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MODELLING UNCERTAINTIES IN MULTI-CRITERIA DECISION MAKING USING DISTANCE MEASURE AND TOPSIS FOR HESITANT FUZZY SETS

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Abstract

A notion for distance between hesitant fuzzy data is given. Using this new distance notion, we propose the technique for order preference by similarity to ideal solution for hesitant fuzzy sets and a new approach in modelling uncertainties. An illustrative example is constructed to show the feasibility and practicality of the new method.

Keywords: uncertainty modelling, multiple criteria analysis, group decisions and negotiations, hesitant fuzzy set, TOPSIS

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1 Introduction

Real world decision making problems are quite challenging because of the difficulty of modeling and coping with uncertain situation. One of the most commonly used approaches in multiple criteria decision making problems is the technique for order preference by similarity to ideal solution (TOPSIS). Hwang and Yoon [11] developed TOPSIS for multiple criteria based decision making (MCDM) problems. TOPSIS is an effective technique for the selection of suitable alternative and to rank the alternatives from best to worst or vice versa [1, 2, 5, 13, 14]. The MCDM provides a framework for comparison of different alternatives based on different criteria. Ranking of alternatives in the TOPSIS work on the concept of distances between alternatives and ideal solutions. Kim et al. [15] and Shih et al. [22] addressed four advantages of TOPSIS:

– It has sound logic to represent the rationale of human choice;

– It has scalar value to consider the best and worst alternative simultaneously;

– It has a simple computation process and can be easily programmed;

– It has ability of the performance measures of all alternatives on attributes to be visualized on a polyhedron, at least for any two dimensions.

Representation of human preference is not suitably possible with exact numeric values for real world decision problems. To handle uncertainty, fuzzy set theory and its different generalizations have been developed and used. Bellman and Zadeh [4] proposed the concept of fuzzy set theory in decision making for the solution of ambiguity in information from human preference. Dubois [9] gave a comparison about some old and new techniques for fuzzy decision analysis. Fuzzy numbers are applied to establish a fuzzy TOPSIS [7, 18] and fuzzy TOPSIS has been further developed by several authors [3, 6, 8, 12, 16, 17, 20, 24, 25, 26]. Hesitant fuzzy sets that have been recently introduced in

[23] provide a very interesting extension of fuzzy sets. They try to manage those situations, where a set of values are possible in the definition process of the membership of an element. In [19, 21, 27] hesitant fuzzy sets are used to obtain multiple attribute decision making. The aim of this paper is to two fold; the first one is to extend fuzzy TOPSIS for hesitant fuzzy sets under the opinion of decision makers, and the second one is the new approach in modelling uncertainties. In the proposed method TOPSIS and HFS is, for the first time used simultaneously.

This article is organized as follows: In Section 2, some preliminary concepts are given to understand our proposal. In Section 3, we gave a notion of distance between HFE's and fuzzy TOPSIS is constructed for HFS. Then in Section 4, the proposed fuzzy TOPSIS method is applied to see its feasibility. Conclusion of the paper is given in the last Section.

2 Preliminaries

A *fuzzy set B* in the universe X is a mapping from X to $[0,1]$. The value $B(x)$ is called the *degree of membership of* x in B. Torra [23] introduced hesitant fuzzy sets (HFS) as a generalization of fuzzy sets.

Definition 1 *[23] A hesitant fuzzy set on X is a function h that when applied to X returns a subset of* $[0,1]$, *which can be represented as the following mathematical symbol:*

$$E = \{(x, h(x)) | x \in X\},$$

where $h(x)$ *is a set of some values in* $[0,1]$, *denoting the possible membership degrees of the element* $x \in X$ *to the set* E. *For convenience, Xia and Xu [28] named* $h(x)$ *a hesitant fuzzy element (HFE).*

A typical hesitant fuzzy set is a fuzzy set where $h(x)$ is a finite subset of $[0,1]$. Examples of hesitant fuzzy sets are given below where $h(x)$ represents the possible membership values of the set at x.

