = 0, 1, \dots, T$. Let $\mathbf{n} = (n_1, \dots, n_K)$ be the vector of all in-service calls and $\mathbf{b} = (b_1, \dots, b_K)$ the vector of peak-bandwidth requirements, then $j = \mathbf{nb}$.

The decision to accept a new service-class k call in the system is based on the following constraints:

(a) the number of in-service calls of service-class k, n_k, together with the new call, should not exceed a maximum threshold n_k^*, i.e. $n_k + 1 \le n_k^*$. Otherwise the call is blocked. This constraint expresses the TH policy;

(b) if constraint (a) is met then:

  (b1) if $j + b_k \le C$, the call is accepted in the system with b_k b.u. and remains in the system for an exponentially distributed service time with mean μ_k^{-1};

  (b2) if $T \ge j + b_k > C$ the call is accepted by compressing its b_k together with the bandwidth of all in-service calls of all service-classes.

The compressed bandwidth of the new service-class k call is:

$$b_k' = r b_k = \frac{C b_k}{j + b_k}, \tag{1}$$

where $r \equiv r(\mathbf{n}) = \frac{C}{\mathbf{nb} + b_k} = \frac{C}{j + b_k}$.

The product *service time by bandwidth per call* is kept constant by changing the mean value of the service time of the new service-class k call to $\frac{1}{\mu_k'} = \frac{j + b_k}{C \mu_k}$. The compressed bandwidth of all in-service calls becomes $\frac{C b_i}{j + b_k}$ for $i = 1, \dots, K$. When all calls have compressed their bandwidth, then $j = C$. Note that the minimum bandwidth that a call tolerates is:

$$b_{k,\min}' = r_{\min} b_k = \frac{C b_k}{T}, \tag{2}$$

where $r_{\min} = \frac{C}{T}$ is the min. proportion of the required peak-bandwidth and is common for all service-classes.

A new service-class k call is blocked if $j + b_k > T$.

When an in-service call, with compressed bandwidth b_i' departs from the system then the remaining calls expand their bandwidth to b_i^* in proportion to their b_i, as follows:

$$b_i'' = \min\left(b_i,\ b_i' + \frac{b_i b_k'}{\sum\limits_{k=1}^{K} n_k b_k} \right). \tag{3}$$

2.2. The Analytical Model

The existence of the bandwidth compression mechanism destroys reversibility in the E-EMLM/TH and therefore the steady state probabilities have no PFS. To circumvent this problem, the state-dependent factors $\phi_k(\mathbf{n})$ are used, which lead to a reversible Markov chain:

$$\phi_k(\mathbf{n}) = \begin{cases} 1, & \text{when } \mathbf{nb} \le C \text{ and } \mathbf{n} \in \mathbf{\Omega} \\ \dfrac{x(\mathbf{n}_k^-)}{x(\mathbf{n})}, & \text{when } C < \mathbf{nb} \le T \text{ and } \mathbf{n} \in \mathbf{\Omega} \end{cases}, \tag{4}$$

where:

$$\mathbf{\Omega} = \left\{ \mathbf{n} : 0 \le \mathbf{nb} \le T,\ n_k \le n_k^*,\ k = 1,\dots,K \right\},$$

$$\mathbf{n} = (n_1, \ldots, n_k, \ldots, n_K),$$
$$\mathbf{n}_k^- = (n_1, \ldots, n_k - 1, \ldots, n_K)$$

and

$$x(\mathbf{n}) = \frac{1}{C}\sum_{k=1}^{K} n_k b_k x(\mathbf{n}_k^-), \quad \text{when } C < \mathbf{nb} \le T, \quad \mathbf{n} \in \mathbf{\Omega}. \quad (5)$$

To prove a recursive formula for the link occupancy distribution, $G(j)$, initially the global balance equation for state $\mathbf{n}$, expressed as *rate into state* $\mathbf{n}$ = *rate out of state* $\mathbf{n}$ is considered:

$$\sum_{k=1}^{K} \lambda_k P(\mathbf{n}_k^-) + \sum_{k=1}^{K} (n_k + 1)\mu_k \phi_k(\mathbf{n}_k^+) P(\mathbf{n}_k^+) = $$
$$= \sum_{k=1}^{K} \lambda_k P(\mathbf{n}) + \sum_{k=1}^{K} n_k \mu_k \phi_k(\mathbf{n}) P(\mathbf{n}),$$

where $\mathbf{n}_k^+ = (n_1, \ldots, n_k + 1, \ldots, n_K)$ and $P(\mathbf{n})$, $P(\mathbf{n}_k^-)$, $P(\mathbf{n}_k^+)$ are the probability distributions of states $\mathbf{n}$, $\mathbf{n}_k^-$, $\mathbf{n}_k^+$, respectively.

