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VANISHING PLANE IN ANALYSING GEOMETRICAL ILLUSIONS IN INTERIORS

Marcin GÓRKO

Department of Land Surveying, Cartography and Descriptive Geometry.
Technical University of Lodz,
6 Politechniki ave. , 90-924 Lodz, Poland
email: marcingorko@poczta.onet.pl

Abstract. Vanishing plane, together with projection plane and projection point, is one of the fundamental objects in the theory of central projection. Its position is determined by the position of an "eye" and the projection plane. Some properties of the vanishing plane may be useful to restore the often unknown position of punto stabile and an "eye". One simple example presented in this paper shows how this can be done.

Key Words: central projection, perspective, vanishing plane, geometric illusion.

1.Introduction

The properties of central projection, together with the properties of human vision, make it possible to create illusions and influence the way we see an interior. One of the well known methods to influence our perception of the architecture is adding some fictive elements – which exists not as a 3D body, but as its perspective. The effect of illusion will occur if the observer stays in precisely selected place in the interior, called punto stabile. Than, the elements of fictive and real architecture seem to be in harmony – they don't show any distortions [1]. Observing the interior from any other point will create some changes in the perspective – in points, where fictive and real architecture meets, an abrupt break will be visible. The observer can interpret it as a deformation of the fictive part of the interior.

One of the most important problems in analyzing geometrical illusions is restitution of the position of punto stabile and projection centre ('eye') from which the illusion is properly seen. This paper shows how to find proper position of punto stabile, using properties of vanishing plane applied to vertical lines, which are continued in fictive architecture.

2. Vanishing plane

Vanishing plane is a plane which include projection centre and is parallel to projection plane. It has following interesting property: lines that intersect each other on the vanishing plane, have parallel perspectives [2]. If a chosen interior exhibits real vertical lines, which seem to be continued in fictive architecture, their perspectives on a vertical projection plane will be analysed. Parallel appearance of fictive lines can be explained if one considers the mutual position of the intersection point and vanishing plane.

3. Geometrical conditions and examples of perspectives

The use on vanishing plane to determine position of punto stabile will be demonstrated on an example of interior, where two columns are 'extended above' – their fictive parts are visible on the ceiling. Axonometric view of this interior, together with one chosen vanishing plane are shown on the Fig 1.

To observe proper continuation of real lines in fictive ones, three conditions need to be fulfilled:

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- Projection plane must be in such a position that real vertical lines appear to be parallel to each other.
- The S point (see also Fig 1), where fictive lines cross each other, must belong to the vanishing plane.
- Perspective ray, which create the perspective of point S must be vertical.

First condition requires the projection plane (and also vanishing plane) to be vertical. Second condition will be fulfilled if it connects the vanishing plane with the interior. The drawing below shows two position of an 'eye' – both belong to the vanishing plane. For a chosen position of the 'eye' O_1, the perspective of ***S*** point will be an in-infinity point $S_1'^{\infty}$. Two first of the three conditions are fulfilled here. The lack of the last conditions will show itself in diagonal (but parallel!) perspectives of fictive lines. If an 'eye', like O_2 will be placed precisely below the ***S*** point, the perspective ray O_2-***S***-$S_2'^{\infty}$ will get in a vertical position. This is enough to see fictive lines parallel and vertical – as the real ones appear.