It is noted that the number of values in different HFEs may be different, let $l_{h(x)}$ be the number of values in $h(x)$. In case values in an HFE are out of order; we can arrange them in such a order, that an HFE h, let $\sigma : (1,2,...,n) \to (1,2,...,n)$ be a permutation satisfying $h_{\sigma(i)} \leq h_{\sigma(i+1)}$, $i = 1,2,...,l_h - 1$. Xu and Xia [29] proposed that two HFEs h_1 and h_2 have the same length l and $h_{1_{\sigma(i)}} = h_{2_{\sigma(i)}}$ if and only if $h_1 = h_2$, for $i = 1,2,...,l$.

Example 1 *[23] Let Xbe a reference set, then following are some hesitant fuzzy sets;*

Empty Set: $h(x) = \{0\}$ *for all* x *in* X.

Full Set: $h(x) = \{1\}$ *for all* x *in* X.

Complete ignorance for a $x \in X$ *(all is possible):* $h(x) = [0,1]$.

Nonsense for a $x \in X : h(x) = \emptyset$.

Definition 2 *[23] Let X be a reference set and h be a HFS. The upper bound* $h^+(x)$ *and lower bound* $h^-(x)$ *for a* $x \in X$ *are defined as*

1) Upper bound: $h^+(x) = max h(x)$.

2) Lower bound: $h^-(x) = min h(x)$.

Example 2 *Consider a hesitant fuzzy set A given by*

$A = \{(x_1, (0.2, 0.3, 0.6, 0.9)), (x_2, (0.1, 0.4, 0.5, 0.7))\}$

Then

$h_A^+(x_1) = max(0.2, 0.3, 0.6, 0.9) = 0.9,$ *and*
$h_A^+(x_2) = max(0.1, 0.4, 0.5, 0.7) = 0.7,$ *and*
$h_A^-(x_1) = min(0.2, 0.3, 0.6, 0.9) = 0.2,$ *and*
$h_A^-(x_2) = min(0.1, 0.4, 0.5, 0.7, 1) = 0.1.$

Definition 3 *[23] For a hesitant fuzzy set represented by its membership function h, we define its complement as follows:*

$$h^c(x) = \bigcup_{\gamma \in h(x)} \{1 - \gamma\}.$$

Example 3 *Consider a hesitant fuzzy set A such that*

$A = \{(x_1, (0.2, 0.3, 0.6, 0.9)), (x_2, (0.1, 0.4, 0.5, 0.7, 1))\}$
Then complement of A is given by

$A^c = \{(x_1, (1 - 0.2, 1 - 0.3, 1 - 0.6, 1 - 0.9)),$
$(x_2, (1 - 0.1, 1 - 0.4, 1 - 0.5, 1 - 0.7, 1 - 1))\}$
$= \{(x_1, (0.8, 0.7, 0.4, 0.1)), (x_2, (0.9, 0.6, 0.5, 0.3, 0))\}.$

Definition 4 *[23] Given two hesitant fuzzy sets represented by their membership functions* h_1 *and* h_2,

– *their union represented by* $h_1 \cup h_2$ *as*

$$(h_1 \cup h_2)(x) =$$

$$\{h \in (h_1(x) \cup h_2(x)) | h \geq \max(h_1^-(x), h_2^-(x))\}.$$

– *their intersection represented by* $h_1 \cap h_2$ *as*

$$(h_1 \cap h_2)(x) =$$

$$\{h \in (h_1(x) \cap h_2(x)) | h \leq \min(h_1^+(x), h_2^+(x))\}.$$

Example 4 *Consider two hesitant fuzzy sets A and B such that*

$A = \{(x_1,(0.2,0.4,0.6)),(x_2,(0.5,0.7,1))\}$,

and

$B = \{(x_1,(0,0.4,0.8)),(x_2,(0.2,0.3,0.45,0.9))\}$. *Then*

$A \cup B =$
$\{(x_1,(0.2,0.4,0.6,0.8)),(x_2,(0.5,0.7,0.9,1))\}$

and

$A \cap B =$
$\{(x_1,(0,0.2,0.4,0.6)),$
$(x_2,(0.2,0.3,0.45,0.5,0.7,0.9))\}$.

