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Computation of SIFs for cracked FGMs under mechanical and thermal loadings

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ABSTRACT

The objective of this study is to present a numerical modeling of mixed-mode fracture in isotropic functionally graded materials (FGMs), under mechanical and thermal loading conditions. In this paper, a modified displacement extrapolation technique (DET) was proposed to calculate the stress intensity factor (SIFs) for isotropic FGMs. Using the Ansys Parametric Design Language APDL, the continuous variations of the material properties are incorporated by specified parameters at the centroid of each element. Three numerical examples are presented to evaluate the accuracy of SIFs calculated by the proposed method. Comparisons have been made between the SIFs predicted by the DET and the available reference solutions in the current literature. A good agreement is obtained between the results of the DET and the reference solutions.

1. INTRODUCTION

Functionally graded materials (FGMs) are nonhomogeneous composites that possess continuous variations in the thermomechanical properties. Due to their potential usage in high temperature applications as protective coatings and interlayers, fracture mechanics and thermal stress analyses of FGMs have been considered by many researchers in the past. Various techniques have been developed in order to study the behavior of cracks in FGMs under mechanical and thermal loading conditions.

For crack problems subjected to mechanical loading, Anlas et al. [1] have evaluated SIFs in FGMs for an edge-cracked plate under uniform mechanical loading, using both the strain energy release rate and the J-contour integral. Rao and Rahman [2] present a Galerkin-based meshless method for calculating SIFs for a stationary crack in two-dimensional FGMs of arbitrary geometry. Kim and Paulino [3-5] extended various finite elements based approaches for fracture mechanics analysis of FGMs such as modified crack closure method, mixed-mode J-integral and interaction integral. Mirahmadi et al. [6] investigate the calculation of stress intensity factor for functionally graded cylinders with two radial cracks using the weight function method. Gu et al. [7] have proposed a finite element based method for calculating SIFs of graded materials, using the equivalent domain integral (EDI) technique. Guo et al. [8] considered mode I crack problems in a finite width graded orthotropic strip under static loading. Boulenouar [9] evaluated the mixed mode SIFs for FGM circular disk by means of the displacement correlation technique (DCT). Hebbar et al. [10] analyzed the fracture problems under mechanical loading, using FGMs for circular disk and three-point bending specimen. Shojaee and Daneshmand [11] applied the extended isogeometric analysis with orthotropic approach for numerical modeling of stationary cracks in FGM plane bodies. Martinez-Paneda and Gallego [12] evaluated the performance of numerical tools in the computational

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assessment of cracks in FGMs by means of the well-known ABAQUS FE code. This analysis is based on computational results of fracture parameters SIFs and T-stress. Benamara et al. [13-14] performed mixed-mode crack propagation in FGMs subjected to mechanical loads using the finite element method (FEM). Chafi and Boulenouar [15] performed numerical investigation on crack growth in FGMs beam using the maximum circumferential stress criterion. Eskandari[16] analyzed the SIFs for crack located at an arbitrary position in rotating FGM disks, using DCT and J-integral methods.

