HYBRID MODEL OF GEARED ROTOR SYSTEM

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Summary

In the paper a hybrid model of a geared multirotor system has been developed. The model is obtained by application of both the modal decomposition methodology and the spatial discretization method. Reduced modal model was constructed for the system without gyroscopic and damping effects. The gyroscopic interaction, damping and other phenomena which are difficult to include in the modal approach were modeled by application of simply lumping technique. Such approach enables to obtain accurate, low order model of geared rotor system with coupled bending and torsional vibrations. In the model it is possible to include nonproportional or nonlinear damping. Obtained hybrid model has been compared with high order FEM model. Simulation results prove that proposed method of modeling is efficient and relatively easy to use.

Keywords: mechanical system, modelling, vibration, modal analysis, model reduction.

HYBRYDOWY MODEL UKŁADU WIRNIKÓW Z PRZEKŁADNIĄ

Streszczenie

W artykule przedstawiono hybrydowy model układu wielowirnikowego z przekładnią. Otrzymano go stosując dwie metody: dekompozycji modalnej oraz dyskretyzacji przestrzennej. Zredukowany model modalny zbudowano dla układu bez efektu żyroskopowego i tłumienia. Oddziaływania żyroskopowe, tłumienie oraz inne zjawiska, które są trudne do uwzględnienia w modelu modalnym modelowano stosując metodę elementów skończonych. Takie podejście umożliwia otrzymanie dokładnego modelu niskiego rzędu uwzględniającego sprzężone drgania giętno-skrętne. W modelu można uwzględnić nieproporcjonalne lub nieliniowe tłumienie. Skonstruowany model hybrydowy został porównany z modelem referencyjnym wysokiego rzędu otrzymanym metodą sztywnych elementów skończonych. Wyniki symulacji potwierdzają skuteczność zastosowanej metody.

Słowa kluczowe: układy mechaniczne, modelowanie, drgania, analiza modalna, redukcja modeli.

1. INTRODUCTION

Rotor systems are constructed from components, some of them are lumped parameter elements and others distributed ones. Such systems are composed of rigid disks mounted on a flexible shafts [3]. Avoiding the mathematical difficulties arising from the manipulation of sets of mixed ordinary and partial differential equations, different approximate lumped models of distributed-lumped systems are usually applied. By using the finite-element method it is possible to obtain an accurate model and final results. However, obtaining a sufficiently accurate result requires a very fine mesh size and therefore a high order model.

For the response analysis of large systems, the use of a high order model requires considerable computer run time and memory. Additionally, in many cases a high order model is not very useful, e.g. in control systems analysis and design. In such cases designers greatly benefit from the availability of very small, low order models that capture the behaviour of a complex system with appropriate accuracy. However a simple but adequate model of a complex distributed-lumped parameter system should reflects the basic properties and provides good insight into the modelled process.

In this paper the method of modelling a geared-rotor system is presented. The proposed approach enables to obtain an accurate, low-order, lumped parameter representation of the investigated system. The final model consists of: 1- reduced modal model of an undamped, linear beam subsystem without gyroscopic phenomena and 2 - spatially lumped model of the gyroscopic effect and non-proportional and/or nonlinear damping. The gear mesh is modelled using a spring (spring – damper) element along the pressure line [10]. A gear transmission error can be introduced as a displacement excitation. Nonlinearity of the gear mesh (backlash) can be also easily introduced in such model.

2. RFEM MODEL OF GEARED ROTOR SYSTEM

Let us consider the rotor system presented in Fig. 1.

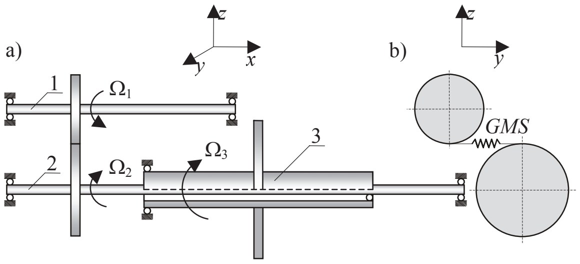

Fig. 1. General view of the considered rotor system

The considered geared rotor system consists of three shafts rotating with angular velocities: $\Omega_1 \neq 0$, $\Omega_2 \neq 0$, $\Omega_3 = 0$. Two of them 1 and 2 (Fig. 1) are coupled by a gear. Another pair of rotors 2 and 3 are coupled by bearing.