Assume now, the existence of Local Balance (LB) between adjacent states. Then the following LB equations can be extracted, for $k = 1, \ldots, K$ and $\mathbf{n} \in \mathbf{\Omega}$:

$$\lambda_k P(\mathbf{n}_k^-) = n_k \mu_k \phi_k(\mathbf{n}) P(\mathbf{n}), \quad (6)$$

$$\lambda_k P(\mathbf{n}) = (n_k + 1)\mu_k \phi_k(\mathbf{n}_k^+) P(\mathbf{n}_k^+). \quad (7)$$

Based on the assumption of LB, $P(\mathbf{n})$ can be determined by

$$P(\mathbf{n}) = G^{-1}\left(x(\mathbf{n}) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!}\right), \quad (8)$$

where $a_k = \frac{\lambda_k}{\mu_k}$ is the offered traffic-load (in Erlangs) of service-class k and $G \equiv G(\Omega) = \sum_{n \in \mathbf{\Omega}} \left(x(\mathbf{n}) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!}\right)$.

Since j is the occupied link bandwidth, $G(j)$ is defined as:

$$G(j) = \sum_{\mathbf{n} \in \mathbf{\Omega}_j} P(n), \quad \mathbf{\Omega}_j = \{\mathbf{n} \in \mathbf{\Omega} : \mathbf{nb} = j\}, \quad (9)$$

Consider now two sets: 1) $0 \le j \le C$ and 2) $C < j \le T$. For set 1), we have the EMLM/TH and $G(j)$'s are given by the following formula [41]:

$$G(j) = \frac{1}{j}\sum_{k=1}^{K} a_k b_k \left[G(j - b_k) - T_k(j - b_k)\right], \text{ for } j = 1, \ldots, C, \quad (10)$$

where

$$T_k(x) := Pr\left[j = x, \; n_k = n_k^*\right]. \quad (11)$$

In Eq. (11) the fact that $n_k = n_k^*$ implies that $j \ge n_k^* b_k$.

When $C < j \le T$, Eq. (4) is substituted in Eq. (6) to have:

$$a_k x(\mathbf{n}) P(\mathbf{n}_k^-) = n_k x(\mathbf{n}_k^-) P(\mathbf{n}). \quad (12)$$

Multiplying both sides of Eq. (12) by b_k and summing over k we obtain:

$$x(\mathbf{n}) \sum_{k=1}^{K} a_k b_k P(\mathbf{n}_k^-) = P(\mathbf{n}) \sum_{k=1}^{K} n_k b_k x(\mathbf{n}_k^-). \quad (13)$$

Equation (13), due to Eq. (5) is written as:

$$P(\mathbf{n}) = \frac{1}{C}\sum_{k=1}^{K} a_k b_k P(\mathbf{n}_k^-). \quad (14)$$

Summing both sides of Eq. (14) over $\mathbf{\Omega}_j = \{n \in \mathbf{\Omega} : \mathbf{nb} = j\}$ and based on Eq. (9), we obtain:

$$G(j) = \frac{1}{C}\sum_{k=1}^{K} a_k b_k \sum_{\mathbf{n} \in \mathbf{\Omega}_j} P(\mathbf{n}_k^-). \quad (15)$$

Since $n_k \le n_k^*$ then

$$\sum_{\mathbf{n} \in \mathbf{\Omega}_j} P(\mathbf{n}_k^-) = G(j - b_k) - Pr\left[x = j - b_k, \; n_k = n_k^*\right].$$

Thus, Eq. (15) can be written as:

$$G(j) = \frac{1}{C}\sum_{k=1}^{K} a_k b_k \left[G(j - b_k) - T_k(j - b_k)\right], \text{ for } j = C+1, \ldots, T, \quad (16)$$

where $T_k(x)$ is given by Eq. (11).