In the area of crack problems in FGMs under thermal loads, many researchers are considered using different approaches: Garg and Pant [17] modified and implemented the element-free Galerkin method (EFGM) to simulate thermoelastic fracture in FGMs. Shafiei et al. [18] extended nonlinear thermal buckling of axially functionally graded micro and nanobeams. Azimi et al. [19] present vibration of rotating functionally graded Timoshenko nano-beams with nonlinear thermal distribution. Yildirim [20] investigated the equivalent domain integral method to evaluate the SIFs in FGM under steady-state and transient thermal loading conditions. Dag [21] developed the computational method based on the Jk-integral in order to calculate crack tip parameters for FGMs, subjected to mixed-mode thermal loading. Yildirim et al. [22] analyzed the 3D surface crack problems in functionally graded coatings subjected to mode-I mechanical and transient thermal loadings. Chen et al. [23] analyzed the influence of nonhomogeneity on the standard J-integral and defines a modified J-integral for cracked FGM. KC and Kim [24] evaluated of the non-singular T-stress and mixed-mode SIFs in FGMs under steady-state thermal loads by means of interaction integral. Rangaraj and Kokini [25] investigated the two-dimensional finite element models with a cohesive zone to study quasi-static crack extension in functionally graded thermal barrier coatings (TBC). Jin and Paulino [26] considered an edge crack in a strip of a FGM to calculate the thermal stress intensity factors (TSIFs) under transient thermal loading conditions. Dag et al. [27] introduced a computational method based on the Jk-integral for mixed-mode fracture analysis of orthotropic FGMs subjected to thermal stresses. Yildirim and Erdogan [28] have used the enriched element technique to evaluate mixed-mode SIFs under uniform thermal loading. Kosker et al. [29] investigated three dimensional FEM in order to evaluate the mixed-mode SIFs around the front of an inclined semi-elliptical crack located in an FGM coating, using the displacement correlation technique under the effect of transient thermal stresses. Dag [30] proposed a new computational method based on the equivalent domain integral for mode-I fracture analysis in orthotropic FGMs subjected to thermal stresses.

Some researchers examined the crack problems in FGMs under thermomechanical loading conditions. In this direction, thermo-mechanical vibration of rotating axially functionally graded nonlocal Timoshenko beam by Azimi et al [31] are considered. Jain et al. [32] developed quasi-static stress and displacement fields for a crack in an infinite FGM medium under thermomechanical loading. Kidane et al. [33] developed the stress fields near the crack tip for mixed-mode crack propagation under thermomechanical loading in FGM. Nami and Eskandari [34] investigated 3D-FEM to evaluate the SIFS for semi-elliptical circumferential surface crack in FGM cylinder subjected to internal pressure and temperature gradient. Takabi [35] presented an analytical and a numerical thermomechanical investigation of a thick-walled cylinder made of the FGMs, subjected to a pressure and a thermal load. Matthew et al. [36] developed a general domain integral method to obtain J-values along crack fronts in three-dimensional configurations of isotropic FGMs, subjected to thermomechanical loading. Moghaddam et al. [37] analyzed the mixed mode SIFs of three-dimensional curved non-planar cracks in FGMs, using the interaction energy integral. The FEM is employed to extract the SIFs along the front of the lens shaped crack in an FGM. Lee et al. [38] developed analytical expressions for dynamic crack-tip stress and displacement fields under thermo-mechanical loading in FGM. Zhang et al. [39] exploited the numerical manifold method (NMM) to study the fracture behavior of two-dimensional FGMs subjected to thermo-mechanical loadings. Firstly, the steady-state heat conduction simulation of the cracked FGMs is performed, and then the computed temperatures are input into the thermoelastic modeling. Moghaddam and Alfano [40] deployed the interaction energy integral in the finite element framework and carried out an un-coupled thermomechanical analysis to extract the mixed-mode SIFs for surface cracks in FGM hollow cylinders. Mahbadi [41] estimated SIFs of rotating solid disks in isotropic functionally graded with a radial crack subjected to a uniform tension at their outer surface and a uniform temperature change through the body. Abotula et al. [42] studied mixed-mode dynamic crack growth in FGMs under thermomechanical loading.

The objective of this study is to present a numerical modeling of mixed-mode fracture in FGMs. Using the APDL code [43], the displacement extrapolation technique (DET) is used to determine numerically the SIFs for isotropic FGMs subjected to mechanical and thermal loading conditions. In this paper, three numerical examples including both mode-I and mixed-mode problems are presented to evaluate the SIFs calculated using proposed method. Comparisons have been made between the SIFs predicted by displacement extrapolation technique DET and available reference solutions in the literature.