In the case of a geared rotor system a coupled phenomenon of torsional and lateral vibrations can appear as the result of the gear meshing effect. There is a large number of modelling methods related to geared rotor system coupled vibrations [1,10]. The purpose of this paper is to present a simplified hybrid model of a geared rotor system obtained by application of both: modal decomposition and spatial discretization methods.

Assuming that the axial motion of the shafts are negligible, each shaft can be considered as a simply supported beam vibrating in two perpendicular planes: *xy*, *xz* and as a torsional shaft vibrating around *x* axis. Torsional and transverse vibrations are coupled by the gear mesh spring (GMS-Fig. 1), which represents the gear mesh stiffness.

The model of presented structure was built based on the Timoshenko beam model. It includes: rotary inertia, shear deformation, the gyroscopic effect as well as internal and/or external damping.

By applying the rigid finite-element method (RFEM) [2,4] one can obtain the following equations for the rotor system:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{B}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{G}\dot{\boldsymbol{q}} = \boldsymbol{f} \, , \qquad (1)$$

where:

$\boldsymbol{q}$ – vector of generalized displacement, $\boldsymbol{f}$ – vector of generalized forces, $\boldsymbol{M},\boldsymbol{B},\boldsymbol{K},\boldsymbol{G}$ – matrices of inertia, damping, stiffness and gyroscopic respectively,

$\boldsymbol{M} = diag(\boldsymbol{M}_y, \boldsymbol{M}_z, \boldsymbol{M}_{xz}, \boldsymbol{M}_{xy}, \boldsymbol{M}_{yz})$,

$\boldsymbol{M}_y = diag(\boldsymbol{M}_{y1}, \boldsymbol{M}_{y2}, \boldsymbol{M}_{y3})$,

$\boldsymbol{M}_z = diag(\boldsymbol{M}_{z1}, \boldsymbol{M}_{z2}, \boldsymbol{M}_{z3})$,

$\boldsymbol{M}_{xz} = diag(\boldsymbol{M}_{xz1}, \boldsymbol{M}_{xz2}, \boldsymbol{M}_{xz3})$,

$\boldsymbol{M}_{xy} = diag(\boldsymbol{M}_{xy1}, \boldsymbol{M}_{xy2}, \boldsymbol{M}_{xy3})$,

$\boldsymbol{M}_{yz} = diag(\boldsymbol{M}_{yz1}, \boldsymbol{M}_{yz2}, \boldsymbol{M}_{yz3})$,

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_y & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_0 \\ \boldsymbol{0} & \boldsymbol{B}_z & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_{xz} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_{xy} & \boldsymbol{0} \\ \boldsymbol{B}_0 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_{yz} \end{bmatrix},$$

$$\boldsymbol{B}_y = \begin{bmatrix} \boldsymbol{B}_{y11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_{y22} & \boldsymbol{B}_{y23} \\ \boldsymbol{0} & \boldsymbol{B}_{y32} & \boldsymbol{B}_{y33} \end{bmatrix}, \boldsymbol{B}_z = \begin{bmatrix} \boldsymbol{B}_{z11} & \boldsymbol{B}_{z12} & \boldsymbol{0} \\ \boldsymbol{B}_{z21} & \boldsymbol{B}_{z22} & \boldsymbol{B}_{z23} \\ \boldsymbol{0} & \boldsymbol{B}_{z32} & \boldsymbol{B}_{z33} \end{bmatrix}$$

$$\boldsymbol{B}_{yz} = \begin{bmatrix} \boldsymbol{B}_{yz11} & \boldsymbol{B}_{yz12} & \boldsymbol{0} \\ \boldsymbol{B}_{yz12} & \boldsymbol{B}_{yz22} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{B}_{yz33} \end{bmatrix}, \boldsymbol{B}_0 = \begin{bmatrix} \boldsymbol{B}_{011} & \boldsymbol{B}_{012} & \boldsymbol{0} \\ \boldsymbol{B}_{021} & \boldsymbol{B}_{022} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$\boldsymbol{B}_{xz} = diag(\boldsymbol{B}_{xz1}, \boldsymbol{B}_{xz2}, \boldsymbol{B}_{xz3})$,