Xu and Xia [29] gave six different distance formulae for HFE's. But in their distance formulae two HFE's should have the same length, so their distance formulae are not applicable for any two HFE's with different length. Motivated by the Hausdorff distance, we give a distance notion for any two HFE's.

Definition 5 *Let x and y be the two HFEs, such that* $x = \{a_1,a_2,\ldots,a_n\}$ *and* $y = \{b_1,b_2,\ldots,b_m\}$, *then distance* $'d'$ *between x and y is defined as*

$$d(x,y) =$$

$$\max\left\{\max_{a_j \in x}\left\{\min_{b_i \in y}(|a_j - b_i|)\right\}, \max_{b_i \in y}\left\{\min_{a_j \in x}(|a_j - b_i|)\right\}\right\}.$$

It is easy to show that this distance 'd' satisfies the following properties.

1. $d(x,y) = 0$ if and only if $x = y$;

2. $d(x,y) = d(y,x)$.

Example 5 *As in above example* $A(x_1) = \{0.2,0.4,0.6\}$ *and* $B(x_1) = \{0,0.2,0.4,0.6\}$, *Now we want to calculate distance between them.*

$d(A(x_1),B(x_1)) = \max\{\max\{\min(|0.2-0|, |0.2-0.2|, |0.2-0.4|, |0.2-0.6|), \min(|0.4-0|, |0.4-0.2|, |0.4-0.4|, |0.4-0.6|), \min(|0.6-0|, |0.6-0.2|, |0.6-0.4|, |0.6-0.6|)\}, \max\{\min(|0-0.2|, |0-0.4|, |0-0.6|), \min(|0.2-0.2|, |0.2-0.4|, |0.2-0.6|), \min(|0.4-0.2|, |0.4-0.4|, |0.4-0.6|), \min(|0.6-0.2|, |0.6-0.4|, |0.6-0.6|)\}\}$

$d(A(x_1),B(x_1)) = \max\{\max\{\min(0.2,\ 0,\ 0.2,\ 0.4), \min(0.4,\ 0.2,\ 0,\ 0.2), \min(0.6,\ 0.4,\ 0.2,\ 0)\}, \max\{\min(0.2,\ 0.4,\ 0.6), \min(0,\ 0.2,\ 0.4), \min(0.2,\ 0,\ 0.2), \min(0.4,\ 0.2,\ 0)\}\}$

$d(A(x_1),B(x_1)) = \max\{\max\{0,\ 0,\ 0\}, \max\{0.2,\ 0,\ 0,\ 0\}\}$

$d(A(x_1),B(x_1)) = \max\{\ 0,0.2\} = 0.2.$

3 TOPSIS for HFS

We give construction of TOPSIS using the proposed notion of distance, which is then used for multi-criteria group decision making where the opinions about the criteria values are expressed as HFS. We suppose that in this group decision making problem, $E = \{e_1,\ e_2,\ \ldots,\ e_K\}$, $A = \{A_1,\ A_2,\ \ldots,\ A_m\}$ and $C = \{C_1, C_2, \ldots, C_n\}$ are the set of the decision makers, alternatives and criteria, respectively.

Step 1. Let $\tilde{X}^l = [H^l_{S_{ij}}]_{m \times n}$ be a hesitant fuzzy decision matrix for the multi-criteria group decision making problem where performance of alternative A_i with respect to decision maker e_l and criterion C_j is denoted as $H^l_{S_{ij}}$.

