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ANALYSIS OF SELECTED TRANSPORTATION NETWORK STRUCTURES BASED ON GRAPH MEASURES

Summary. The structure of transportation networks has been the subject of analysis for many years, due to the important role that it plays in assessing the efficiency of transportation systems. One of the most common approaches to representing this structure is to use graph theory, in which elements of transportation infrastructure are depicted by a set of vertices and edges. An approach based on graph theory allows us to assess the structure of a transportation network in terms of connectivity, accessibility, density or complexity. In the paper, different transportation network structures are assessed and compared, based on graph measures.

Keywords: transportation network, graph theory, graph and topology measures

1. INTRODUCTION

A transportation network is usually understood as a set of transportation points, with connections between them, in the form of paths or routes, designed for travel by people, cargo shipments and the passage of vehicles [20]. The spatial structure of such a network corresponds to the connections that exist between the elements of transportation infrastructure

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in the geographical space. This means that elements of the transportation network are also elements of land use in the area in which they are located [5]. The volume of traffic flows, expressed by the number of travelling people, as well as moving vehicles, or by the mass of carried goods in a given unit of time, is one of the measures of transportation network performance.

The efficiency of the entire transportation system in the area under analysis is largely determined by the structure of its transportation network. The denser and more consistent the network, the greater the number of connections between two selected vertices. This has a significant impact on the possibility of reducing traffic congestion by moving traffic onto alternative roads, which in turn means shorter travel time. This explains why the analysis of transportation network structures has been the subject of intensive research for many decades [1,6-9,22,24].

The article analyses the assessment of selected structures of a transportation network based on graph measures. The network models correspond to real transportation systems. The analysis is carried out in terms of the possibility of using different types of graph measures when assessing the propagation of disturbances in the transportation network.

2. REPRESENTATION OF THE TRANSPORTATION NETWORK STRUCTURE

The physical topology of a network, which is understood as the arrangement of nodes and links in the network, is based on point and linear transportation infrastructure objects. The aim of the study is to define the scope of the representation of the infrastructure's elements and the connections between them. Therefore, the structure of the network may be both very simplified and particularly complex. Thus, depending on the adopted criteria for the classification of transportation systems, scales and aggregation level, a transportation node may be a single intersection, a bus stop, a railway station, a road junction, an airport, a logistics centre or even a whole city. In turn, the link may be a single traffic lane, a railway track, a communication route or a corridor connecting important transportation nodes. When designing a network model, the proper representation of location, direction and connections is of particular importance. It is also worth noting that the topology of a network model should be as close as possible to the structure of the real network it represents [18].

The structure of the transportation network can be mapped by using various mathematical tools. One of the most commonly and intuitive approaches is to represent the transportation system of the studied area using graph theory [12,25]. Graph methods has been used to map and study the spatial structure of transportation networks since the 1960s [10,13]. In Poland, they have been used, for example, to assess the topological accessibility of the railway network of the West Pomeranian Voivodeship [19], former Poznań Province [15,16] and Silesia [21].

The main requirement of topological analysis is to represent an existing transportation network as an abstract set of points (nodes or vertices), connected by a set of lines (segments, edges or arcs). In the graph theory approach, attention is primarily centred on the arrangement of connections between nodes, which allows for the use of undirected graphs. Metric and capacity characteristics are also often ignored [2].

Two basic approaches to the representation of a transportation network structure using graph theory are found in the literature [23]:
-   Primal, in which the nodes of the network are represented in the form of vertices, and links in the form of arcs or edges

- Dual, in which the sections of the network are represented in the form of vertices, and nodes in the form of arcs or edges