$\boldsymbol{B}_{xy} = diag(\boldsymbol{B}_{xy1}, \boldsymbol{B}_{xy2}, \boldsymbol{B}_{xy3})$,

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_y & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_0 \\ \boldsymbol{0} & \boldsymbol{K}_z & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{xz} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{xy} & \boldsymbol{0} \\ \boldsymbol{K}_0 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{yz} \end{bmatrix},$$

$$\boldsymbol{K}_0 = \begin{bmatrix} \boldsymbol{K}_{011} & \boldsymbol{K}_{012} & \boldsymbol{0} \\ \boldsymbol{K}_{021} & \boldsymbol{K}_{022} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{K}_y = \begin{bmatrix} \boldsymbol{K}_{y11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{y22} & \boldsymbol{K}_{y23} \\ \boldsymbol{0} & \boldsymbol{K}_{y32} & \boldsymbol{K}_{y33} \end{bmatrix},$$

$$\boldsymbol{K}_z = \begin{bmatrix} \boldsymbol{K}_{z11} & \boldsymbol{K}_{z12} & \boldsymbol{0} \\ \boldsymbol{K}_{z21} & \boldsymbol{K}_{z22} & \boldsymbol{K}_{z23} \\ \boldsymbol{0} & \boldsymbol{K}_{z32} & \boldsymbol{K}_{z33} \end{bmatrix},$$

$$\boldsymbol{K}_{yz} = \begin{bmatrix} \boldsymbol{K}_{yz11} & \boldsymbol{K}_{yz12} & \boldsymbol{0} \\ \boldsymbol{K}_{yz21} & \boldsymbol{K}_{yz22} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{yz33} \end{bmatrix},$$

$\boldsymbol{K}_{xz} = diag(\boldsymbol{K}_{xz1}, \boldsymbol{K}_{xz2}, \boldsymbol{K}_{xz3})$,

$\boldsymbol{K}_{xy} = diag(\boldsymbol{K}_{xy1}, \boldsymbol{K}_{xy2}, \boldsymbol{K}_{xy3})$,

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{G}_0 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \boldsymbol{G}_0 = \begin{bmatrix} \boldsymbol{G}_{01} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_{02} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G}_{03} \end{bmatrix},$$

$\boldsymbol{G}_{01} = \boldsymbol{G}_1^*\Omega_1$, $\boldsymbol{G}_{02} = \boldsymbol{G}_2^*\Omega_2$, $\boldsymbol{G}_{03} = \boldsymbol{G}_3^*\Omega_3$,

$\boldsymbol{G}_1^*, \boldsymbol{G}_2^*, \boldsymbol{G}_3^*$ - diagonal, gyroscopic matrices of FE-s moment of inertia (bending),

$\boldsymbol{q} = col(\boldsymbol{q}_y, \boldsymbol{q}_z, \boldsymbol{q}_{xz}, \boldsymbol{q}_{xy}, \boldsymbol{q}_{yz})$,

$\boldsymbol{q}_y = col(\boldsymbol{q}_{y1}, \boldsymbol{q}_{y2}, \boldsymbol{q}_{y3})$, $\boldsymbol{q}_z = col(\boldsymbol{q}_{z1}, \boldsymbol{q}_{z2}, \boldsymbol{q}_{z3})$,

$\boldsymbol{q}_{xz} = col(\boldsymbol{q}_{xz1}, \boldsymbol{q}_{xz2}, \boldsymbol{q}_{xz3})$, $\boldsymbol{q}_{xy} = col(\boldsymbol{q}_{xy1}, \boldsymbol{q}_{xy2}, \boldsymbol{q}_{xy3})$,

$\boldsymbol{q}_{yz} = col(\boldsymbol{q}_{yz1}, \boldsymbol{q}_{yz2}, \boldsymbol{q}_{yz3})$,

following applied subscripts are related to the generalized displacements:

y – transverse displacement along *y* axis,

z – transverse displacement along *z* axis,

xy – angular bending displacement in *xy* plane,

xz – angular bending displacement in xz plane,
yz – torsional displacement,
subscripts 1, 2, 3 denote subsystems (rotors).