Equations (10) and (16) result in the following approximate but recursive formula for the calculation of $G(j)$'s in the E-EMLM/TH:

$$G(j) = \frac{1}{\min(C, j)}\sum_{k=1}^{K} a_k b_k \left[G(j - b_k) - T_k(j - b_k)\right],$$
$$\text{for } j = 1, \ldots, T. \quad (17)$$

Having determined $G(j)$'s the CBP of service-class k, B_k, and the link utilization, U, are calculated as:

$$B_k = \sum_{j=T-b_k+1}^{T} G^{-1} G(j) + \sum_{j=n_k^* b_k}^{T-b_k} G^{-1} T_k(j), \quad (18)$$

$$U = \sum_{j=1}^{C} j G^{-1} G(j) + \sum_{j=C+1}^{T} C G^{-1} G(j), \quad (19)$$

where $G = \sum_{j=0}^{T} G(j)$ is the normalization constant.

In Eqs. (17) and (18) the knowledge of $T_k(j)$ is required. Since $T_k > 0$ when $j = n_k^* b_k, \ldots, T - b_k$, two subsets are considered: 1) $n_k^* b_k \le j \le C$ and 2) $C + 1 \le j \le T - b_k$.

For the first subset, let a system of capacity $F_k = T - b_k - n_k^* b_k$ that accommodates all service-classes but service-class k. For this system, $r_k(j)$ is defined as:

$$r_k(j) = \frac{1}{j}\sum_{\substack{i=1 \\ i \ne k}}^{K} a_i b_i \left[r_k(j - b_i) - T_i(j - b_i)\right],$$
$$\text{for } j = 1, \ldots, F_k. \quad (20)$$

Based on $r_k(j)$'s, $T_k(j)$ is computed via the formula

$$T_k(j) = \frac{a_k^{n_k^*}}{n_k^*!} r_k(j - n_k^* b_k). \quad (21)$$

For the second subset, $T_k(j)$ can be determined by

$$T_k(j) = \frac{a_k^{n_k^*}}{n_k^*!} \sum_{\mathbf{n}\in\mathbf{\Omega}} x(\mathbf{n}) \prod_{\substack{i=1\\ i\neq k}}^{K} \frac{a_i^{n_i}}{n_i!}, \tag{22}$$

where $\mathbf{\Omega} = \left\{\mathbf{n} \in \mathbf{\Omega} : n_k^* b_k + \sum\limits_{i=1, i\neq k}^{K} n_i b_i = j,\ C+1 \le j \le T - b_k\right\}$.

In Eq. (22), $T_k(j)$ is determined only for a subset of $\mathbf{\Omega}$, defined by $C+1 \le j \le T-b_k$ and only under the assumption that $n_k = n_k^*$. This means that enumeration of the subset of Ω is needed for those states $\mathbf{n} = (n_1, n_2, \ldots, n_k = n_k^*, \ldots, n_K)$ where $C+1 \le \mathbf{nb} \le T-b_k$. Based on the fact that the value of T should not be much higher than the corresponding value of C (otherwise the increase of delay for elastic calls may be unacceptable for some applications) the subset of $\mathbf{\Omega}$ will not become large. In general, the computational complexity of Eq. (22) grows exponentially with $K-1$ (since for service-class k we have $n_k = n_k^*$) and can be in the order of $O\big((T-b_k-C)^{(K-1)}\big)$. Assuming the existence of the CS policy and ignoring the bandwidth compression mechanism, then the computational complexity becomes $O(C^K)$ [1].

To further reduce the computational complexity of the proposed model, the application of convolutional algorithms may be considered [48], but this is left for future work.

3. The Elastic EMLM/TH-BR

Consider again a link of capacity C b.u. that accommodates K elastic service-classes of Poisson arriving calls. A new service-class k ($k = 1, \ldots, K$) call has a peak-bandwidth requirement of b_k b.u. and a BR parameter t_k that expresses the reserved b.u. used to benefit calls of all other service-classes except k. If $j + b_k \le T - t_k$ and $n_k^* + 1 \le n_k^*$ then the call is accepted in the link and remains for an exponentially distributed service time with mean μ_k^{-1}. Otherwise the call is blocked and lost.