For the purpose of analysing the values of selected graph measures for various structures of a transportation network, a set of numbers relating to the types of structures has been determined as follows:

$$I = \{1, \dots, i, \dots \bar{I}\} \tag{1}$$

where i is the number for structure type, and $\bar{I}$ is the number for all structure types under analysis. Therefore, using the primal approach to mapping the structure of the transportation network, the i-th structure of the network may be described in the form of a graph:

$$G_i = (V_i, E_i), \quad i \in I \tag{2}$$

where V_i is the set of vertices of graph G_i, E_i is the set of edges of graph G_i. Both the vertices and the edges are sequentially numbered. Therefore, the set V_i contains subsequent numbers for the vertices of graph G_i, i.e.:

$$V_i = \{1, \dots, v_i, \dots, \bar{V}_i\}, \quad i \in I \tag{3}$$

where v_i is the number of a vertex of graph G_i, $\bar{V}_i$ is the number of the last vertex (in the set of vertex numbers in ascending order) corresponding to the size of the set V_i, and the set E_i contains subsequent numbers for the edges of graph G_i, i.e.:

$$E_i = \{1, \dots, e_i, \dots, \bar{E}_i\}, \quad i \in I \tag{4}$$

where e_i is the number for an edge of graph G_i, and $\bar{E}_i$ is the number for the last edge (in the set of edges numbers in ascending order) corresponding to the size of the set E_i.

The mathematical model of the transportation network should be constructed in such a way as to enable the identification of its elements, the description of its spatial structure and the assignment of specific characteristics to its individual elements [26].

3. MEASURES OF A TRANSPORTATION NETWORK STRUCTURE

There are many measures that can be used to assess the structure of the transportation network and analyse its efficiency. Some of them take into account spatial features (distance, surface), as well as the level of activity (traffic), while others solely rest on the topological dimension of the network. They may be applied to [18]:
- The expression of the relationship between values and the network structures they represent
- The comparison of different transportation networks at a specific point in time

- The comparison of the evolution of a transportation network at different points in time

The representation of the structure of the transportation network in the form of graph enables a set of functions to be assigned, which correspond to the properties (characteristics) of the elements of this structure in relation to each vertex and/or edge of the graph. These characteristics are also used to assess the structure of the transportation network. In order to conduct comparative analyses of the topology of entire networks or their parts, the measures at the network level are particularly important. Among the groups of indices for assessing the structure of a transportation network are graph measures, which are particularly important because of the representation of the network in the form of a graph.

One of the most important characteristics of a transportation network is its **level of connectivity**, which may be described as the degree of connections in a particular area or a measure of how many components of the transportation network are connected to each other [3,17]. The more connected networks there are, the shorter the travel times and costs. Moreover, connectivity plays an important role in the social and economic development of regions [17]. Three measures, based on graph theory, were initially developed by Kansky [10,11] and can be used to assess the connectivity of a transportation network [4,11,18]: alpha, beta and gamma measures. It is worth noting that all of them are ratios, that is, they represent a relation between distinguishable elements of a network [11,18].

The **alpha measure** $\alpha(G_i)$ compares the number of cycles in the network represented by the graph G_i with maximum number of cycles [11,18]. This measure can range from 0 to 1. Values close to 1 indicate a well-connected network; however, $\alpha(G_i)$ does not usually equal 1. For simple and less connected networks (for example, tree networks), values of $\alpha(G_i)$ are close to 0.

The alpha measure may be calculated based on the following formula:

$$\alpha(G_i) = \frac{e(G_i) - v(G_i) + p(G_i)}{2 \cdot v(G_i) - 5} \qquad [-], \quad i \in I \tag{5}$$

where $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$; $v(G_i)$ is the number of vertices in a graph G_i, wherein $v(G_i) = \bar{V}_i$; and $p(G_i)$ is the number of isolated subgraphs in a graph G_i. The alpha measure is often expressed as a percentage value, which denotes the percentage of maximum connectivity.

The second graph measure, the **beta measure** $\beta(G_i)$, expresses the relation between a certain number of edges and the number of vertices in a graph G_i [4,18]. It is one of the simplest measures used to evaluate the connectivity of transportation networks [11]. It may be calculated based on the following formula:

$$\beta(G_i) = \frac{e(G_i)}{v(G_i)} \qquad [-], \quad i \in I \tag{6}$$

where $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$; and $v(G_i)$ is the number of vertices in a graph G_i, wherein $v(G_i) = \bar{V}_i$.

Similar to the alpha measure, higher values of the beta index characterize well-connected networks [4,11]. For planar graphs, the maximum value of the beta index is 3, whereas, for non-planar graphs, its values are infinite. All disconnected graphs have values of the beta measure smaller than 1, while a perfect grid network may have values of the beta measure around 2.5. For network planning purposes, values of about 1.4 for the beta measure are acceptable [3].