Step 2. We produce the single decision matrix X by aggregating the opinions of all the DMs involved in the group decision making problem. $X = [x_{ij}]$, where $x_{ij} = \{x \mid x \in H^l_{S_{ij}}$ and $s_{p_{ij}} \leq x \leq s_{q_{ij}}$ for all $l\}$ where

$$s_{p_{ij}} = \min\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\}$$

and

$$s_{q_{ij}} = \max\left\{\min_{l=1}^{K}(\max H^l_{S_{ij}}), \max_{l=1}^{K}(\min H^l_{S_{ij}})\right\}.$$

$$\begin{aligned}\tilde{A}^{+} = & \Big[x|x\in H^{l}_{S_{ij}}\forall i\; and \max_{l=1}^{K}\left(\max_{i}(\min H^{l}_{S_{ij}})\right)\leq x\leq \max_{l=1}^{K}\left(\max_{i}(\max H^{l}_{S_{ij}})\right)|j\in\Omega_b,\\ & x|x\in H^{l}_{S_{ij}}\forall i\; and \min_{l=1}^{K}\left(\min_{i}(\min H^{l}_{S_{ij}})\right)\leq x\leq \min_{l=1}^{K}\left(\min_{i}(\max H^{l}_{S_{ij}})\right)|j\in\Omega_c\Big]\end{aligned} \quad (1)$$

i=1, 2, ..., m, and j = 1, 2, ..., n.

$$\tilde{A}^{+} = (\tilde{V}_1^{+}\;\; \tilde{V}_2^{+}\;\; \dots\;\; \tilde{V}_n^{+})$$

$$\begin{aligned}\tilde{A}^{-} = & \Big[x|x\in H^{l}_{S_{ij}}\forall i\; and \max_{l=1}^{K}\left(\max_{i}(\min H^{l}_{S_{ij}})\right)\leq x\leq \max_{l=1}^{K}\left(\max_{i}(\max H^{l}_{S_{ij}})\right)|j\in\Omega_c,\\ & x|x\in H^{l}_{S_{ij}}\forall i\tilde{a}nd \min_{l=1}^{K}\left(\min_{i}(\min H^{l}_{S_{ij}})\right)\leq x\leq \min_{l=1}^{K}\left(\min_{i}(\max H^{l}_{S_{ij}})\right)|j\in\Omega_b\Big]\end{aligned} \quad (2)$$

i=1, 2, ..., m, and j = 1, 2, ..., n.

$$\tilde{A}^{-} = (\tilde{V}_1^{-}\tilde{V}_2^{-}\dots\tilde{V}_n^{-})$$

Aggregated performance of alternative A_i for criterion C_j is denoted as x_{ij}, in the final aggregated matrix X.

Step 3. Let Ω_b be the collection of all the benefit criteria and Ω_c be the collection of all the cost criteria. The HFS positive-ideal solution (HFS-PIS), denoted as $\tilde{A}^{+} = (\tilde{V}_1^{+}\;\; \tilde{V}_2^{+}\;\; \dots\;\; \tilde{V}_n^{+})$, and the HFS negative-ideal solution (HFS-NIS), denoted as $\tilde{A}^{-} = (\tilde{V}_1^{-}\;\; \tilde{V}_2^{-}\;\; \dots\;\; \tilde{V}_n^{-})$, are defined as follows Eq. 1 and 2:

Step 4. The construction of positive ideal separation matrix (D^{+}) and negative ideal separation matrix (D^{-}) are defined as follows Eq. 3 and 4:

Step 5. Calculate the relative closeness (RC) of each alternative to the ideal solution as follows:

$$RC(A_i) = \frac{D_i^{-}}{D_i^{+} + D_i^{-}},\; i = 1,2,\dots,m,$$

where $D_i^{-} =_{j=1}^{n} d(x_{ij}, \tilde{V}_j^{-})$

and $D_i^{+} =_{j=1}^{n} d(x_{ij}, \tilde{V}_j^{+})$.