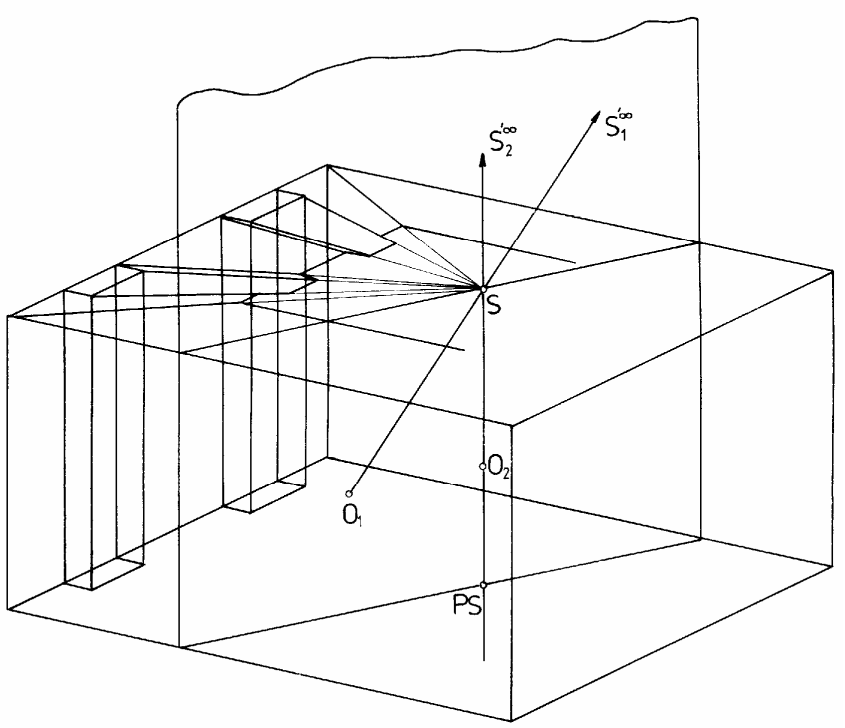

Figure 1: Two examples for an 'eye' O_1 and O_2 on the vanishing plane

It is easy to notice, there are infinite many vanishing planes which fulfil second condition. All vanishing planes that go through the ***S*** point create a bundle of planes and the planes will have a common vertical edge, which goes through ***S***. In other words the edges of all vanishing planes with the floor go through the punto stabile (***PS***), which leads to an important conclusion: In perspective on a vertical projection plane, fictive architecture can be observed as parallel, even if the 'eye' does not match with punto stabile. Following two drawings present this interior in perspective on vertical projection plane, with the ***S*** point laying on it. Fig.2 shows how this interior would be observed, if the 'eye' is not placed below the point ***S*** – the observer sees, that fictive elements are parallel to each other, but they are not vertical. The inclination of those lines in respect to vertical direction informs the observer where to move to get a proper view of the illusion.

A perspective obtained for an 'eye' below point ***S*** is shown on the Fig. 3.