The third graph measure developed by Kansky, the **gamma measure** $\gamma(G_i)$, expresses the relation between an observed number of edges in the network and the maximum possible number of edges [12,19]. Similar to the alpha measure, it ranges from 0 to 1, with 1 denoting a completely connected network and 0 denoting a poor level of connectivity [11]. It is determined as follows:

$$\gamma(G_i) = \frac{e(G_i)}{3 \cdot (v(G_i) - 2)} \qquad [-], \quad i \in I \tag{7}$$

where $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$; and $v(G_i)$ is the number of vertices in a graph G_i, wherein $v(G_i) = \bar{V}_i$. It is worth noticing that Formula (7) is applicable to planar graphs [11]. Usually, the gamma measure is expressed as percentage values.

Different graph-based measures take into consideration the network as a whole. Example of such measures may be the **eta measure** $\eta(G_i)$, which is an average length of a link in a transportation network [4,11]. It may be calculated based on the following formula:

$$\eta(G_i) = \frac{L(G_i)}{e(G_i)} \qquad [\text{km}], \quad i \in I \tag{8}$$

where $L(G_i)$ is the total length of the graph G_i, i.e., the sum of the length of all edges from the set E_i; and $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$. The total length $L(G_i)$ of the graph G_i is a very important characteristic of the transportation network structure from the point of view of its efficiency. According to [4], the longer the edges in the network, the better it is to ensure the maximum speed of a given link. The total length $L(G_i)$ is also taken into account in the calculation of the **pi measure** $\pi(G_i)$, which expresses the relationship between the total mileage of a transportation network and its diameter [11,18]. It may be determined based on the following formula:

$$\pi(G_i) = \frac{L(G_i)}{d(G_i)} \qquad [-], \quad i \in I \tag{9}$$

where $L(G_i)$ is the total length of the graph G_i, and $d(G_i)$ is the length of a diameter of a graph G_i, i.e., the length of the shortest path between the two most distanced vertices from the set V_i.

The pi measure equals 1 for less complicated networks. Greater values are ascribed to more complex transportation networks [11].

Another important measure based on graph theory is the **graph density** $D(G_i)$, which is understood as the ratio of the number of its edges to the largest possible number of edges that may be stretched on the vertices of the graph G_i [26]. Therefore, this measure is calculated based on the following formula:

$$D(G_i) = \frac{2 \cdot e(G_i)}{v(G_i) \cdot (v(G_i) - 1)} \qquad [-], \quad i \in I \tag{10}$$

where $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$; and $v(G_i)$ is the number of vertices in a graph G_i, wherein $v(G_i) = \bar{V}_i$.

The density of the graph may also be determined in relation to the real area $A(G_i)$ occupied by the transportation network, as represented by the graph G_i. Hence, nodal and edge graph density can be distinguished. The vertices and edges of the graph G_i may represent point and linear elements of the transportation infrastructure belonging to different transportation subsystems.

Nodal graph density $X(G_i)$ is the relation between a number of vertices and the area of the network. It may be calculated based on the following formula:

$$X(G_i) = \frac{v(G_i)}{A(G_i)} \qquad [1/\text{km}^2], \quad i \in I \tag{11}$$

where $v(G_i)$ is the number of vertices in a graph G_i, wherein $v(G_i) = \bar{V}_i$; and $A(G_i)$ is the size of the area occupied by the transportation network, as represented by the graph G_i.

Edge graph density $Y(G_i)$ is the relation between a number of edges and the area of the network. It may be calculated based on the following formula:

$$Y(G_i) = \frac{e(G_i)}{A(G_i)} \qquad [1/\text{km}^2], \quad i \in I \tag{12}$$

where $e(G_i)$ is the number of edges in a graph G_i, wherein $e(G_i) = \bar{E}_i$; and $A(G_i)$ is the size of the area occupied by the transportation network represented by the graph G_i.