The paper consists of four sections. Besides this introduction, Section 2 presents the numerical evaluation of SIFs using displacement extrapolation method. In Section 3, we present several numerical examples to examine the accuracy and performance of the displacement extrapolation technique in evaluating mixed-mode SIFs for isotropic FGMs subjected to thermal and mechanical loading conditions. Finally, the major conclusions are summarized in Section 4.

2. NUMERICAL EVALUATION OF SIFS

Among the computational methods developed to study the fracture problems for homogeneous and non-homogeneous materials, we can mention modified crack closure method [3], mixed-mode J-integral [4], interaction integral [5], equivalent domain integral [44], displacement correlation technique DCT [45] and generalized displacement correlation method [46]. In this work, the displacement extrapolation technique DET proposed for homogeneous materials is modified to calculate the SIFs for isotropic FGMs, as follows [47-50]:

$$K_I = \frac{E_{tip}}{3\,(1+\upsilon_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\left[4\left(v_j - v_l\right) - \frac{(v_k - v_m)}{2}\right] \quad (1)$$

$$K_{II} = \frac{E_{tip}}{3\,(1+\upsilon_{tip})(1+k_{tip})}\sqrt{\frac{2\pi}{L}}\left[4\left(u_j - u_l\right) - \frac{(u_k - u_m)}{2}\right] \quad (2)$$

where:

E_{tip} and ν_{tip} are the Young's modulus and the Poisson's ratio given at the crack-tip.

$k_{tip}=(3-\nu_{tip})/(1+\nu_{tip})$ for plane stress and $k_{tip}=(3-4\nu_{tip})$for plane strain.

L is the length of the singular element side.

u_n and v_n(n=j, k, l and m) are the nodal displacements at nodes j, k, l and m in the x and y directions, respectively.

In this work, the special quarter point finite elements were proposed to obtain a better approximation of the field around the crack-tip (Fig. 1), where the mid-side node of the element connected to the crack-tip is moved to (1/4) of the length of this element [51-53].

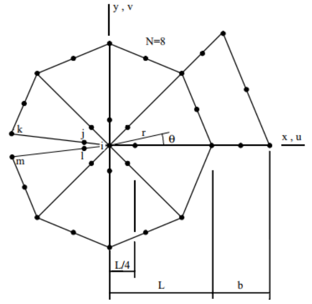


Fig. 1. Singular element used for present study

3. NUMERICAL RESULTS AND DISCUSSION

The performance of the extrapolation technique for SIFs evaluation in isotropic FGMs subjected to mechanical and thermal loading conditions is examined by means of numerical examples. The following examples are presented:

1) Three-Point bending specimen with crack parallel to material gradation, subjected to mechanical loading.
2) FGM disk with an inclined center crack, subjected to mechanical loading.
3) Crack in functionally graded thermal barrier coating.

In this study, the Ansys Parametric Design Language has been investigated for creating the subroutine to simulate the fracture of isotropic FGMs, mechanical and thermal loadings. An example of the implementation is given in Appendix section.

3.1. Three-Point bending specimen with crack parallel to material gradation

In this example, three-point bend specimen are considered with length L=54 units, depth 2H=10 units, and thickness t=1unit. A crack of length (a) is assumed to initiate parallel to the material gradation as shown in Fig.2.

A concentrated load P=1 unit was applied at the middle of the beam and two supports were symmetrically placed with respect to an edge crack of length a. The three-point bend specimen consists of 2h units deep FGM sandwiched between two distinct homogeneous materials, each of which has depth (H-h).

