



DOI: 10.5604/01.3001.0016.1190

Analytical evaluation of the influence of adding rubber layers on free vibration of sandwich structure with presence of nano-reinforced composite skins

M. Al-Shable ^a, M. Al-Waily ^{a,*}, E.K. Njim ^b

^a Department of Mechanical Engineering, Faculty of Engineering, University of Kufa, Iraq

^b Ministry of Industry and Minerals, State Company for Rubber and Tires Industries, Iraq

* Corresponding e-mail address: muhanedl.alwaeli@uokufa.edu.iq

ORCID identifier:  <https://orcid.org/0000-0002-7630-1980> (M.A.-W.)

ABSTRACT

Purpose: Developing structural designs that offer superior vibration properties is still a major challenge, but they stay solid and lightweight simultaneously. Composite faces are frequently used in insulating constructions as an alternative to sheet metal roofs. Rubber overlays have been added to reduce waves' natural frequency and fade time.

Design/methodology/approach: The mechanical properties and the natural frequency calculation of the materials that make up the composite structural panels designed for structural applications with the addition of rubber layers were studied in this study.

Findings: The results showed the addition of rubber layers with SiO₂ nanoparticles with a density of 1180 kg/m³, and the optimal decrease (VF = 2.5%) is 38.5% in the natural frequency while at a density of 1210 kg/m³, it is 40.2% in the natural frequency. While the addition of rubber layers with Al₂O₃ nanoparticles shows a density of 1180 kg/m³, the optimum reduction (VF = 2.5%) is 41% in HF while at a density of 1210 kg/m³ 36.8% in an NF 41% during a density of 1210 kg/m³ 38.4%.

Research limitations/implications: Certain hypotheses were used to apply Kirchhoff's theory to solve the mathematical model of the structure.

Practical implications: The work was carried out on the faces of nanocomposites made of SiO₂/epoxy and Al₂O₃/epoxy with different densities and polylactic acid core. The inclusion of nanoparticles as a percentage of the fraction size ranges from 0% to 2.50%.

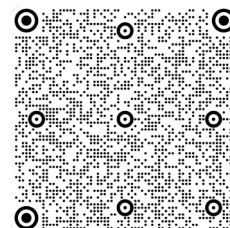
Originality/value: This study's results shed light on the fundamental behaviour of the components that make up the sandwich in the presence of rubber layers.

Keywords: Sandwich panel, Analytical solution, Composite-face, Nanoparticles, Rubber layer

Reference to this paper should be given in the following way:

M. Al-Shable, M. Al-Waily, E.K. Njim, Analytical evaluation of the influence of adding rubber layers on free vibration of sandwich structure with presence of nano-reinforced composite skins, Archives of Materials Science and Engineering 116/2 (2022) 57-70.

DOI: <https://doi.org/10.5604/01.3001.0016.1190>



METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

In engineering fields such as aerospace, automotive, energy, and biomedical, sandwich structures are helpful for their excellent strength-to-weight ratio and high energy absorption. In these structures, thin, stiff face sheets are coupled with thick, low-density cores. Nowadays, many applications widely use the construction of layered structures consisting of viscoelastic layers embedded between elastic layers [1]. Insulating structures are created by placing a low-density core between two steel faces, resulting in a system with improved stiffness and a strength-to-mass ratio. Functionally graded materials (FGM), piezoelectric materials, and rubber are the types widely used in today's technologies [2-5]. Rubber is currently used in various fields, including cables, tires, home appliances, paint, packaging materials, textiles, sporting goods for transportation, infrastructure construction, medical and optical equipment, etc., due to its light weight, flexibility, and other benefits. Rubber comes in many forms: natural rubber (NR), synthetic rubber (SBR), nitrile rubber, etc. [6]. Surface panels with a two-line structure are used for various purposes, but they always keep the majority of applied loads in place [7]. The sandwich structure consists of cores, surface plate materials, adhesives, and two layers of rubber, with varying mechanical properties depending on the application. The properties of the stable core face plates affect the component's hardness, stability, composition, and strength; Face sheets shall be appropriately connected and of acceptable quality to perform these duties [8].

Rubber layers are added to improve mechanical properties and reduce vibration and fading time. The chemical composition of rubber affects its properties. Vulcanization and mixing raw rubber with additives are essential to changing rubber to meet specific requirements. Vulcanization is one of the most critical parts of rubber technology. A cross-link between the rubber chains is required to turn raw rubber into a web. This mesh is not as sticky as natural rubber, does not harden in the cold or soften much except at extremely high temperatures, becomes elastic, wear-resistant, compresses, and reduces the stresses necessary to achieve a specific deformation increase [9]. Requirements a face sheet can also be used as a dynamic, smooth profile, a non-slip rough surface, or a solid and long-lasting floor covering. One sandwich panel may be thicker or have a somewhat different structure than the other to perform these different roles better. As a sandwich face plate, any thin, metallic, non-metallic, or composite material can be used to reduce deformation [10].