Network density $N(G_i)$ is yet another measure, which is understood as the relation between the total length of the graph G_i and the area of the network. It may be calculated based on the following formula:

$$N(G_i) = \frac{L(G_i)}{A(G_i)} \qquad [\text{km/km}^2], \quad i \in I \tag{13}$$

where $L(G_i)$ is the total length of the graph G_i, and $A(G_i)$ is the size of the area occupied by the transportation network, as represented by the graph G_i.

Network density is a measure of the development of a transportation network. Highly developed networks have higher values of $N(G_i)$ [11]. Density may be also important in determining the accessibility of a transportation system [14].

4. CASE STUDY

Four different transportation network structures (mesh, tree, hub and spoke, and linear) have been chosen in order to conduct a comparative analysis. The network models have been developed on the basis of real communication systems and chosen on the basis that the size of their respective area was similar. This has allowed us to compare the values of network density and graph-based measures. The analysed structures are presented in Figure 1.
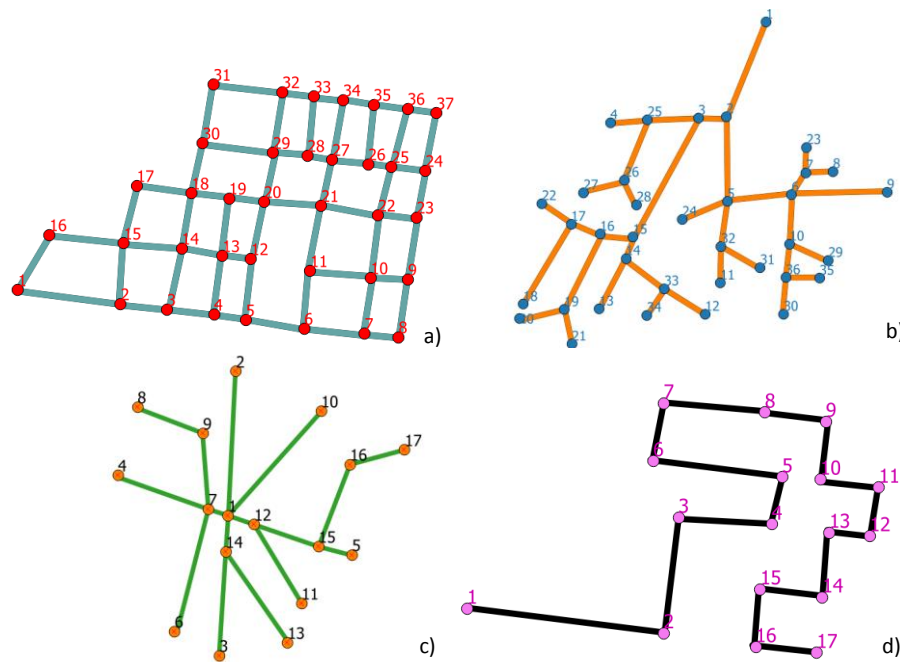

Fig. 1. Different types of transportation network structure chosen for analysis:
a) mesh, b) tree, c) hub and spoke, d) linear
Source: Own research

The selected characteristics for each transportation network structure, as presented in Figure 1), are set out in Table 1. They have been used to calculate the measures according to Formulas (5)-(13).

Tab. 1

Values of selected characteristics for the analysed types of transportation network structure

Characteristics	Type of transportation network structure			
	Mesh G_1	Tree G_2	Hub and spoke G_3	Linear G_4
Number of vertices $v(G_i)$ [-]	37	36	17	17
Number of edges $e(G_i)$ [-]	58	35	16	16
Length of graph $L(G_i)$ [km]	5.990	5.079	3.091	2.456
Size of the area $L(G_i)$ [km²]	0.264	0.293	0.252	0.270
Diameter $d(G_i)$ [km]	1.181	1.047	0.884	2.456

The results of the analysis are presented in Table 2. The measures allow us to assess the transportation network structures from different aspects, i.e., connectivity of the network, and its complexity or accessibility.