Submatrices in: $\boldsymbol{B}_y$, $\boldsymbol{B}_z$, $\boldsymbol{B}_{yz}$, $\boldsymbol{B}_{xz}$, $\boldsymbol{B}_{xy}$, $\boldsymbol{B}_0$ and in $\boldsymbol{K}_y$, $\boldsymbol{K}_z$, $\boldsymbol{K}_{yz}$, $\boldsymbol{K}_{xz}$, $\boldsymbol{K}_{xy}$, $\boldsymbol{K}_0$, $\boldsymbol{G}_0$ have dimensions related to number of finite elements applied in the RFEM model.
All matrices were obtained from RFE model [3,4] by rearrangement according to general displacement $\boldsymbol{q}$.

In general, $\boldsymbol{f}$ can be a function of Ω_1, Ω_2 and Ω_3 (e.g. centrifugal forces).
By substituting:

$$\boldsymbol{f}_B = \boldsymbol{B}\dot{\boldsymbol{q}}\ ,\ \boldsymbol{f}_G = \boldsymbol{G}\dot{\boldsymbol{q}}\ , \tag{2}$$

we can present equation (1) in the form

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f} + \boldsymbol{f}_B + \boldsymbol{f}_G \tag{3}$$

or

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f}_\Sigma\ , \tag{4}$$

where

$$\boldsymbol{f}_\Sigma = \boldsymbol{f} + \boldsymbol{f}_B + \boldsymbol{f}_G\ . \tag{5}$$

3. MODAL DECOMPOSITION

The model described by equation (4) can be written in modal representation as:

$$\boldsymbol{M}_m\ddot{\boldsymbol{q}}_m + \boldsymbol{K}_m\boldsymbol{q}_m = \boldsymbol{f}_m\ , \tag{6}$$

where:

$$\boldsymbol{M}_m = \boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi} = diag(m_1,\ldots,m_r,\ldots,m_n)\ ,$$
$$\boldsymbol{K}_m = \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} = diag(k_1,\ldots,k_r,\ldots,k_n)\ ,$$
$$\boldsymbol{q}_m = col(q_{m1}\ \cdots\ q_{mr}\ \cdots\ q_{mn})\ ,\ \boldsymbol{f}_m = \boldsymbol{\Phi}^T\boldsymbol{f}_\Sigma\ ,$$
$$\boldsymbol{\Phi} = col(\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_r,\cdots,\boldsymbol{\varphi}_n)\ ,$$

in which:
m_i – modal coefficients of inertia, k_i – modal coefficients of stiffness $\boldsymbol{\varphi}_i$ – eigenvectors of matrix $\boldsymbol{M}^{-1}\boldsymbol{K}$.
By solving (6) we can next obtain the solution of (1) in the following form:

$$\boldsymbol{q} = \boldsymbol{\Phi}\boldsymbol{q}_m\ ,\ \dot{\boldsymbol{q}} = \boldsymbol{\Phi}\dot{\boldsymbol{q}}_m\ . \tag{7}$$

4. REDUCED MODAL MODEL

Modal model (6) can be reduced by removing those rows and columns in $\boldsymbol{M}_m$, $\boldsymbol{K}_m$ which are insignificant to the system's dynamics. Thus, in such approach we obtain:

$$\boldsymbol{M}_{mr}\ddot{\boldsymbol{q}}_{mr} + \boldsymbol{K}_{mr}\boldsymbol{q}_{mr} = \boldsymbol{f}_{mr}\ , \tag{8}$$

where:

$$\left.\begin{array}{l}\boldsymbol{M}_{mr} = diag(m_1,\ldots,m_r)\ ,\ \boldsymbol{K}_{mr} = diag(k_1,\ldots,k_r)\ ,\\ \boldsymbol{q}_{mr} = col(q_{m1},\ldots,q_{mr})\ ,\ \boldsymbol{f}_{mr} = \boldsymbol{\Phi}_r^T\boldsymbol{f}_\Sigma\ ,\\ \boldsymbol{\Phi}_r = col(\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_{r1})\ .\end{array}\right\} \tag{9}$$