To determine $G(j)$'s in the E-EMLM/TH-BR the authors propose the following approximate but recursive formula:

$$G(j) = \left\langle \begin{array}{ll} 1, & \text{for } j=0 \\ \dfrac{1}{\min(C,j)} \sum\limits_{k=1}^{K} a_k D_k(j-b_k) \times & \\ \times\big[G(j-b_k) - T_k(j-b_k)\big] & \text{for } j=1,\ldots,T \\ 0, & \text{otherwise} \end{array} \right\rangle, \tag{23}$$

where

$$D_k(j-b_k) = \begin{cases} b_k & \text{for} \quad j \le T - t_k \\ 0 & \text{for} \quad j > T - t_k \end{cases}. \tag{24}$$

A characteristic of the BR policy is that it ensures CBP equalization among different service-classes by a proper selection of the BR parameters. If, for example, CBP equalization is required between calls of two service-classes with $b_1 = 1$ and $b_2 = 10$ b.u., respectively, then $t_1 = 9$ b.u. and $t_2 = 0$ b.u. so that $b_1 + t_1 = b_2 + t_2$.

The application of the BR policy in the E-EMLM/TH-BR is based on the assumption that the number of service-class k calls is negligible in states $j > T - t_k$ and is incorporated in Eq. (23) by the variable $D_k(j-b_k)$ given in Eq. (24). The states $j > T - t_k$ belong to the so-called reservation space. Note that the population of calls of service-class k in the reservation space may not be negligible. In [6], [11] a complex procedure is implemented that takes into account this population in the EMLM and Engset multirate state-dependent loss models, respectively. However, this procedure may not always increase the accuracy of the CBP results compared to simulation [11].

Similarly to the E-EMLM/TH, the CBP of service-class k, B_k, is determined based on two groups of states:

- those where the available link bandwidth is less than $b_k + t_k$ b.u. when the new call arrives in the system; this happens when $T - b_k - t_k + 1 \le j \le T$;
- those where the available link bandwidth is enough to accept the new call, i.e. $j \le T - b_k - t_k$ but $n_k = n_k^*$.

The latter implies that $j \ge n_k^* b_k$, or $n_k^* b_k \le j \le T - b_k - t_k$. Thus, the values of B_k are calculated by:

$$B_k = \sum_{j=T-b_k-t_k+1}^{T} G^{-1} G(j) + \sum_{j=n_k^* b_k}^{T-b_k-t_k} G^{-1} T_k(j), \tag{25}$$

where $G = \sum\limits_{j=0}^{C} G(j)$ is the normalization constant.

As far as U is concerned, it can be determined by Eq. (19). In Eqs. (23) and (25) the knowledge of $T_k(j)$ is required for $n_k^* b_k \le j \le T - b_k - t_k$. The authors consider again two subsets: 1) $n_k^* b_k \le j \le C$ and 2) $C+1 \le j \le T - b_k - t_k$. For the first subset, authors use Eqs. (20), (21) where $F_k = T - b_k - t_k - n_k^* b_k$, while for subset (2) we use Eq. (22) where $x(\mathbf{n})$ is given by Eq. (5) and

$$\mathbf{\Omega'} = \left\{ \mathbf{n} \in \mathbf{\Omega'} : n_k^* b_k + \sum_{\substack{i=1\\ i\neq k}}^{K} n_i b_i = j,\ C+1 \le j \le T - b_k - t_k \right\}.$$

If $C = T$ and both the TH and the BR policies are considered, then calls are not allowed to compress their bandwidth. In this case, the proposed E-EMLM/TH-BR coincides with the EMLM/TH-BR of [45]. The values of $G(j)$'s and CBP are given by Eqs. (26), (27), respectively:

$$G(j) = \left\langle \begin{array}{ll} 1, & \text{for } j=0 \\ \dfrac{1}{j} \sum\limits_{k=1}^{K} a_k D_k(j-b_k) \times & \\ \times\big[G(j-b_k) - T_k(j-b_k)\big] & \text{for } j=1,\ldots,C \\ 0, & \text{otherwise} \end{array} \right\rangle, \tag{26}$$

$$B_k = \sum_{j=C-b_k-t_k+1}^{C} G^{-1} G(j) + \sum_{j=n_k^* b_k}^{C-b_k-t_k} G^{-1} T_k(j), \tag{27}$$

where

$$D_k(j-b_k)=\begin{cases} b_k & \text{for} \quad j \leq C-t_k \\ 0 & \text{for} \quad j > C-t_k \end{cases}. \tag{28}$$