The variation of Young's modulus in the material gradient region is linear, expressed by [9]:

$$E(x_2) = Ax_2 + B \quad (3)$$

$$\text{with: } \begin{cases} A = \frac{(E_2 - E_1)}{2h} \\ B = \frac{(E_2 + E_1)}{2} \end{cases} \quad (4)$$

The Poisson's ratio ν is assumed to be constant, where E_1, E_2 and 2h are material parameters. The following data were used in the present analysis:2h =1 unit, E_1=1 unit, and E_2/E_1=0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, and 20. For each E_2/E_1 ratio, three different crack lengths with a/2H=0.45, 0.5 and 0.55 were examined such that the crack tips were either at the middle of the FGM layer (a/2H =0.5) or at the material interfaces (a/2H =0.45 or 0.55), as shown in Fig. 3a and Fig. 3b. The structure considered is meshed by quadratic elements with 8 nodes and particularly, special elements were used to characterize the singularity around the crack-tip. The number of elements used in this analysis is 841 elements with 2248 nodes (for a/2H=0.45).

The determination of stress intensity factors $K_I$for three crack sizes (a=4.5, 5 and 5.5 units) is performed under plane stress condition. Table 1 compares the normalized mode-I SIF$\left(\frac{K_I\sqrt{H}}{P}\right)$ obtained by present technique for various combinations of E_2/E_1 and a/2H, with the FEM results reported by Kim and Paulino [54] using J*-Integral method and the results obtained by Rao and Rahman [2] using modified interaction integrals based on element-free Galerkin method (EFGM). Good agreements are observed between three approaches.

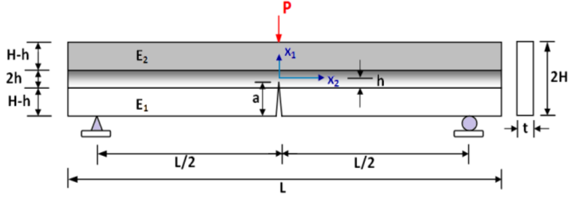
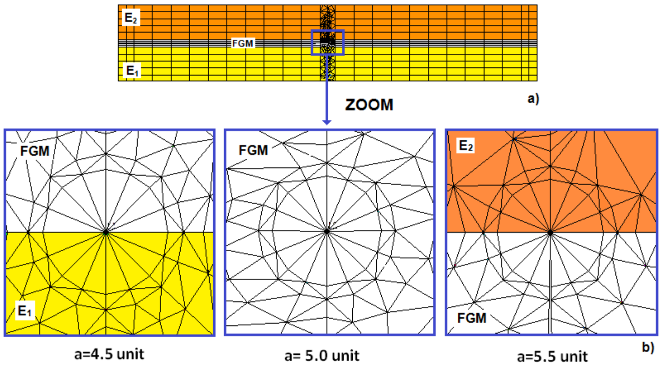


Fig. 2. Three-point bend specimen with crack parallel to material gradation



a=4.5 unit a= 5.0 unit a=5.5 unit

Fig. 3. FE mesh for three-point bending specimen: (a) Global FE mesh, (b) Detail of the mesh around the crack tip

Table 1. Normalized SIF for three-Point bending specimen

E_2/E_1	a/2H=0.45		
	Present study	Rao and Rahman[2]	Kim and Paulino[54]
0.1	23.69	23.61	23.47
0.2	17.51	17.28	17.36
0.5	11.73	11.45	11.65
1	8.18	7.95	8.13
2	5.23	5.15	5.23
5	2.50	2.51	2.54
10	1.28	1.31	1.33
20	0.61	0.65	0.66

E_2/E_1	a/2H=0.50		
	Present study	Rao and Rahman[2]	Kim and Paulino[54]
0.1	24.89	23.96	23.92
0.2	19.04	18.36	18.32
0.5	13.08	12.30	12.57
1	9.85	9.20	9.46
2	7.61	7.33	7.31
5	5.71	5.46	5.49
10	4.77	4.61	4.58
20	4.11	3.98	3.93

E_2/E_1	a/2H=0.55		
	Present study	Rao and Rahman[2]	Kim and Paulino[54]
0.1	13.07	13.40	13.73
0.2	12.48	12.16	12.79
0.5	11.69	11.29	11.76
1	11.14	10.85	11.15
2	10.63	10.44	10.62
5	9.97	9.93	9.96
10	9.52	9.58	9.50
20	9.13	9.27	9.12

3.2. FGM disk with an inclined center crack

We consider a circular FGM disk with a center crack inclined by θ=30° (Fig. 4a). The disk is meshed by quadratic and triangular elements, as shown in (Fig. 4b). A special mesh is used to characterize the singularity around the two crack-tips (Fig. 4c). FGM disk was meshed by 2688 elements with 6180 nodes.