Prerequisites face sheets can be used in many different ways. The face sheets and the rubberized layers help protect

the core from external influences such as moisture in many circumstances; therefore, impermeable materials are required for the face coverings. For example, surface panel materials may not be necessary for some acoustic panel applications to seal core components [11]. As a result, it should have high toughness, tensile and compressive strength, impact resistance, surface finish, and environmental resistance. Hence the requirements of rubber layers and face plates can be used in various ways. Composite face panels for aircraft sandwich installations are generally made of unidirectional glass strips that are pre-impregnated or layers of carbon fibres in a polymer matrix (epoxy, bismalimide, etc.) [12,13]. The performance of sandwich panels with polymer cores and aluminium alloy face-sheets for static and dynamic analysis have been investigated by numerous researchers both experimentally and numerically.

The analytical analysis of dynamic problems of layered beams and plates can also be found in [14-16]. Moreover, numerical methods, based mainly on classical plate theory and the finite element method, are presented and used in [17-18].

Al-Waily et al. (2017) investigated the effect of glass and carbon powder reinforcement on the mechanical properties and vibration response of an Isotropic hyper composite materials plate [19]. Gupta et al. (2020) conducted an analytical and experimental study on natural frequency ranges for a type of sandwich beam that has been manufactured using natural rubber (NR) in combination with skin material of aluminium alloy 1050 grade [20]. A new type of mesh auxiliary chemicals, with a 3D-printed polymeric core and two unidirectional Carbon Fibre Reinforced (CFRP) front sheets, was investigated by Hou et al. (2018) [21]. Lewandowski et al. (2021) evaluated the performance of free vibration of laminate plates using a theoretical solution [22]. Karim et al. (2018) discovered that if the steel components were replaced with hybrid materials consisting of 95% epoxy, 5% fibreglass fiberglass, 5% rubber, and fiberglass, improving safety, comfort, and durability. The high strength-to-weight ratio and abrasion resistance was enhanced [23].

Over the past few decades, a great deal of attention has been paid to sandwich panels and laminated composites, considering their load and boundary conditions. For example, many researchers studied the analysis of static and dynamic responses and buckling of these structures [24-28].

Naji et al. (2018) employed the layerwise theory to develop a finite element formulation to investigate the vibration characteristics of laminated composite beams with a magnetorheological layer. Experimental tests under different magnetic fields are carried out to verify the

numerical result [29]. Nayak et al. (2018) investigated the forced vibration for the sandwiched beam structure made of a magnetorheological elastomer core; there, experimental work was presented to calculate the vibration beam response with different parameters effect [30].

Vishwas et al. (2019) explained that rubber affects the mechanical behaviour of structures. It absorbs a lot of energy. Changing the angle makes the composite panel better. It also serves as a benchmark for how well low-speed composite panels can be used [31]. Bonthu et al. (2020) used high-density polyethylene (HDPE) and a glass-reinforced balloon (GMB) fed to the extruder to generate filament for the 3DP printer, which was used for sandwich printing. A high-quality, once-in-a-lifetime product free from defects [32].

Ju et al. (2020) proved that with or without LRBs, the mobile crane has little effect on the plant's primary ambient vibrations. LRBs with a considerable initial strength should be employed for high-tech industries to handle wind loads in the usual design [33]. Mastalygina et al. (2020) tested low-density polyethylene (PE) composite films with a variety of natural rubber (NR) compositions (10-30 per cent by weight). The effect of NR on the structural properties, water absorption, and mechanical properties of composite materials was studied [34]. Dobrotá et al. (2020) fabricated conveyor belts with two types of rubber and PVC in their structure. The use of PVC in the construction of the rubber matrix belt is beneficial, but it operates in environments that may accelerate the ageing of the material [35]. Barrios et al. (2020) manufactured compounds based on natural rubber epoxide (ENR) and thermo-reducing graphene oxide (TRGO) with high efficiency of up to 85% at room temperature without the need for external catalysts [36].

To predict the vibration-damping performance of sandwich panels with lattice truss cores and investigate the underlying enhancement mechanisms, Wang et al. (2020) developed a combined finite element-modal strain energy method [37]. The layers of curved surface EPDM rubber were joined to CNC-processed curved surface metal dies with fibreglass spunbond, and the composites were heat treated by Albayrak and Kaman (2021). It was revealed that the layers separated in composites created at ambient temperature [38].