The link utilization can be calculated by

$$U=\sum_{j=1}^{C} jG^{-1}G(j). \tag{29}$$

Finally, if $C=T$ and we do not consider the TH and the BR policies, then the proposed E-EMLM/TH-BR coincides with the classical EMLM of [1], [2]. In that case, the link occupancy distribution is determined by the well-known Kaufman-Roberts recursion:

$$G(j)=\left\langle \begin{array}{ll} 1, & \text{for } j=0 \\ \dfrac{1}{j}\sum\limits_{k=1}^{K} a_k b_k G(j-b_k) & \text{for } j=1,\ldots,C \\ 0, & \text{otherwise} \end{array} \right\rangle, \tag{30}$$

The CBP of service-class k is given by [1], [2]:

$$B_k=\sum_{j=C-b_k+1}^{C} G^{-1}G(j), \tag{31}$$

while the link utilization can be determined by Eq. (29).

4. The Elastic-Adaptive EMLM/TH-BR

Adaptive traffic is a variant of elastic traffic since adaptive calls can tolerate bandwidth compression without altering their service time. To include adaptive traffic in the E-EMLM/TH, authors assume that K_e and K_a are the number of elastic and adaptive service-classes, respectively. The single link accommodates K service-classes of Poisson arriving calls, where $K=K_e+K_a$.

The existence of the bandwidth compression mechanism destroys reversibility in the proposed model and therefore the steady state probabilities have no PFS. To circumvent this problem, we use $\phi_k(\mathbf{n})$ based on Eq. (4) while the values of $x(\mathbf{n})$ are given by:

$$x(\mathbf{n})=\begin{cases} 1, & \text{when } \mathbf{nb} \leq C, \ \mathbf{n}\in\mathbf{\Omega} \\ \dfrac{1}{C}\left(\sum\limits_{k\in K_e} n_k b_k x(\mathbf{n}_k^-)+\right. & \\ \left. +r(\mathbf{n})\sum\limits_{k\in K_a} n_k b_k x(\mathbf{n}_k^-)\right) & \text{when } C<\mathbf{nb}\leq T, \ \mathbf{n}\in\mathbf{\Omega} \\ 0, & \text{otherwise} \end{cases}. \tag{32}$$

The derivation of Eq. (32) is based on the assumptions:

- The bandwidth of all in-service calls of service-class $k\in K$ (elastic or adaptive) is compressed by a factor $\phi_k(\mathbf{n})$, in state $C<\mathbf{nb}\leq T$, so that:

$$\sum_{k\in K_e} n_k b'_k+\sum_{k\in K_a} n_k b'_k=C \tag{33}$$

- The product *service time by bandwidth per call* of service-class k calls, $k\in K$, remains the same in state $\mathbf{n}$ either of the irreversible or the reversible Markov chain. In other words:

  For elastic service-classes:

$$\frac{b_k r(\mathbf{n})}{\mu_k r(\mathbf{n})}=\frac{b'_k}{\mu_k\phi_k(\mathbf{n})}\Rightarrow b'_k=b_k\phi_k(\mathbf{n}). \tag{34}$$

  For adaptive service-classes:

$$\frac{b_k r(\mathbf{n})}{\mu_k}=\frac{b'_k}{\mu_k\phi_k(\mathbf{n})}\Rightarrow b'_k=b_k\phi_k(\mathbf{n})r(\mathbf{n}). \tag{35}$$

Under these assumptions, Eq. (32) can be derived by substituting Eqs. (34), (35) and Eq. (4) into Eq. (33).