The FGM Disk is considered under plane stress condition and the variation of Young's modulus along the radial direction is given as follows:

$$E(r) = \bar{E}e^{\beta r} \tag{5}$$

r is the disk radius, with $r = \sqrt{X_1^2 + X_2^2}$, $X_1$and X_2 are cartesian coordinates.

A point load P = ± 100 units is applied to the top and the bottom of the disk, at the coordinate nodes (0, ± 10), respectively.

The displacement boundary conditions are defined as follows: $(X_1, X_2) = (-10, 0)$, $(u_1, u_2) = (0, 0)$, $(X_1, X_2) = (10, 0)$ and $u_2 = 0$.

Fig. 4 shows the applied boundary conditions used for this example. The determination of stress intensity factors K_I is performed under plane stress condition, for following numerical values:
a=1, R=10, βa= (-0.5, -0.25, 0, 0.25, 0.5), $\bar{E}$=1, ν=0.3

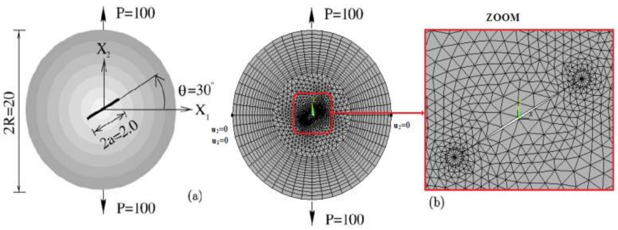


Fig. 4. (a) Geometry of FGM disc with a central crack, (b) Complete mesh configuration (c) Detailed mesh around the two crack-tips

Table 2 presents FEM results for the mode-I SIF obtained by present approach for various values of βa, with those reported by other methods using M-integral method [5] and modified crack closure method (MCC)[3]. There is a good agreement between present evaluation of SIFs results and the other available reference in the literature.

The results obtained in examples 1 and 2 allow us to conclude that the displacement extrapolation technique modified for non-homogeneous materials, correctly described the stress-strain field around the crack-tip, for plates subjected to mechanical loading.

Table 2. Mode- I SIF for an inclined center crack in a circular FGM disk

	M-Integral [5]		**MCC [3]**		**Present study**	
β	K_I	K_{II}	K_I	K_{II}	K_I	K_{II}
-0.50	22.91	15.19	22.54	14.76	22.95	14.84
-0.25	17.53	13.21	17.37	12.92	17.56	12.94
0	11.47	9.73	11.45	9.596	11.48	9.57
0.25	5.862	5.651	5.898	5.602	5.86	5.56
0.50	2.205	2.417	2.236	2.412	2.21	2.39

3.3. Crack in functionally graded thermal barrier coating TBC

In the present problem, an edge crack in functionally graded thermal barrier coating under thermal loading has been modeled and analyzed using the present technique. Fig. 6 shows a functionally graded thermal barrier coating deposited on the bond coat and the metallic substrate. The FGM coating consists of 100% zirconium-yttria at $X_1=0$ and 100%nickel-chromium-aluminium-zirconium(NiCrAlY) bond coat at $X_1=W_1$. The metallic substrate is made up of a nickel-based super-alloy.

The dimensions of FGM TBC along with thermal loading and boundary conditions are shown in Fig. 5. Initially the system is assumed to be at a uniform temperature (T_0 =1000° C).

The top and bottom edges of the TBC system are assumed to be insulated. Application of temperature boundary conditions drives the system to a steady state condition with temperature $T_1 =0.2\times T_0$ and $T_2=0.5\times T_0$ at left and right edges, respectively.

