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Safety and resilience of Global Baltic Network of Critical Infrastructure Networks related to cascading effects

Keywords

critical infrastructure, critical infrastructures network, cascading effect, impact, safety

Abstract

The main aim of the paper is analysis of safety of the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*), taking into account interactions among particular critical infrastructure networks this network consists of. The safety function and other safety characteristics: the risk function and mean values and the standard deviations of the lifetimes in the safety state subsets are determined assuming the particular critical infrastructure networks have exponential safety functions. Finally, the coefficients of cascading effect impact on the intensities of degradation of the *GBNCIN*, and the indicator of that network resilience to cascading effect impact, are presented.

1. Introduction

The Baltic Sea is the area covering relatively high concentration of different kind of installations, that in case of their malfunctions, can cause significant negative influence on societies and natural environment within the region and ashore around. Some of installations and systems, showing interconnections, interdependencies and interactions, can be qualified as Critical Infrastructure Networks (CI networks), defined as a set of interconnected and interdependent critical infrastructures, interacting directly and indirectly at various levels of their complexity and operating activity [EU-CIRCLE Report D1.1, EU-CIRCLE Taxonomy, 2015].

Works performed within the scope of EU-CIRCLE Report [EU-CIRCLE Report D1.2, 2016], allowed to distinguish eight CI networks operating in the Baltic Sea region. The networks, abbreviated as BCIN (Baltic Critical Infrastructure Networks), have been described and analysed in the report. Further, their operation process model has been developed and introduced in reports [EU-CIRCLE Report D3.3, 2016] and [Kołowrocki, et al., 2017].

Recognised networks, operating within the Baltic Sea region, and their interconnections and interdependencies, lead to qualify them as forming a Network of CI Networks. This resulted with a concept of the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*).

The article is aiming to analyse above mentioned interconnections and interdependencies, by introducing an approach to show influence of one of networks malfunctions, on the other networks. Basic assumption adopted is particular CI network departure from the safety state subsets, causes decreasing lifetimes of the other CI networks. The impact of each of distinguished CI networks, on the other CI networks, has been illustrated by specifying of respective coefficients.

Above mentioned assumptions allowed to determine basic safety indicators of the *GBNCIN*, in case one of CI networks malfunctions have influence on other CI networks functioning e.g. their lifetimes in the safety state subsets: the *GBNCIN* safety function, the *GBNCIN* risk function, the mean values and the standard deviations of the *GBNCIN* lifetimes in the safety state subsets and the intensities of the *GBNCIN* departure from the safety state subsets.

2. Global Baltic Network of Critical Infrastructure Networks and Its Safety Parameters

The *GBNCIN* is a network comprising eight ($n^{GBNCIN} = 8$) interconnected and interdependent Baltic Critical Infrastructure Networks (*BCINs*), distinguished and analysed in [Dziula, Kołowrocki, 2017a-b], [EU-CIRCLE Report D1.2, 2016]:

– *BECCIN* – the Baltic Electric Cable Critical Infrastructure Network E_1^{GBNCIN};
– *BGPCIN* – the Baltic Gas Pipeline Critical Infrastructure Network E_2^{GBNCIN};
– *BOPCIN* – the Baltic Oil Pipeline Critical Infrastructure Network E_3^{GBNCIN};
– *BWFCIN* – the Baltic Wind Farm Critical Infrastructure Network E_4^{GBNCIN};
– *BORCIN* – the Baltic Oil Rig Critical Infrastructure Network E_5^{GBNCIN};
– *BPCIN* – the Baltic Port Critical Infrastructure Network E_6^{GBNCIN};
– *BSCIN* – the Baltic Shipping Critical Infrastructure Network E_7^{GBNCIN};
– *BSTPOICIN* – the Baltic Ship Traffic and Port Operation Information Critical Infrastructure Network E_8^{GBNCIN}.

To assess the safety and resilience of the *GBNCIN*, it has been assumed that:

– all E_i^{GBNCIN}, $i = 1,2,...,8$, and the *GBNCIN* under consideration have the safety state set $\{0,1,...,z^{GBNCIN}\}$, $z^{GBNCIN} \geq 1$,
– the safety states are ordered, the safety state 0 is the worst and the safety state z^{GBNCIN} is the best,
– $T_i^{GBNCIN}(u)$, $i = 1,2,...,8$, are independent random variables representing the lifetimes of E_i^{GBNCIN}, in the safety state subset $\{u, u+1,..., z^{GBNCIN}\}$, while they were in the safety state z^{GBNCIN} at the moment $t = 0$,
– $T^{GBNCIN}(u)$ is a random variable representing the lifetime of a *GBNCIN* in the safety state subset $\{u, u+1,..., z^{GBNCIN}\}$, while it was in the safety state z^{GBNCIN} at the moment $t = 0$,
– the BCINs and the *GBNCIN* states degrades with time t,
– $s_i^{GBNCIN}(t)$, $i = 1,2,...,8$, is the E_i^{GBNCIN} safety state at the moment t, $t \in < 0, \infty)$, given that it was in the safety state z^{GBNCIN} at the moment $t = 0$,
– $s^{GBNCIN}(t)$ is the S^{GBNCIN} safety state at the moment t, $t \in < 0, \infty)$, given that it was in the safety state z^{GBNCIN} at the moment $t = 0$.