Based on Eqs. (32)–(35) and following the analysis of Section 2, it can be proved that $G(j)$'s are given by the following formula for the EA-EMLM/TH:

$$G(j)=\left\langle \begin{array}{ll} 1, & \text{for } j=0 \\ \frac{1}{\min(C,j)}\sum\limits_{k=1}^{K_e} a_k b_k\big[(G(j-b_k)-T_k(j-b_k)\big] & \\ +\dfrac{1}{j}\sum\limits_{k=1}^{K_a} a_k b_k\big[G(j-b_k)-T_k(j-b_k)\big], & \text{for } j=1,\ldots,T \\ 0, & \text{otherwise} \end{array} \right\rangle. \tag{36}$$

Having determined $G(j)$'s, the CBP of service-class k, B_k, and the link utilization, U, can be calculated via Eqs. (18) and (19), respectively.

To determine $G(j)$'s, in the EA-EMLM/TH-BR the following approximate but recursive formula is proposed:

$$G(j)=\left\langle \begin{array}{ll} 1, & \text{for } j=0 \\ \frac{1}{\min(C,j)}\sum\limits_{k=1}^{K_e} a_k D_k(j-b_k)\big[G(j-b_k)-T_k(j-b_k)\big] & \\ +\dfrac{1}{j}\sum\limits_{k=1}^{K_a} a_k D_k(j-b_k)\big[G(j-b_k)-T_k(j-b_k)\big], & \text{for } j=1,\ldots,T \\ 0, & \text{otherwise} \end{array} \right\rangle, \tag{37}$$

where the values of $D_k(j-b_k)$ are given by Eq. (24).

Similar to the E-EMLM/TH-BR, the CBP of service-class k, B_k, and the link utilization, U are determined, via Eqs. (25) and (19), respectively.

5. Numerical Examples – Evaluation

In this section, an application example of the proposed E-EMLM/TH-BR and the model of [49] (EMLM/TH-BR) is presented. Through the proposed model authors obtain analytical CBP and link utilization results, and compare them with the corresponding simulation results, in order to reveal the accuracy of the proposed model. Similar accuracy appears in the case of the EA-EMLM/TH-BR and therefore these results are not presented herein. The simulation model is based on the bandwidth compression/expansion mechanism described by $r(\mathbf{n})$'s. On the other hand, the proposed analytical models are based on $\phi_k(\mathbf{n})$'s. In that sense, the comparison of analytical with simulation results shows how satisfactory the approximation of $\phi_k(\mathbf{n})$'s is. Simulation results are mean values of 7 runs. Each run is based on the generation of four million calls. To account for a warm-up period, the blocking events of the first 5% of these generated calls are not considered in the results. Due to the fact that reliability ranges are very small, they are not presented in the figures that follow. The simulation language used is Simscript III [50].

As an application example, a link of capacity $C = 70$ b.u. is considered and three values of T:

1) $T = C = 70$ b.u.,
2) $T = 75$ b.u. with $r_{\max} = 70/75$,
3) $T = 80$ b.u. with $r_{\max} = 70/80$.

The link accommodates three service-classes, with the following characteristics:

- 1st service-class: $a_1 = 5.0$ Erl, $b_1 = 2$, $n_1^* = 25$, $t_1 = 7$,
- 2nd service-class: $a_2 = 1.5$ Erl, $b_2 = 5$, $n_2^* = 11$, $t_2 = 4$,
- 3rd service-class: $a_3 = 1.0$ Erl, $b_3 = 9$, $n_3^* = 6$, $t_2 = 0$.

In the x axis of all figures, traffic loads α_1, α_2 and α_3 increase in steps of 1, 0.5 and 0.25 Erl, respectively. So, Point 1 refers to $(a_1, a_2, a_3) = (5.0, 1.5, 1.0)$ while Point 7 is $(a_1, a_2, a_3) = (11.0, 4.5, 2.5)$.

In Figs. 1–3, authors consider the proposed E-EMLM/TH-BR and present the analytical and simulation CBP results of the three service-classes, respectively, for all values of T. For comparison, the corresponding analytical results of the EMLM/TH-BR (when $T = C = 70$) are presented. According to Figs. 1–3, authors deduce that:

- the results obtained by the proposed formulas are very close to the simulation results;
- the bandwidth compression mechanism reduces CBP as expected (higher reduction is achieved for $T = 80$ b.u.);
- the analytical CBP results obtained by the existing EMLM/TH-BR fail to approximate the simulation CBP results of the E-EMLM/TH-BR;
- the application of the BR policy in the E-EMLM/TH-BR results in the CBP increase of the 1st and 2nd service-classes and the CBP decrease of the 3rd service-class. This behavior is expected since the BR parameters are chosen to favor the 3rd service-class.