Step 6. Rank all the alternatives A_i ($i = 1,2,\dots,m$) according to the closeness coefficient $RC(A_i)$, the greater the value $RC(A_i)$, the better the alternative $A_{i.}$

4 Example

In this Section, we give an example to illustrate the usefulness of the method proposed in Section 3 to get the best alternative.There is an investment company, which wants to invest money in the best option (adapted from [10]). There are five possible alternatives in which to invest the money: A_1 is a car industry, A_2 is a food company, A_3 is a computer company, A_4 is an arms company, A_5 is a TV company. The investment company must take a decision according to the following four criteria: C_1 is the risk analysis; C_2 is the growth analysis; C_3 is the social-political impact analysis, C_4 is the environmental impact analysis. The decision is to be taken by company board of directors i.e. decision makers e_K ($K = 1, 2, \dots, 10$).

Step 1. The five possible alternatives A_i ($i = 1$, 2, 3, 4, 5) are to be evaluated using the HFS by ten decision makers e_K ($K = 1, 2, \dots, 10$), as listed in Table 1-3.

Step 2. The decision matrix constructed by utilize Table 1-3 is listed in Table 4;

Step 3. For cost criteria C_1, C_4 and benefit criteria C_2, C_3 HFS-PIS A^{+} and HFS-NIS A^{-} is as follows Eq. 5:

Step 4. Positive ideal matrix (D^{+}) Eq. 6. Negative ideal matrix (D^{-}) Eq. 7:

$$D^{+}=\begin{bmatrix} d(x_{11},\tilde{V}_1^{+}) & + & d(x_{12},\tilde{V}_2^{+}) & + & \cdots & + & d(x_{1n},\tilde{V}_n^{+}) \\ d(x_{21},\tilde{V}_1^{+}) & + & d(x_{22},\tilde{V}_2^{+}) & + & \cdots & + & d(x_{2n},\tilde{V}_n^{+}) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1},\tilde{V}_1^{+}) & + & d(x_{m2},\tilde{V}_2^{+}) & + & \cdots & + & d(x_{mn},\tilde{V}_n^{+}) \end{bmatrix} \quad (3)$$

$$D^{-}=\begin{bmatrix} d(x_{11},\tilde{V}_1^{-}) & + & d(x_{12},\tilde{V}_2^{-}) & + & \cdots & + & d(x_{1n},\tilde{V}_n^{-}) \\ d(x_{21},\tilde{V}_1^{-}) & + & d(x_{22},\tilde{V}_2^{-}) & + & \cdots & + & d(x_{2n},\tilde{V}_n^{-}) \\ \vdots & & \vdots & & \vdots & & \vdots \\ d(x_{m1},\tilde{V}_1^{-}) & + & d(x_{m2},\tilde{V}_2^{-}) & + & \cdots & + & d(x_{mn},\tilde{V}_n^{-}) \end{bmatrix} \quad (4)$$

Table 1. Decision matrix ($\tilde{X}^1$) with respect to e_1, e_2, e_3 and e_4.

	C_1	C_2	C_3	C_4
A_1	{0.5,0.6,0.8}	{0.6,0.8}	{0.1,0.3}	{0.1,0.3}
A_2	{0.1,0.3}	{0.5,0.7,0.8}	{0.5,0.6}	{0.5,0.6}
A_3	{0.5,0.7}	{0.5,0.6}	{0.7,0.9}	{0.1,0.2}
A_4	{0.7,0.9}	{0.1,0.2}	{0.1,0.3}	{0.5,0.6,0.7}
A_5	{1}	{0.1,0.3}	{0,0.2}	{0.4,0.7}

Table 2. Decision matrix ($\tilde{X}^2$) with respect to e_5, e_6 and e_7.