The above assumptions mean that the safety states of the *GBNCIN* with degrading *BCINs* may be changed in time only from better to worse [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011, 2012], [Xue, 1985], [Xue, Yang, 1995].

Further, following four safety states $(z^{GBNCIN} = 3)$, of the *GBNCIN* and each of *BCINs* have been distinguished:

– *GBNCIN* / *BCINs* state of full ability: z_3^{GBNCIN},
– *GBNCIN* / *BCINs* impendency over safety state: z_2^{GBNCIN},
– state of *GBNCIN* / *BCINs* unreliability: z_1^{GBNCIN},
– state of full inability of the *GBNCIN* / *BCIN*: z_0^{GBNCIN}.

The critical safety state of the *GBNCIN* and the *BCINs* is $r^{GBNCIN} = 2$.

The probability that the E_i^{GBNCIN} is in the safety state subset $\{u, u+1,..., z^{GBNCIN}\}$, at the moment t, $t \in < 0, \infty)$, while it was in the safety state $z^{GBNCIN} = 3$ at the moment $t = 0$, determined as the safety function of E_i^{GBNCIN}, is a vector

$$S_i^{GBNCIN}(t,\cdot) = [S_i^{GBNCIN}(t,0), S_i^{GBNCIN}(t,1), S_i^{GBNCIN}(t,2), S_i^{GBNCIN}(t,3)], \quad t \in < 0, \infty), \; i = 1,2,...,8, \tag{1}$$

where

$$S_i^{GBNCIN}(t,u) = P(s_i^{GBNCIN}(t) \geq u \mid s_i^{GBNCIN}(0) = 3) = P(T_i^{GBNCIN}(u) > t), \; u = 0,1,2,3. \tag{2}$$

Under this definition $S_i^{GBNCIN}(t,0) = 1$, $i = 1,2,...,8$.

Further, it is assumed that the *BCINs* have identical exponential safety functions

$$S_i^{GBNCIN}(t,\cdot) = [1, S_i^{GBNCIN}(t,1), S_i^{GBNCIN}(t,2), S_i^{GBNCIN}(t,3)], \quad t \in < 0, \infty), \; i = 1,2,...,8, \tag{3}$$

with the coordinates

$$S_i^{GBNCIN}(t,u) = \exp[-\lambda_i^{GBNCIN}(u)t], \; u = 1,2,3. \tag{4}$$

The *GBNCIN* will be analysed under the assumption it is a multistate series network. That means that the *GBNCIN* is in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, if and only if all its *BCIN* networks are in this subset of safety states. As it has been previously stated, there are four safety states and the best state $z^{GBNCIN} = 3$.

The *BCIN* lifetimes in the safety states are expressed in years and they have the exponential safety functions (3)-(4) with the intensities of departure from the safety subsets, by the assumption, given by

$$\lambda_i^{GBNCIN}(1) = 0.02, \ \lambda_i^{GBNCIN}(2) = 0.05,$$
$$\lambda_i^{GBNCIN}(3) = 0.1, \ i = 1,2,...,8. \tag{5}$$

3. Interactions between Baltic Critical Infrastructure Networks

Baltic Critical Infrastructure Networks are interconnected and interdependent [EU-CIRCLE Report D3.1, 2016]. They are interacting each other so as the *BCIN* departure from the safety state subsets causes decreasing lifetimes of the remaining *BCINs*. Then, if the network E_j^{GBNCIN} (j = 1,2,…,8) leaves the safety state subset {u,u+1,…,3} (u = 1,2,3), then the safety parameters of the remaining *BCIN* networks worsen depending on the type of the network E_j^{GBNCIN} with the coefficients of the *BCIN* impact on the other *BCINs*. Namely, the *BCINs* lifetimes and their mean values in the subset {υ,υ+1,…,3} (υ = u,u-1,…,1 and u = 1,2) decrease according to the formulas

$$T_{i/j}^{GBNCIN}(\upsilon) = [1 - q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN})] \cdot T_i^{GBNCIN}(\upsilon), \tag{6}$$