In a study by Homkhiew et al. (2021), RWS was added 30-50 per cent by weight to blended TPNR composites to boost rupture modulus and tensile strength. The proportions of NR and HDPE are 60/40 and 50/50, respectively. TPNR composites reinforced with RWS 80 net RWS 40 enhanced tensile strength and stiffness [39]. Suspension is the most effective and simplest cost-effective way to reduce vehicle vibrations. Flexible rubber damping parts provide significant structural damping at a fraction of the cost of the

traditional method [40]. Kalsoom et al. (2021) investigated the stiffness and damping characteristics of the sandwich beam with 3D printed thermoplastic composite face sheets using higher-order beam theory based on various parameters such as support conditions, non-homogeneous magnetic flux, geometrical properties [41].

Lewandowski et al. (2021) conducted numerical studies on dynamic characteristics for composite sandwich beams made from elastic and viscoelastic layers based on refined zig-zag theory [42].

Zheng et al. (2022) proposed the dynamic analytical model of the composite sandwich structure by arranging rubber layers to determine damping characteristics based on the Rayleigh-Ritz method [43]. Using experimental and numerical analysis, Singh and Khan (2022) studied the vibration characteristics of polymer composites with viscoelastic material properties [44]. Mohammadi et al. (2022) proposed isogeometric analysis to analyse the free and forced vibration analyses of sinusoidally corrugated functionally graded carbon nanotube reinforced thin composite panels based on Kirchhoff-Love theory [45].

Selvaraj et al. (2022) studied the mechanical behaviour and dynamic characteristics of natural fibre reinforced composite sandwich plates with multiple-core layers using experimental work. The results are verified using a finite element method [46]. The experimental investigation is carried out to validate the frequency vibration bandgaps by combining the design concepts of locally resonant 3D printed meta-structures sandwich structures [47]. Vibration behaviours of a composite plate resting on the nonlinear elastic foundation using the finite element method are also discussed in [48].

Through the analysis of earlier literature, some research employed rubber layers numerous times, once as a heat insulator, once to absorb energy, again to improve the physical and mechanical qualities, and once as a vibration damper. As can be observed, the earlier research was primarily experimental and numerical, with only a few theoretical investigations. In addition, some were free to use, while others had to be used with force.

The present paper use displacement based on classical plate theory to solve the dynamic problems for rubber-layered composite panel constructions. The linear constitutive relations are used to model the characteristics of the sandwich panel layers. A new analytical solution is constructed to describe the free vibration problem of plates with composite faces in the presence of rubber layers and nanomaterials. The eigenvalue formula calculates the panel's natural frequency and dynamic response of the whole structure based on different core parameters and skins. Following the mathematical formulation applied to the

structure using the thin-sheet impulse theory, alumina Al₂O₃ and silica SiO₂ were used in the theoretical equations to investigate the impact of reinforced skins on the natural frequency results. In addition, some conclusions are formed based on several numerical examples.

2. Analytical solution

A classical thin plate theory (CPT) or Kirchhoff plate theory is generally used to analyse composite structures for vibration and stability. Modal analysis of thin plates can be computed using CPT assumptions, but they can be inaccurate if the ratio of thickness-to-plane dimensions is high. This is because the effect of transverse shear deformation, which is not considered in the Kirchhoff theory, becomes significant in thick plates [49].

The small deflection theory of thin plates is based on the following assumptions [50]:

1. The thickness of the plate (h) is small compared to its lateral dimensions.
2. Normal stresses in the direction transverse to the plate are taken to be negligibly small.
3. Effect of rotatory inertia is negligible. This implies that the plane sections are normal to the mid-surface before deformation and remain normal to the mid-surface even after deformation or bending. This assumption suggests that the transverse shear strains, ϵ_{xz} , and ϵ_{yz} , are negligible, where z denotes the thickness direction.
4. The transverse normal strain ϵ_{zz} under transverse loading can be neglected. The transverse normal stress σ_{zz} is small and can be neglected compared to the other stress components.

Depending on the assumptions made above, according to thin plate theory, the way of achieving equilibrium for the differential equation of force, the undamped motion of plates (equilibrium in the z direction) with composite-face sandwich plate strengthened by nano Al₂O₃ and SiO₂ fraction can be written in the following form [51].