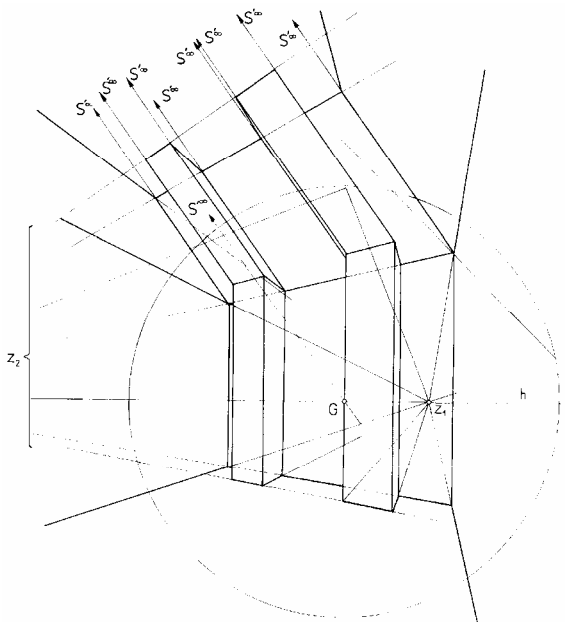

Figure 2: Description in text

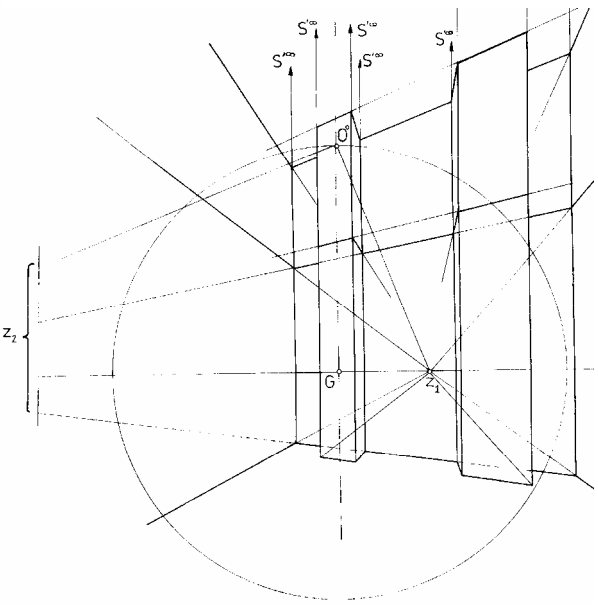

Figure 3: Description in text

The properties of vanishing plane to search for punto stabile can be easily used if a common camera is used. If projection centre means the same as the main point of the lens, vanishing plane will go through this point perpendicularly to the optical axis (it will cross the lens, parallel to the film plane). The drawing below shows a cross section of a single-lens-reflex camera with positions of projection's plane τ in its "negative" position and vanishing plane $\bar{\tau}$.

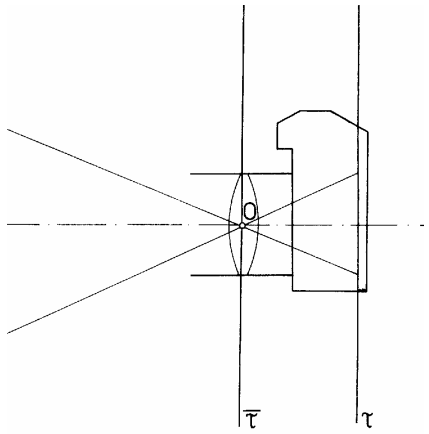

Figure 4: Projection plane τ and vanishing plane $\bar{\tau}$ interpreted by a camera

Vertical position of the vanishing plane can be obtained, if the lens' axis will be aimed horizontally which is an easy job if one uses a photographic tripod equipped with high-quality panoramic head, even without a water level. If the camera permits to see the real image from the lens (this can be a single-lens-reflex, view- or a digital camera), its proper position in the interior can be found empirically – by try-and-repeat method. If one can find two different vanishing planes, where fictive lines appear parallel, the edge of both planes will be a vertical line that goes through the floor in punto stabile. This means every point on this line can be an

'eye' – projection centre which produces proper effect of illusion. This in turn leads to another important conclusion: illusions which exhibits only vertical lines don't allow to set the height of an 'eye' over punto stabile.

When a camera is used to analyse illusions in the way described above, two things from the technical point of view shall also be considered:

- the need for use of a super wide-angle lens with perfectly corrected distortion.
- vanishing plane connected to the camera must be projected somehow on the interior floor.

References

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[2] Bartel K.: *Perspektywa malarska.* Państwowe Wydawnictwo Naukowe, Warszawa 1960.

PŁASZCZYZNA ZNIKNIENIA W ANALIZIE ILUZJI GEOMETRYCZNYCH WE WNĘTRZACH

Płaszczyzna zniknienia, razem z płaszczyzną tła i środkiem rzutów („okiem"), stanowią podstawowe elementy w teorii rzutu środkowego. Jej położenie uwarunkowane jest położeniem środka rzutów i płaszczyzny tła. Pewne własności płaszczyzny zniknienia mogą być pomocne w odtwarzaniu często nieznanego położenia punto stabile i „oka". Prosty przykład omówiony w artykule ilustruje tę metodę.

Reviewer: Prof. Jerzy MROCZKOWSKI, DSc, Arch

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