$$E[T_{i/j}^{GBNCIN}(\upsilon)]$$
$$= [1 - q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN})] \cdot E[T_i^{GBNCIN}(\upsilon)],$$
$$i = 1,2,\ldots,8, \ j = 1,2,\ldots,8, \tag{7}$$

where $q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN})$ are the coefficients of the network E_j^{GBNCIN} impact on the functioning of other networks E_i^{GBNCIN} (i = 1,2,…,8, $i \neq j$),

$$q(\upsilon, E_i^{GBNCIN}, E_i^{GBNCIN}) = 0, \ i = 1,2,...,8, \tag{8}$$

and

$$0 \le q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN}) < 1 \tag{9}$$

for i = 1,2,…,8, j = 1,2,…,8, υ = u, u–1,…,1 and u = 1,2.

These coefficients, existing in (6)-(7), take by the assumption following values

$$q(1, E_i^{GBNCIN}, E_1^{GBNCIN}) = 0.5,$$
$$q(2, E_i^{GBNCIN}, E_1^{GBNCIN}) = 0.2, \ i = 2,3,\ldots,8, \tag{10}$$

$$q(1, E_i^{GBNCIN}, E_2^{GBNCIN}) = 0.1,$$
$$q(2, E_i^{GBNCIN}, E_2^{GBNCIN}) = 0.05, \ i = 1,3,4,\ldots,8, \tag{11}$$

$$q(1, E_i^{GBNCIN}, E_3^{GBNCIN}) = 0.2,$$
$$q(2, E_i^{GBNCIN}, E_3^{GBNCIN}) = 0.1, \ i = 1,2,4,5,6,7,8, \tag{12}$$

$$q(1, E_i^{GBNCIN}, E_4^{GBNCIN}) = 0.03,$$
$$q(2, E_i^{GBNCIN}, E_4^{GBNCIN}) = 0.02, \ i = 1,2,3,5,6,7,8, \tag{13}$$

$$q(1, E_i^{GBNCIN}, E_5^{GBNCIN}) = 0.02,$$
$$q(2, E_i^{GBNCIN}, E_5^{GBNCIN}) = 0.01, \ i = 1,2,3,4,6,7,8, \tag{14}$$

$$q(1, E_i^{GBNCIN}, E_6^{GBNCIN}) = 0.35,$$
$$q(2, E_i^{GBNCIN}, E_6^{GBNCIN}) = 0.15, \ i = 1,2,3,4,5,7,8, \tag{15}$$

$$q(1, E_i^{GBNCIN}, E_7^{GBNCIN}) = 0.4,$$
$$q(2, E_i^{GBNCIN}, E_7^{GBNCIN}) = 0.2, \ i = 1,2,3,4,5,6,8, \tag{16}$$

$$q(1, E_i^{GBNCIN}, E_8^{GBNCIN}) = 0.5,$$
$$q(2, E_i^{GBNCIN}, E_8^{GBNCIN}) = 0.25, \ i = 1,2,\ldots,7, \tag{17}$$

and (8) holds.

Consequently, the safety function of E_i^{GBNCIN} (i = 1,…,8) network after the departure of E_j^{GBNCIN} (j = 1,2,…,8) from the subset {u,u+1,…,3} (u = 1,2,3), is defined as a vector

$$S_{i/j}^{GBNCIN}(t,\cdot)$$
$$= [1, S_{i/j}^{GBNCIN}(t,1), S_{i/j}^{GBNCIN}(t,2), S_{i/j}^{GBNCIN}(t,3)],$$
$$t \ge 0, \ i = 1,2,\ldots,8, \ j = 1,2,\ldots,8, \tag{18}$$

with the coordinates given by

$$S_{i/j}^{GBNCIN}(t,\upsilon) = P(T_{i/j}^{GBNCIN}(\upsilon) > t),$$
$$\upsilon = u, u-1,\ldots,1, \ u = 1,2, \tag{19}$$

$$S_{i/j}^{GBNCIN}(t,\upsilon) = P(T_{i/j}^{GBNCIN}(\upsilon) > t)$$
$$= P(T_i^{GBNCIN}(\upsilon) > t) = S_i^{GBNCIN}(t,\upsilon),$$
$$\upsilon = u+1,\ldots,3, \ u = 1,2. \tag{20}$$

Under the assumption about the exponential distribution, the conditional intensities of the E_i^{GBNCIN} network departure from the subset $\{\upsilon,\upsilon+1,\ldots,3\}$ after the departure of the E_j^{GBNCIN} network, by (7), are

$$\lambda_{i/j}^{GBNCIN}(\upsilon) = \frac{\lambda_i^{GBNCIN}(\upsilon)}{1-q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN})}, \qquad (21)$$

for $i = 1,2,\ldots,8,$ $j = 1,2,\ldots,8,$ $\upsilon = u, u-1,\ldots,1,$ $u = 1,2.$