	C_1	C_2	C_3	C_4
A_1	{0.1,0.2}	{0.4,0.9}	{0,0.2}	{0.4,0.6}
A_2	{0,0.2}	{0.1,0.3}	{0.4,0.5}	{0.6,1}
A_3	{0.4,0.6}	{0.1,0.2}	{0.4,0.6}	{0,0.2}
A_4	{0.6,1}	{0.4,0.7}	{0,0.1}	{0.5,0.7}
A_5	{0.5,0.7}	{0.4,0.6}	{0,0.1}	{0.6,1}

Table 3. Decision matrix ($\tilde{X}^3$) with respect to e_8, e_9 and e_{10}.

	C_1	C_2	C_3	C_4
A_1	{0.4,0.6}	{0.6,1}	{0.3,0.5}	{0,0.3}
A_2	{0.3,0.6}	{0.1,0.3}	{0.5,0.9}	{0.3,0.5}
A_3	{0.1,0.3}	{0.6,0.9}	{0.3,0.7}	{0,0.1}
A_4	{0.6,0.9}	{0.5,0.7}	{0,0.2,0.4}	{0.5,0.6,0.8}
A_5	{0.5,0.6}	{0.1,0.3}	{0.2,0.4}	{1}

Table 4. Decision matrix (X).

	C_1	C_2	C_3	C_4
A_1	{0.2,0.4,0.5}	{0.6,0.8}	{0.2,0.3}	{0.3,0.4}
A_2	{0.2,0.3}	{0.3,0.5}	{0.5}	{0.5,0.6}
A_3	{0.3,0.4,0.5}	{0.2,0.5,0.6}	{0.6,0.7}	{0.1}
A_4	{0.7,0.9}	{0.2,0.4,0.5}	{0.1}	{0.5,0.6,0.7}
A_5	{0.6,0.7,1}	{0.3,0.4}	{0.1,0.2}	{0.7,1}

$$\begin{array}{lllll} A^{+}=[& \{0,0.1,0.2\} & \{0.6,0.7,0.8,0.9,1\} & \{0.7,0.8,0.9\} & \{0,0.1\} \quad] \\ A^{-}=[& \{0.7,0.8,0.9\} & \{0.1,0.2\} & \{0,0.1\} & \{1\} \quad] \end{array} \quad (5)$$

$$D^+ = \begin{bmatrix} 0.3 & + & 0.2 & + & 0.6 & + & 0.3 \\ 0.2 & + & 0.5 & + & 0.4 & + & 0.5 \\ 0.3 & + & 0.4 & + & 0.2 & + & 0.1 \\ 0.7 & + & 0.5 & + & 0.8 & + & 0.6 \\ 0.8 & + & 0.6 & + & 0.7 & + & 0.9 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 1.6 \\ 1.0 \\ 2.6 \\ 3.0 \end{bmatrix} \quad (6)$$

$$D^- = \begin{bmatrix} 0.5 & + & 0.6 & + & 0.2 & + & 0.7 \\ 0.6 & + & 0.3 & + & 0.5 & + & 0.5 \\ 0.4 & + & 0.4 & + & 0.6 & + & 0.9 \\ 0.1 & + & 0.3 & + & 0.1 & + & 0.5 \\ 0.1 & + & 0.2 & + & 0.1 & + & 0.3 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.9 \\ 2.3 \\ 1.0 \\ 0.7 \end{bmatrix} \quad (7)$$

Step 5. Relative closeness (RC) of each alternative to the ideal solutions:

$RC(A_1) = 2/(1.4+2) = 0.5882;$

$RC(A_2) = 1.9/(1.9+1.6) = 0.5429;$

$RC(A_3) = 2.3/(2.3+1) = 0.6970;$

$RC(A_4) = 1.0/(1.0+2.6) = 0.2778;$

$RC(A_5) = 0.7/(0.7+3) = 0.1892.$

Step 6. Rank all the alternatives $A_i(i = 1,2,\ldots,5)$ according to the closeness coefficient $RC(A_i)$:

$A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5.$

So the most suitable alternative is A_3.

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