An approximate solution of (1) by the application of reduced order model (8) can be obtained from the formulas:

$$\boldsymbol{q} = \boldsymbol{\Phi}_r\boldsymbol{q}_{mr}\ ,\ \dot{\boldsymbol{q}} = \boldsymbol{\Phi}_r\dot{\boldsymbol{q}}_{mr}\ . \tag{10}$$

However, in order to obtain better static accuracy of reduced model (see chapter 6) one can apply also modal stiffness coefficients of modes r+1, … , n (static correction). In such case instead of (9) we have:

$$\left.\begin{array}{l}\boldsymbol{M}_{mr} = diag(m_1,\ldots,m_r,0,\ldots,0)\ ,\ \boldsymbol{K}_{mr} = \boldsymbol{K}_m\ ,\\ \boldsymbol{q}_{mr} = \boldsymbol{q}_m\ ,\ \boldsymbol{f}_{mr} = \boldsymbol{f}_{mr}\ ,\ \boldsymbol{\Phi}_r = \boldsymbol{\Phi}_r\ .\end{array}\right\} \tag{11}$$

It should be mentioned that applying formulas (11) the order of the model is the same as by application (9).

5. HYBRID REDUCED MODEL

By taking into account (2) and (5) we can transform (8) into the following form:

$$\boldsymbol{M}_{mr}\ddot{\boldsymbol{q}}_{mr} + \boldsymbol{K}_{mr}\boldsymbol{q}_{mr} = \boldsymbol{\Phi}_r^T\boldsymbol{f} - \boldsymbol{\Phi}_r^T\boldsymbol{B}\boldsymbol{\Phi}_r\dot{\boldsymbol{q}}_{mr} - \boldsymbol{\Phi}_r^T\boldsymbol{G}\boldsymbol{\Phi}_r\dot{\boldsymbol{q}}_{mr} \tag{12}$$

Equations (12) and (10) present the final hybrid model, in which $\boldsymbol{f}$ is the input data and $\boldsymbol{q}$ is the response of the system. A block diagram describing the above hybrid model is presented in Fig. 2. This shows that in the hybrid model matrices $\boldsymbol{M}_{mr}$, $\boldsymbol{K}_{mr}$ are taken from modal reduced models (8) and matrices $\boldsymbol{B}$, $\boldsymbol{G}$ origin from the initial FEM model (1).

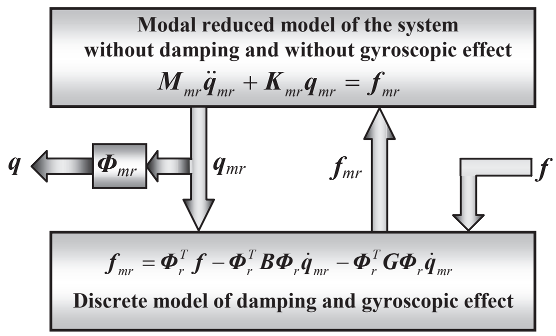

Fig. 2. Block diagram of the hybrid model

Proposed method of hybrid modelling of geared rotor system was previously applied by authors for successful modelling of other (non geared) rotor systems [5-9].

6. NUMERICAL CALCULATIONS AND RESULTS

The continuous structure (Fig. 1) is divided into 203 rigid finite elements (RFE) and 209 spring damping elements (SDE). Discrete model obtained by using the RFE method is shown in Fig. 3b. Each RFE has fife degrees of freedom, i.e. transverse displacement along axes y and z and angular displacement around all three axes.