Thus, considering (4), (18)-(20) and (21), the networks E_i^{GBNCIN} (i = 1,2,…,8) after the departure of E_j^{GBNCIN} (j = 1,2,…,8) from the safety subset have the safety functions (18) with the coordinates

$$S_{i/j}^{GBNCIN}(t,\upsilon) = \exp[-\frac{\lambda_i^{GBNCIN}(\upsilon)}{1-q(\upsilon, E_i^{GBNCIN}, E_j^{GBNCIN})}t],$$
$$\upsilon = u, u-1,\ldots,1,\ u = 1,2, \qquad (22)$$

$$S_{i/j}^{GBNCIN}(t,\upsilon) = \exp[-\lambda_i^{GBNCIN}(\upsilon)t],\ \upsilon = u+1,\ldots,3,$$
$$u = 1,2. \qquad (23)$$

4. Safety and Resilience of Global Baltic Network of Critical Infrastructure Networks

Assuming the *GBNCIN* is a multistate series network and dependence between *BCINs*, expressed in (6)-(7), in case the *BCINs* have exponential safety functions (3)-(4) and considering (22)-(23), the safety function of the *GBNCIN* related to cascading effects is given by the vector [Blokus-Roszkowska, Kolowrocki, 2017a-b]

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,\cdot)$$
$$= [1, \boldsymbol{S}_{CE}^{GBNCIN}(t,1), \boldsymbol{S}_{CE}^{GBNCIN}(t,2), \boldsymbol{S}_{CE}^{GBNCIN}(t,3)], \qquad (24)$$

where

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,1) = \exp[-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(2)t]$$
$$+\sum_{j=1}^{8}\frac{\lambda_j^{GBNCIN}(2)-\lambda_j^{GBNCIN}(1)}{\sum_{i=1}^{8}\lambda_i^{GBNCIN}(2)-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(1)}$$
$$\cdot[\exp[-\sum_{i=1}^{8}\frac{\lambda_i^{GBNCIN}(1)}{1-q(1, E_i^{GBNCIN}, E_j^{GBNCIN})}t]$$
$$-\exp[-(\sum_{i=1}^{8}\lambda_i^{GBNCIN}(2)-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(1)$$
$$+\sum_{i=1}^{8}\frac{\lambda_i^{GBNCIN}(1)}{1-q(1, E_i^{GBNCIN}, E_j^{GBNCIN})})t]], \qquad (25)$$

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,2) = \exp[-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(3)t]$$
$$+\sum_{j=1}^{8}\frac{\lambda_j^{GBNCIN}(3)-\lambda_j^{GBNCIN}(2)}{\sum_{i=1}^{8}\lambda_i^{GBNCIN}(3)-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(2)}$$
$$\cdot[\exp[-\sum_{i=1}^{8}\frac{\lambda_i^{GBNCIN}(2)}{1-q(2, E_i^{GBNCIN}, E_j^{GBNCIN})}t]$$
$$-\exp[-(\sum_{i=1}^{8}\lambda_i^{GBNCIN}(3)-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(2)$$
$$+\sum_{i=1}^{8}\frac{\lambda_i^{GBNCIN}(2)}{1-q(2, E_i^{GBNCIN}, E_j^{GBNCIN})})t]], \qquad (26)$$

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,3) = \exp[-\sum_{i=1}^{8}\lambda_i^{GBNCIN}(3)t], \qquad (27)$$

for $t \geq 0$.

Next, applying (24)-(27) and substituting the assumed values of *BCIN* intensities (5) and the abovementioned values of coefficients of the *BCIN* impact on other networks' functioning (10)-(17), the coordinates of the *GBNCIN* safety function are

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,1) = \exp[-0.4t] + \frac{1}{8}[\exp[-0.3t]$$
$$-\exp[-0.54t] + \exp[-0.176t] - \exp[-0.416t]$$
$$+\exp[-0.195t] - \exp[-0.435t] + \exp[-0.164t]$$
$$-\exp[-0.404t] + \exp[-0.163t] - \exp[-0.403t]$$
$$+\exp[-0.235t] - \exp[-0.475t] + \exp[-0.253t]$$
$$-\exp[-0.493t] + \exp[-0.3t] - \exp[-0.54t]], \qquad (28)$$