In Fig. 4, the link utilization results (in b.u.) are presented. Again, the analytical results are very close to simulation, while the existing EMLM/TH-BR fails to approximate the results obtained by the proposed model.

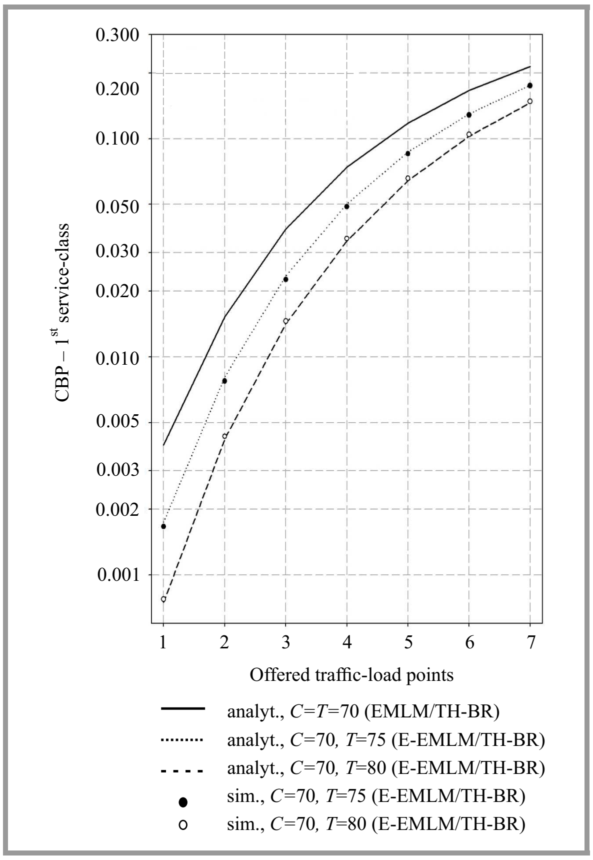

Fig. 1. CBP – 1st service-class.

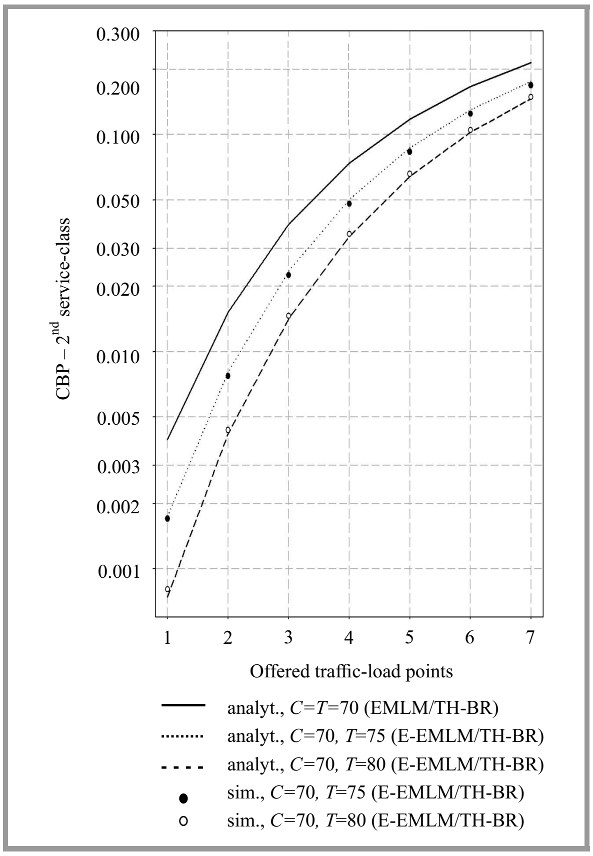

Fig. 2. CBP – 2nd service-class.