Tab. 2

Values of the selected graph measures for different structures of transportation networks

Graph measure	Type of transportation network structure			
	Mesh G_1	Tree G_2	Hub and spoke G_3	Linear G_4
Alpha measure $\alpha(G_i)$ [-]	0.32	0.00	0.00	0.00
Beta measure $\beta(G_i)$ [-]	1.57	0.97	0.94	0.94
Gamma measure $\gamma(G_i)$ [-]	0.55	0.34	0.36	0.36
Eta measure $\eta(G_i)$ [km]	0.10	0.15	0.19	0.15
Pi measure $\pi(G_i)$ [-]	5.07	4.85	3.50	1.00
Graph density $D(G_i)$ [-]	0.09	0.06	0.12	0.15
Nodal graph density $X(G_i)$ [1/km²]	140.15	122.87	67.46	62,69
Edge graph density $Y(G_i)$ [1/km²]	219.70	119.45	63.49	59,26

Network density $N(G_i)$ [km/km²]	22.69	17.33	12.27	9.10

According to the calculated values of the alpha, beta and gamma measures, the network with the highest level of connectivity is the mesh network. Only in instances involving such a structure does the beta measure exceed 1.00 and alpha measure exceed 0.00. The value of the beta measure for the mesh network (1.57) is characteristic for connected graphs and relatively close to 1.40, which is a good value for planning purposes. However, it is still significantly smaller that the value for a perfect grid. In terms of the other structures, the beta measures for the tree network, hub and spoke network, and linear network are smaller than 1.00 and similar to each other. Moreover, for these network structures, the alpha measure equals 0.00, which is normal for tree networks and other less connected networks (such as linear networks) [11]. Furthermore, the value of the gamma index for the mesh network is significantly higher than for other networks. There is no significant difference in the values of the gamma measure for the tree network, hub and spoke network and linear network.

On the other hand, the value of the eta measure for the mesh network is the smallest. This denotes that, in such a transportation network structure, the average length of a link is shorter than for other structures. However, it is worth noting that the mesh network has a significantly larger number of edges and vertices in comparison to other structures with a relatively similar size of area. Although edges are shorter, they are much better connected to each other.

The mesh network has the densest structure, which relates to the fact that, in a given area of a transportation network, there are many short edges. The second densest network is the tree network; however, there is no significant difference between the tree network and the hub and spoke network. The least dense structure is the linear network.

The most complex network, based on the pi measure, is the mesh network. The second most complex network is the tree network, with a relatively similar pi measure result to the mesh network. The hub and spoke network is the least complex. For the linear network, the pi measure equals 1.00, which is normal for this type of structure as the diameter equals the length of the graph [11].

Mesh and tree networks have a significantly higher number of vertices per square kilometre. Less complex networks, i.e., the hub and spoke network and the linear network, tend to have a smaller number of vertices in a similar area.

The mesh network is characterized by the highest level of connectivity. Such a transportation network structure ensures that the access to all vertices is quite easy; however, it also covers the biggest part of a given area with edges (roads, railway etc.). Tree or hub and spoke networks are not connected at a satisfactory level. This means that users of such transportation networks have to travel longer, which also means higher costs of transportation. Furthermore, in the case of linear networks, connectivity is not satisfactory and accessibility to vertices is limited.

5. CONCLUSIONS

Analysing the structure of transportation networks is an important subject of research. Typically, transportation networks are represented, using graph theory, as a set of vertices and edges or arcs, which represent real objects in the transportation infrastructure, such as roads, intersections and bus stops. Therefore, measures based on graph theory allow us to assess and compare different structures of a transportation network.

Four different structures, which are common in transportation systems all over the world, have been analysed and evaluated. Nine graph-based measures have been used in order to reveal characteristic features of the analysed structures. The study showed significant differences in the level of connectivity or complexity among these structures.

The measures may be also used to assess accessibility within a given transportation network. This in turn may have an influence on the time or cost of transportation. As each transportation network structure has certain unique features, it is important to perform an analysis of these structures in order to reveal these features and make the best use of them. This may influence the whole transportation system, of which a transportation network is an important part.

The issues presented in the article require further research. Of particular importance are aspects related to the method of dividing the transportation network into smaller parts, taking into account the network hierarchy. There is also a need to develop a comprehensive method of assessing the structure of a transportation network, in which other factors (e.g., traffic volume, area scale, level of aggregation) are taken into account.

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