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,2) = \exp[-0.8t] + \frac{1}{8}[\exp[-0.488t]$$
$$-\exp[-0.888t] + \exp[-0.418t] - \exp[-0.818t]$$
$$+\exp[-0.439t] - \exp[-0.839t] + \exp[-0.407t]$$
$$-\exp[-0.807t] + \exp[-0.404t] - \exp[-0.804t]$$
$$+\exp[-0.462t] - \exp[-0.862t] + \exp[-0.488t]$$
$$-\exp[-0.888t] + \exp[-0.517t]$$
$$-\exp[-0.917t]], \qquad (29)$$

$$\boldsymbol{S}_{CE}^{GBNCIN}(t,3) = \exp[-0.8t]. \qquad (30)$$

The safety function coordinates of the *GBNCIN* related to cascading effects, given by (28)-(30), are illustrated in *Figure 1*.

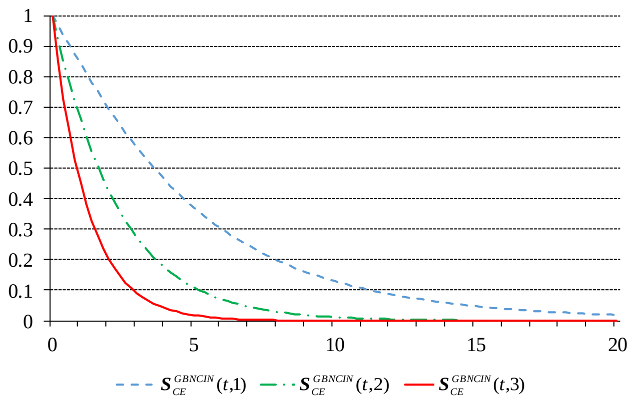

Figure 1. The graphs of the *GBNCIN* safety function coordinates.

If $r^{GBNCIN} = 2$ is the critical safety state, then the second safety indicator of the *GBNCIN* related to cascading effects is the risk function

$$\boldsymbol{r}_{CE}^{GBNCIN}(t) = P(T^{GBNCIN}(r) \le t), \quad t \in <0,\infty), \tag{31}$$

defined as a probability that the GBNCIN related to cascading effects is in the subset of safety states worse than the critical safety state $r^{GBNCIN} = 2$ while it was in the best safety state $z^{GBNCIN} = 3$ at the moment $t = 0$ [Kołowrocki 2014], [Kołowrocki, Soszyńska-Budny, 2011] and given by

$$\boldsymbol{r}_{CE}^{GBNCIN}(t) = 1 - \boldsymbol{S}_{CE}^{GBNCIN}(t,2), \; t \in <0,\infty), \tag{32}$$

where $\boldsymbol{S}_{CE}^{GBNCIN}(t,2)$ is the coordinate of the *GBNCIN* safety function given by (29).
Hence, the moment when the *GBNCIN* risk function exceeds a permitted level δ^{GBNCIN}, is

$$\tau_{CE}^{GBNCIN} = \boldsymbol{r}_{CE}^{GBNCIN^{-1}}(\delta^{GBNCIN}), \tag{33}$$

where $\boldsymbol{r}_{CE}^{GBNCIN^{-1}}(t)$, if exists, is the inverse function of the *GBNCIN* risk function $\boldsymbol{r}_{CE}^{GBNCIN}(t)$, given by (32). For the assumed value $\delta^{GBNCIN} = 0.2$, the moment of exceeding an acceptable level equals

$$\tau_{CE}^{GBNCIN} \cong 0.55 \text{ years} \cong 201 \text{ days}. \tag{34}$$

The graph of the *GBNCIN* risk function, called the fragility curve, is illustrated in *Figure 2*.

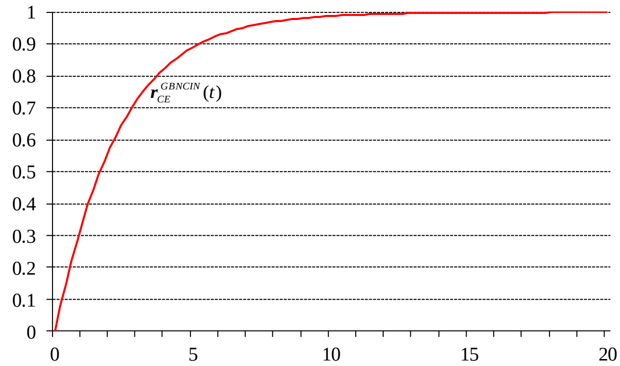

Figure 2. The graph of the *GBNCIN* risk function.