Material property variations of Young's modulus E, Poisson's ratio ν and thermal expansion coefficient α for the FGM coating are represented by power-law type functions, which are given as follows:

$$E(x_1) = E_c + (E_{Bc} - E_c)x^2 \tag{6}$$

$$\nu(x_1) = \nu_c + (\nu_{Bc} - \nu_c)x \tag{7}$$

$$\alpha(x_1) = \alpha_c + (\alpha_{Bc} - \alpha_c)x \tag{8}$$

$$k(x_1) = k_c + (k_{Bc} - k_c)x^2 \tag{9}$$

Where subscript (c) denotes FGM coating and subscript (Bc) symbolizes the bond coat. The thermomechanical properties of different constituents of TBC are listed in table 3. Fig. 6 shows complete mesh configuration with mesh detail around the crack-tip.

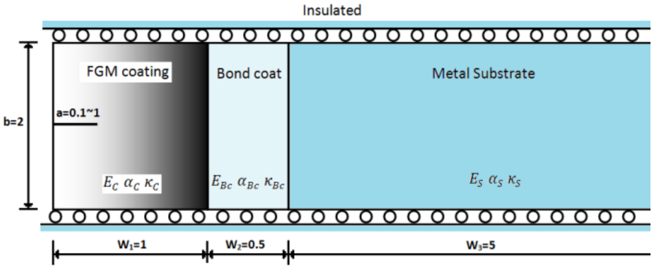


Fig. 5. Crack in a functionally graded thermal barrier coating

Figure 7 compares the variation of mode-I SIF for various a/W ratios in a FGM TBC using present approach with those reported by Garg and Pant [55] using element-free Galerkin method (EFGM) and by KC and Kim [24] using interaction integral method (M-integral); excellent agreement was observed between the three techniques.

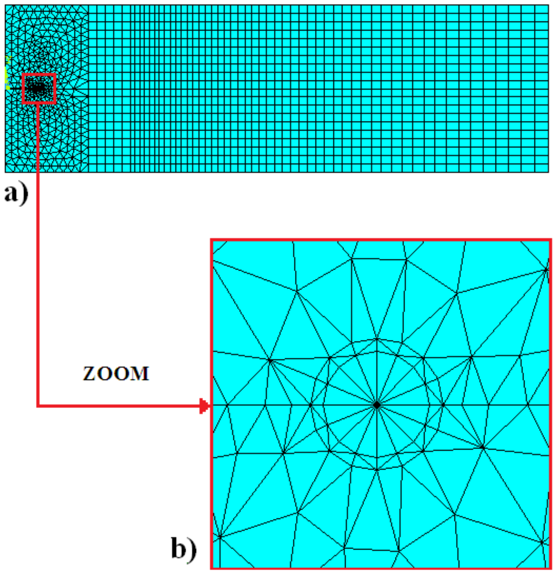


Fig. 6(a) Complete mesh configuration in TBC

(b) Detailed mesh around the crack-tip

Table 3. Thermomechanical properties of TBC constituent

Material	E (GPa)	ν	α ($°C^{-1}$)	k ($W/m^{-1}K^{-1}$)
Zirconia-Yttria	27.6	0.25	10.01× 10^{-6}	1
Bondcoat (NiCrAlY)	137.9	0.27	15.16× 10^{-6}	25
Substrate(Ni)	175.8	0.25	13.91× 10^{-6}	7