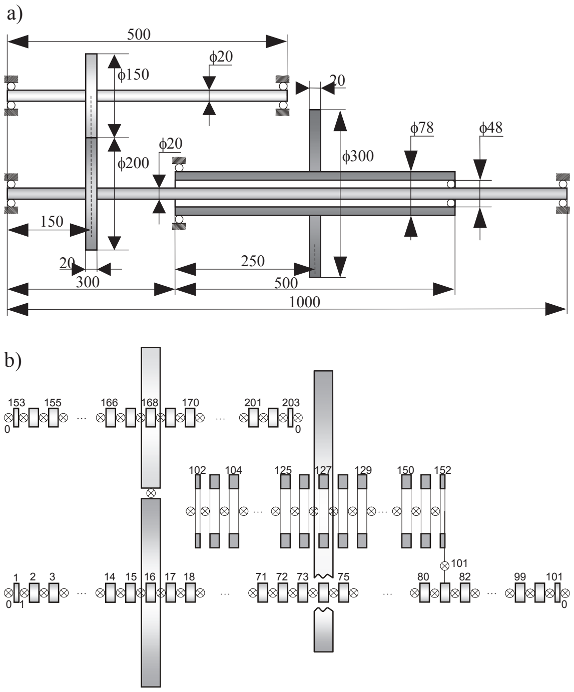

Fig. 3. Investigated continuous structure – a) and its discrete model – b)

Parameters of RFEs are presented in Tabs. 1÷4. They are calculated for the following physical data: modulus of elasticity (Young) $E=2\cdot10^{11}$ [Pa], shear modulus (Kirchhoff) $G=7.8\cdot10^{10}$ [Pa], mass density $\rho=8000$ [kg/m^3].
Denotation of the RFEs parameters one can find in [3,4].

Tab. 1. The inertia coefficients of RFE no. i

i	m_i	$J_{y,i}$	$J_{z,i}$
	[kg]	[kgm^2]	[kgm^2]
1, 101, 153, 203	0.01256	$3.4033\cdot10^{-7}$	$6.2831\cdot10^{-7}$
2÷15, 17÷100, 154-167,169-202	0.02513	$8.3775\cdot10^{-7}$	$1.2566\cdot10^{-6}$
16	5.02654	0.0127339	0.0253840
168	2.77716	0.0040673	0.0079496
102, 152	0.11875	$6.2503\cdot10^{-5}$	$1.2451\cdot10^{-4}$
103÷126, 128÷151	0.23750	$1.2649\cdot10^{-4}$	$2.4902\cdot10^{-4}$
127	10.54519	0.06341441	0.12665307

Tab. 2. The stiffness coefficients of SDE no. k

i	j	k	$c_{k,z}$ [Nm^{-1}]	$c_{k,x}=c_{k,y}$ [Nm^{-1}]	$c_{k,xz}$ [Nm]
1	2	1			
⋮	⋮	⋮	6283185307.179	2067560665.144	157079.633
100	101	100			
102	103	102			
⋮	⋮	⋮	59376101152.847	19538448285.609	31127921.029
151	152	151			
153	154	153			
⋮	⋮	⋮	6283185307.179	2067560665.144	157079.633
202	203	202			
81	152	101	$2\cdot10^{15}$	$2\cdot10^{15}$	0
0	1	0			
101	0	0			
0	102	0			
0	153	0			
203	0	0			

The full RFEM model was used as the reference model for validation of simplified hybrid model of considered system. In this model the gyroscopic effect was included. Fig. 6 presents the influence of the gyroscopic effect on the frequency characteristics, in the case when the angular velocities $\Omega_1=\Omega_2=1000$ [rad/s] were assumed.

Tab. 3. The damping coefficients of SDE no. k

i	j	k	$b_{k,z}$ [Nsm^{-1}]	$b_{k,x}=b_{k,y}$ [Nsm^{-1}]	$b_{k,xz}$ [Nsm]
1	2	1			
⋮	⋮	⋮	2617993.878	861483.61	65.45
100	101	100			
102	103	102			
⋮	⋮	⋮	24740042.147	8141020.119	12969.967
151	152	151			
153	154	153			
⋮	⋮	⋮	2617993.878	861483.61	65.45
202	203	202			
81	152	101	0	0	0
0	1	0			
101	0	0			
0	102	0			
0	153	0			
203	0	0			

Tab. 4. The connection coordinates of SDE no. k to RFE no. i

i	j	k	$s_{r,k,z}$ [m]	$s_{r,k,x}$ [m]	$s_{p,k,z}$ [m]	$s_{r,k,x}$ [m]
1	2	1	0.0025	0	-0.005	0
⋮	⋮	⋮	0.005	0	-0.005	0
100	101	100	0.005	0	-0.0025	0
102	103	102	0.0025	0	-0.005	0
⋮	⋮	⋮	0.005	0	-0.005	0
151	152	151	0.005	0	-0.0025	0
153	154	153	0.0025	0	-0.005	0
⋮	⋮	⋮	0.005	0	-0.005	0
202	203	202	0.005	0	-0.0025	0
81	152	101	0.0025	0	0	0
0	1	0	0	0	-0.0025	0
101	0	0	0.0025	0	0	0
0	102	0	0	0	-0.0025	0