Other safety characteristics of the *GBNCIN* related to cascading effects are the mean values and the standard deviations of the *GBNCIN* lifetime in the safety state subsets {1,2,3}, {2,3}, {3}. In case the *BCINs* have exponential safety functions (3)-(4) and considering (22)-(23) for assumed model of dependency, the mean values can be counted from the formulae

$$\mu_{CE}^{GBNCIN}(1) = \frac{1}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)}$$
$$+ \sum_{j=1}^{8} \frac{\lambda_j^{GBNCIN}(2) - \lambda_j^{GBNCIN}(1)}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(1)}$$
$$\cdot [1 / \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(1)}{1 - q(1, E_i^{GBNCIN}, E_j^{GBNCIN})}$$
$$- 1/[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(1)$$
$$+ \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(1)}{1 - q(1, E_i^{GBNCIN}, E_j^{GBNCIN})}]], \tag{35}$$

$$\mu_{CE}^{GBNCIN}(2) = \frac{1}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3)}$$
$$+ \sum_{j=1}^{8} \frac{\lambda_j^{GBNCIN}(3) - \lambda_j^{GBNCIN}(2)}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)}$$
$$\cdot [1 / \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(2)}{1 - q(2, E_i^{GBNCIN}, E_j^{GBNCIN})}$$
$$- 1/[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)$$
$$+ \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(2)}{1 - q(2, E_i^{GBNCIN}, E_j^{GBNCIN})}]], \tag{36}$$

$$\mu_{CE}^{GBNCIN}(3) = \frac{1}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3)} \tag{37}$$

and the standard deviations from

$$\sigma_{CE}^{GBNCIN}(u) = \sqrt{n_{CE}^{GBNCIN}(u) - [\mu_{CE}^{GBNCIN}(u)]^2} \quad (38)$$

for $u = 1,2,$ where

$$n_{CE}^{GBNCIN}(1) = \frac{2}{[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)]^2}$$
$$+ 2\sum_{j=1}^{8} \frac{\lambda_j^{GBNCIN}(2) - \lambda_j^{GBNCIN}(1)}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(1)}$$
$$\cdot [1/[\sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(1)}{1 - q(1, E_i^{GBNCIN}, E_j^{GBNCIN})}]^2$$
$$- 1/[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(2) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(1)$$
$$+ \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(1)}{1 - q(1, E_i^{GBNCIN}, E_j^{GBNCIN})}]^2], \quad (39)$$

$$n_{CE}^{GBNCIN}(2) = \frac{2}{[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3)]^2}$$
$$+ 2\sum_{j=1}^{8} \frac{\lambda_j^{GBNCIN}(3) - \lambda_j^{GBNCIN}(2)}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)}$$
$$\cdot [1/[\sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(2)}{1 - q(2, E_i^{GBNCIN}, E_j^{GBNCIN})}]^2$$
$$- 1/[\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3) - \sum_{i=1}^{8} \lambda_i^{GBNCIN}(2)$$
$$+ \sum_{i=1}^{8} \frac{\lambda_i^{GBNCIN}(2)}{1 - q(2, E_i^{GBNCIN}, E_j^{GBNCIN})}]^2], \quad (40)$$

and

$$\sigma_{CE}^{GBNCIN}(3) = \frac{1}{\sum_{i=1}^{8} \lambda_i^{GBNCIN}(3)}. \quad (41)$$

According to (35)-(37) and substituting the values of coefficients (10)-(17) and intensities (5), the mean lifetimes of the *GBNCIN* in the subsets {1,2,3}, {2,3}, {3}, in years, are:

$$\mu_{CE}^{GBNCIN}(1) \cong 5.05, \quad \mu_{CE}^{GBNCIN}(2) \cong 2.30,$$
$$\mu_{CE}^{GBNCIN}(3) \cong 1.25. \quad (42)$$

Similarly, applying (38)-(41), the standard deviations of the *GBNCIN* lifetimes can be determined and their values in years are:

$$\sigma_{CE}^{GBNCIN}(1) \cong 4.96, \quad \sigma_{CE}^{GBNCIN}(2) \cong 2.25,$$
$$\sigma_{CE}^{GBNCIN}(3) \cong 1.25. \quad (43)$$

The mean values of the *GBNCIN* lifetimes in the particular states 1,2,3, by (42), in years are:

$$\bar{\mu}_{CE}^{GBNCIN}(1) \cong 2.75, \quad \bar{\mu}_{CE}^{GBNCIN}(2) \cong 1.05,$$
$$\bar{\mu}_{CE}^{GBNCIN}(3) \cong 1.25. \quad (44)$$

Other *GBNCIN* safety indices are the intensities of departure from the safety state subsets {1,2,3}, {2,3}, {3}, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,\cdot) = [0, \boldsymbol{\lambda}_{CE}^{GBNCIN}(t,1), \ \boldsymbol{\lambda}_{CE}^{GBNCIN}(t,2),$$
$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,3)], \ t \geq 0. \quad (45)$$