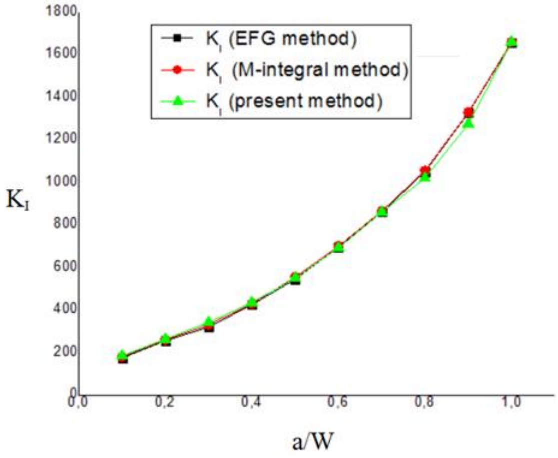


Fig. 7. Variations of SIF K_I along the graded region

In Fig. 8, we have shown the temperature distribution in graded TBCs (for a/W=0.4). It can be clearly seen that the temperature field remains unaffected by the presence of crack and the heat flux is parallel to the crack surface, along x-direction.

The temperature distribution obtained by the present study was compared with that obtained by Garg and Pant [55]. A good agreement is observed with EFG method solution.

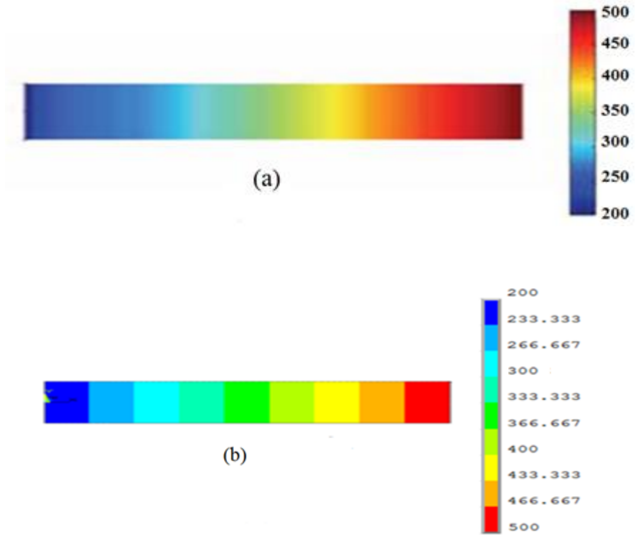


Fig. 8. Temperature distribution in functionally graded:

(a) Garg and Pant [55], (b) Present study

4. CONCLUSION

In this paper, the displacement extrapolation technique used for homogeneous materials has been modified and proposed to determine numerically the SIFs for isotropic FGM. The present method is investigated to analyze the mixed mode fracture problems under mechanical and steady-state thermal loads. In order to obtain a better approximation of the field near the crack tip in graded region, the special quarter point finite elements is investigated.

This paper presents various numerical examples in which the accuracy of the present method is verified. A comparison of SIF values predicted by FEM and available reference solutions generated numerically reveals the applicability of the present technique. This approach has been successfully used in evaluating SIFs in FGMs under mechanical loading and has also been used in the evaluation of mixed-mode SIFs in FGMs under thermal loadings.

The simplicity and accuracy of this implementation show that it can be further extended with the study and analysis of the fracture in graded materials with multi-loading and complex geometry conditions.

Appendix: Example for user subroutine for FGMs under thermal loading

```
/prep7

Ex=Ec+((Ebc-Ec)*(i**2))

mux=muc+((mubc-muc)*i)

bettax=bettac+((bettabc-bettac)*i)

kx=kc+((kbc-kc)*(i**2))

!-----------------------------------------------

mp,ex,h,Ex

mp,nuxy,h,mux

!-----------------------------------------------

MPTEMP,,,,,,,,

MPTEMP,0,0

MPdata,KXX,h,,kx

UIMP,h,REFT,,,T0

MPdata,ALPX,h,,bettax

/MPLIB,STAT

MPCHG,H,H,

!*enddo

FLST,2,2,4,ORDE,2

FITEM,2,3

FITEM,2,10

/GO


DL,P51X, ,TEMP,200,0

FLST,2,2,4,ORDE,2

FITEM,2,14

FITEM,2,-15

/GO

DL,P51X, ,TEMP,500,0

SOLVE

FINISH

!--------------------------------------------------
```

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