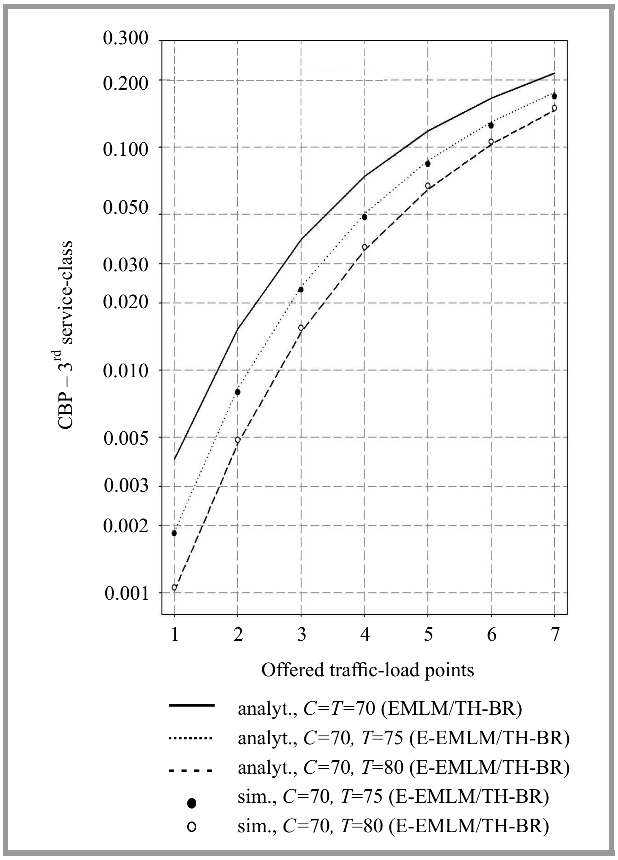

Fig. 3. CBP – 3rd service-class.

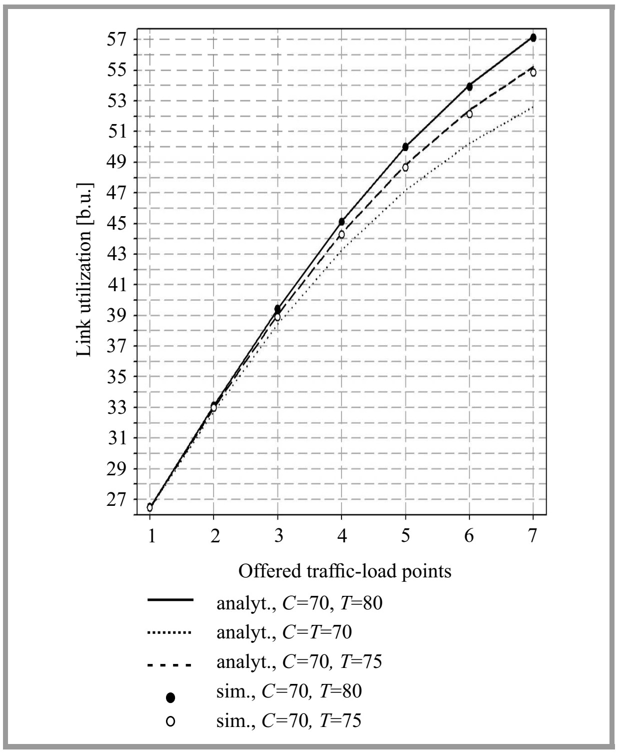

Fig. 4. Link utilization.

As a final comment, the results obtained by the proposed formulas are very close to the simulation results even for quite large values of T compared to C. However, increasing T results in a delay increase of elastic calls, which may be unacceptable for some applications. Thus, T should be chosen so that this delay remains within acceptable levels.

6. Conclusion

In this paper authors propose multirate loss models where Poisson arriving calls compete for the available link bandwidth under the TH and the BR policies. Calls are of elastic or adaptive type, i.e., they can tolerate bandwidth compression while in-service. When bandwidth of in-service elastic calls is compressed then their remaining service time is increased. Adaptive in-service calls do not alter their service time. The analysis of the proposed models leads to approximate but recursive formulas for the calculation of the steady-state probabilities and consequently CBP and link utilization. Simulation results verify the accuracy of the proposed models. In addition, numerical results show the necessity of the proposed models, since existing models fail to approximate the results obtained by the proposed models, and their consistency.

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