These intensities can be determined from the formula

$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,u) = \frac{-\frac{d\boldsymbol{S}_{CE}^{GBNCIN}(t,u)}{dt}}{\boldsymbol{S}_{CE}^{GBNCIN}(t,u)}, \ u = 1,2,3, \quad (46)$$

where $\boldsymbol{S}_{CE}^{GBNCIN}(t,u), \ u = 1,2,3,$ are given by (28)-(30), and after some transformation they take form

$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,1) = \{0.4 + 0.125 \cdot [0.6\exp[0.1t]$$
$$- 1.08\exp[-0.14t] + 0.176\exp[0.224t]$$
$$- 0.416\exp[-0.016t] + 0.195\exp[0.205t]$$
$$- 0.435\exp[-0.035t] + 0.164\exp[0.236t]$$
$$- 0.404\exp[-0.004t] + 0.163\exp[0.237t]$$
$$- 0.403\exp[-0.003t] + 0.235\exp[0.165t]$$
$$+ 2.53\exp[1.47t] - 4.93\exp[-0.93t]]\}$$
$$- 0.475\exp[-0.075t] \ /\{1 + 0.125 \cdot [2\exp[0.1t]$$
$$- 2\exp[-0.14t] + \exp[0.224t] - \exp[-0.016t]$$
$$+ \exp[0.205t] - \exp[-0.035t] + \exp[0.236t]$$
$$- \exp[-0.004t] + \exp[0.237t] - \exp[-0.003t]$$
$$+ \exp[0.165t] - \exp[-0.075t] + \exp[0.147t]$$
$$- \exp[-0.093t]]\}, \ t \geq 0, \quad (47)$$

$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,2) = \{0.8 + 0.125 \cdot [0.976\exp[0.312t]$$
$$- 1.776\exp[-0.088t] + 0.418\exp[0.382t]$$
$$- 0.818\exp[-0.018t] + 0.439\exp[0.361t]$$
$$- 0.839\exp[-0.039t] + 0.407\exp[0.393t]$$
$$- 0.807\exp[-0.007t] + 0.404\exp[0.396t]$$
$$- 0.804\exp[-0.004t] + 0.462\exp[0.338t]$$
$$- 0.862\exp[-0.062t] + 0.517\exp[0.283t]$$
$$- 0.917\exp[-0.117t]]\} \ /\{1 + 0.125 \cdot [2\exp[0.312t]$$

$$-2\exp[-0.088t]+\exp[0.382t]-\exp[-0.018t]$$
$$+\exp[0.361t]-\exp[-0.039t]+\exp[0.393t]$$
$$-\exp[-0.007t]+\exp[0.396t]-\exp[-0.004t]$$
$$+\exp[0.338t]-\exp[-0.062t]+\exp[0.283t]$$
$$-\exp[-0.117t]]\},\ \ t\geq 0, \quad (48)$$

$$\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,3)=0.8,\ t\geq 0, \quad (49)$$

and their graphs are illustrated in *Figure 3*.

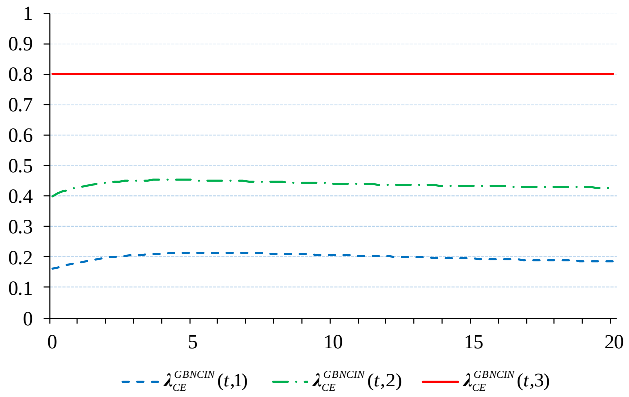

Figure 3. The graphs of the *GBNCIN* intensities.