Taking into account the RFEM model without damping and gyroscopic effect the modal reduced model was built. Fig. 4 presents eigenfunctions corresponding to too selected eigenvalues: $\omega_1=122.62$, $\omega_3=322.81$ of the system (without damping and gyroscopic interactions).
Modal reduced model (9, 11) was built for 8 retained modes. Comparison of frequency characteristics related to full undamped model and reduced model are presented in Fig. 5.
The modal reduced model was next combined with discrete model of gyroscopic interaction and damping. In this way a hybrid model of considered system was obtained.
To verify the obtained reduced hybrid model, its frequency response was compared to that of the full FEM model (reference model). The results are presented in Fig. 7 and 8.

The simulation results prove that obtained hybrid, reduced model presents very nice accuracy in the frequency range related to the number of retained modes.

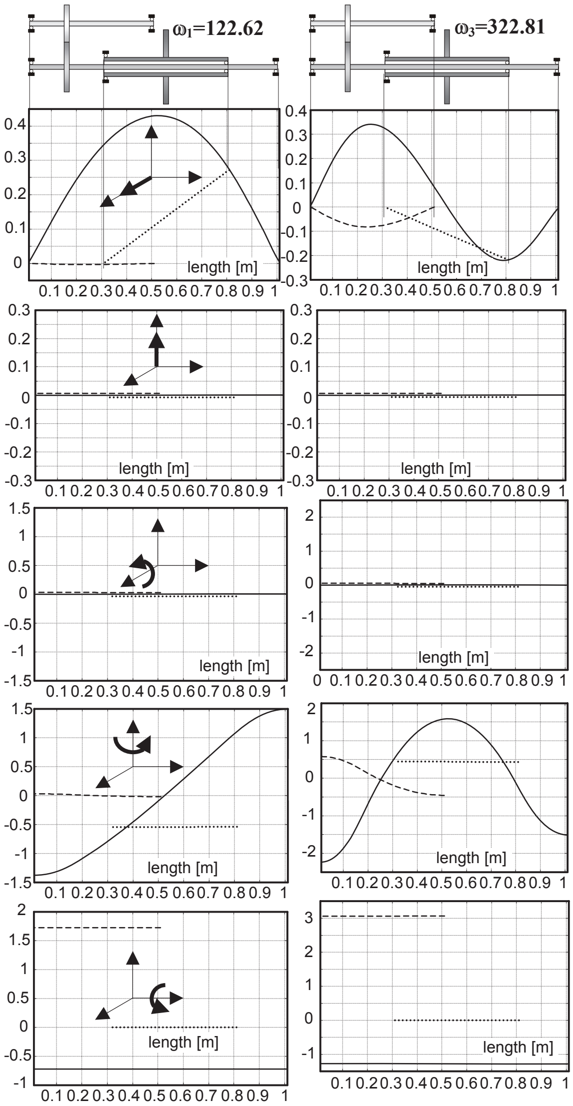

Fig. 4. Examples of eigenfunctions of investigated rotor system

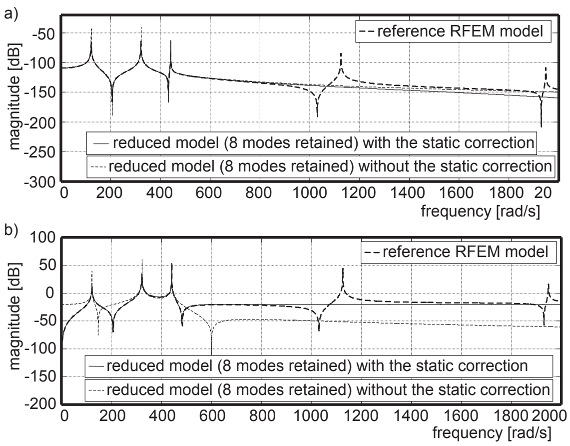

Fig. 5. Modal models validation in frequency domain: a) input –force acting along *z* axis and applied at the 16 RFE, output – transverse displacement at the same point and along the same axis; b) input – kinematic excitation (displacement) along gear pressure line, output – the same as in the case a)