Using these intensities, the coefficients of cascading effect impact on the *GBNCIN* intensities of departure from the safety state subsets {1,2,3}, {2,3}, {3}, can be estimated. Then, the coordinates of the vector

$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,\cdot)=[0,\boldsymbol{\rho}_{CE}^{GBNCIN}(t,1),\ \boldsymbol{\rho}_{CE}^{GBNCIN}(t,2),$$
$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,3)],\ t\geq 0, \quad (50)$$

are given by

$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,u)=\frac{\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,u)}{\boldsymbol{\lambda}_{0}^{GBNCIN}(t,u)},\ u=1,2,3, \quad (51)$$

where $\boldsymbol{\lambda}_{0}^{GBNCIN}(t,u),\ u=1,2,3,$ are the intensities of the departure from the safety state subset {1,2,3}, {2,3}, {3}, without of cascading effect impact.
Since the *GBNCIN* is considered as a series network, the intensities of the *GBNCIN* departure without of cascading effect impact are

$$\boldsymbol{\lambda}_{0}^{GBNCIN}(t,u)=8\cdot\lambda_{i}^{GBNCIN}(u),\ t\geq 0,\ u=1,2,3, \quad (52)$$

and from (5) they take values

$$\boldsymbol{\lambda}_{0}^{GBNCIN}(t,1)=0.16,\ \boldsymbol{\lambda}_{0}^{GBNCIN}(t,2)=0.4,$$
$$\boldsymbol{\lambda}_{0}^{GBNCIN}(t,3)=0.8. \quad (53)$$

Then, applying (51) and from (47)-(49) and (53), the coefficients of the cascading effect impact on the *GBNCIN* intensities of degradation are

$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,1)=6.25\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,1),$$
$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,2)=2.50\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,2),$$
$$\boldsymbol{\rho}_{CE}^{GBNCIN}(t,3)=1,\ t\geq 0, \quad (54)$$

where $\boldsymbol{\lambda}_{CE}^{GBNCIN}(t,1),\ \boldsymbol{\lambda}_{CE}^{GBNCIN}(t,2),$ are respectively given by (47) and (48).
The indicator of the *GBNCIN* resilience to cascading effect impact is defined by

$$\boldsymbol{RI}_{CE}^{GBNCIN}(t,r^{GBNCIN})=\frac{1}{\boldsymbol{\rho}_{CE}^{GBNCIN}(t,r^{GBNCIN})},\ t\geq 0, \quad (55)$$

where $\boldsymbol{\rho}_{CE}^{GBNCIN}(t,r^{GBNCIN})$ is the coefficient of cascading effect impact on the *GBNCIN* intensities of degradation given by (54) and the *GBNCIN* critical safety state is $r^{GBNCIN}=2$. Respective graph is illustrated in *Figure 4*.

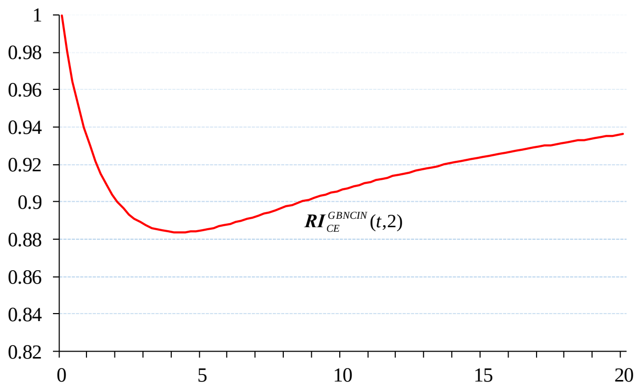

Figure 4. The graph of the *GBNCIN* resilience indicator.

Finally, the stationary coefficient of cascading effect impact on the *GBNCIN* intensities of departure from the safety state subset not worse than a critical safety state $r^{GBNCIN}=2$ i.e. {2,3}, is determined using (42) and (53)

$$\tilde{\boldsymbol{\rho}}_{CE}^{GBNCIN}(2)=\frac{\mu_{0}^{GBNCIN}(2)}{\mu_{CE}^{GBNCIN}(2)}=\frac{2.50}{2.30}\cong 1.087 \quad (56)$$

and the stationary indicator of the *GBNCIN* resilience to cascading effect impact for a critical safety state $r^{GBNCIN}=2$, by (56), takes value

$$\tilde{\boldsymbol{RI}}_{CE}^{GBNCIN}(2) = \frac{1}{\tilde{\boldsymbol{\rho}}_{CE}^{GBNCIN}(2)} \cong 0.92 = 92\%. \qquad (57)$$

5. Conclusions

Analysis of safety of the *GBNCIN*, taking into account interactions of particular *BCINs*, has allowed to determine the *GBNCIN* safety function and other safety characteristics: the risk function, the mean values and standard deviations of lifetimes in the safety state subsets, the coefficients of cascading effect impact on the intensities of degradation of the Global Baltic Network of Critical Infrastructure Networks, and the indicator of that Network resilience to cascading effect impact.

Future researches in this field will include influence of climate-weather change process, and the operation process of the *GBNCIN*. Moreover, researches results will be enhanced with consideration of circumstances when malfunctions of one of networks have various influence on the other networks.

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