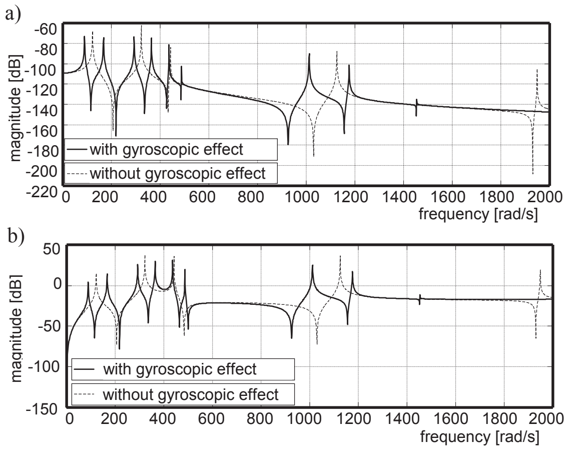

Fig. 6. Influence of the gyroscopic effect on frequency characteristics. Inputs and outputs in a) and b) are the same as in Fig. 5

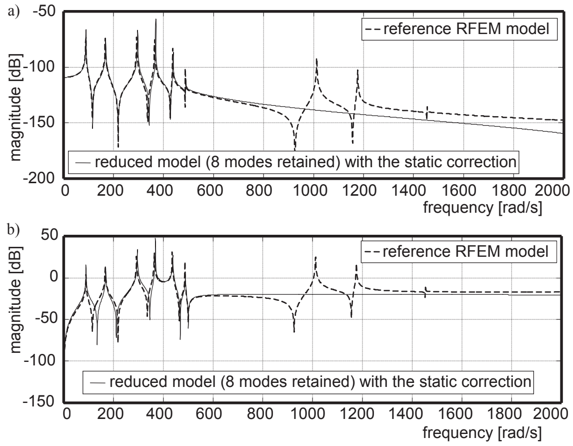

Fig. 7. The frequency characteristics of models with gyroscopic effect and without damping: a) force excitation, b) kinematic excitation. Inputs and outputs are the same as in Fig. 5

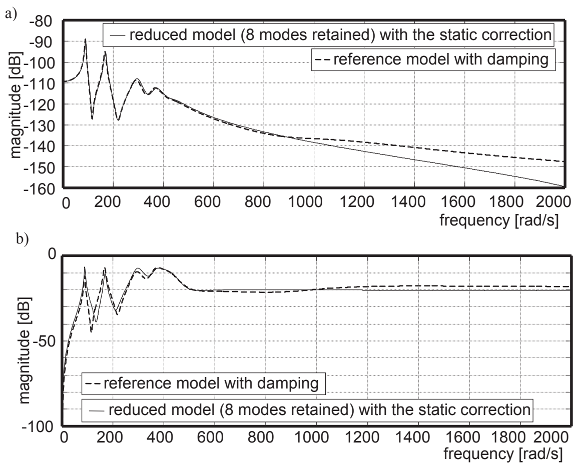

Fig. 8. The frequency characteristics of models with gyroscopic effect and non proportional damping: a) force excitation. Inputs and outputs are the same as in Fig. 5, b) kinematic excitation

7. CONCLUSIONS

In this paper the method of modelling a geared rotor system is presented. The proposed approach enables to obtain an accurate low-order lumped parameter representation of the investigated system. The final model consists of reduced modal models of undamped beam/shaft systems and spatially lumped model of the gyroscopic effect and a non-proportional damping model. The gear mesh was modelled using a spring element along the gear pressure line. The transmission error can be introduced as a displacement excitation. The obtained simulation results, in the form of corresponding frequency characteristics, prove that the proposed method is efficient and can be applied in the case of more complex geared rotor systems. For example, nonlinear damping or nonlinearity of gear mesh can be included. Also unbalanced and speed varying rotors can be considered. In such cases the time domain investigations must be performed. It will be the authors future work in the rotor dynamics modelling area.

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