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \rho h \frac{\partial^2 \omega}{\partial t^2} = p_z \tag{1}$$

The plate's bending and twisting moments per unit length are represented by the symbols, M_{xx} , M_{yy} , and M_{xy} . The displacement of strain, the transverse displacement of the plate's central surface, expresses the composite-face sandwich plate's relationships are:

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_0}{\partial x} - z \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial w_0}{\partial y} - z \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} - 2z \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{2}$$

where, ϵ_{xx} , ϵ_{yy} , and ϵ_{xy} are the normal and shear strain for the plate, respectively,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}} & \frac{E_{yy}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{E_{yy}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} \tag{3}$$

where (σ_{xx} and σ_{yy}) are the normal stresses, σ_{xy} is the shear stress of the plate. Rearranging equations 2 and 3 yields:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{yy}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial y^2} \\ \frac{E_{xx}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial y^2} \\ 2G_{xy} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{4}$$

Mechanical properties of upper and lower skins:

Consider Young's modulus of the upper skin is $E_{xx} = E_{yy} = E_{fu}$ and the rigidity modulus: $G_{xy} = G_{fu} = \frac{E_{fu}}{2(1+\nu_u)}$, while the Poisson's ratio $\nu_{xy} = \nu_{fu}$, then,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{fu}}{1-\nu_u^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{fu} \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{fu}}{1-\nu_u^2} \left(\nu_{fu} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_u}{(1+\nu_{fu})} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{5}$$

For the lower face part, the elastic modulus: $E_{xx} = E_{yy} = E_{fl}$ and the rigidity modulus: $G_{xy} = G_{fl} = \frac{E_{fl}}{2(1+\nu_{fl})}$ also, Poisson's ratio: $\nu = \nu_{fl}$, then,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{fl}}{1-\nu_{fl}^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{fl} \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{fl}}{1-\nu_{fl}^2} \left(\nu_{fl} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{fl}}{(1+\nu_{fl})} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{6}$$

Mechanical properties of the core

The Young's modulus: $E_{xx} = E_{yy} = E_c$, the rigidity modulus:

$G_{xy} = G_c = \frac{E_c}{2(1+\nu_c)}$, and the Poisson's ratio $\nu_{xy} = \nu_c$ then,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_c}{1-\nu_c^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_c \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{fu}}{1-\nu_c^2} \left(\nu_c \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_c}{(1+\nu_c)} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{7}$$

Mechanical properties of upper and lower rubber parts

The Young's modulus of the upper rubber part is $E_{xx}=E_{yy}=E_{ru}$, and the rigidity modulus: $G_{xy}=G_{ru}=\frac{E_{ru}}{2(1+\nu_{ru})}$, also Poisson's ratio $\nu_{xy}=\nu_{ru}$ then,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{ru}}{1-\nu_{ru}^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{ru} \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{ru}}{1-\nu_{ru}^2} \left(\nu_{ru} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{ru}}{(1+\nu_{ru})} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (8)$$

Considering the lower rubber part properties, Young's modulus $E_{xx}=E_{yy}=E_{rl}$ And rigidity modulus: $G_{xy}=G_{rl}=\frac{E_{rl}}{2(1+\nu_{rl})}$ also, Poisson's ratio $\nu_{xy}=\nu_{rl}$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{rl}}{1-\nu_{rl}^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{rl} \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{rl}}{1-\nu_{rl}^2} \left(\nu_{rl} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_{rl}}{(1+\nu_{rl})} \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (9)$$

As noted in the above equation, biharmonic form plate stresses in the mid-plane of the plate change in the z-direction across the thickness of the plate. In terms of stress components, express the bending and twisting moments, as well as the shear pressures, as,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z dz \quad (10)$$

As for the inertia part of the equation (ρh), which is mathematically integrated with respect to thickness as,

$$\rho h = \int_{-h/2}^{h/2} \rho dz \quad (11)$$

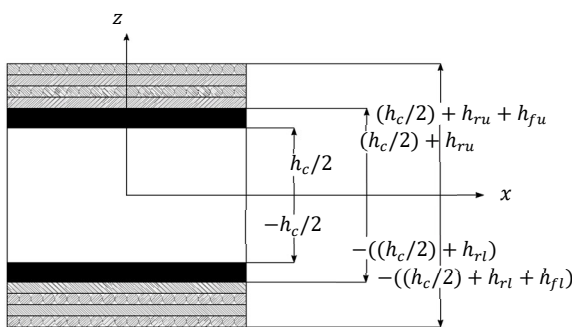


Fig. 1. Configuration of the sandwich structure layers

The layers heights of the sandwich structure are clarified as shown in Fig. 1. There, by using Eqs. 5 to 9 into Eq. 10, and Eq. 1, get,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} \left(\int_{-(h_c/2+h_{rl})}^{-(h_c/2+h_{rl}+h_{fl})} (\sigma_x)_f z dz + \int_{-(h_c/2+h_{rl})}^{-(h_c/2)} (\sigma_x)_r z dz + \int_{-(h_c/2)}^{(h_c/2)} (\sigma_x)_c z dz + \int_{(h_c/2)}^{(h_c/2+h_{ru})} (\sigma_x)_r z dz + \int_{(h_c/2+h_{ru})}^{(h_c/2+h_{ru}+h_{fu})} (\sigma_x)_f z dz \right) \\ \left(\int_{-(h_c/2+h_{rl})}^{-(h_c/2+h_{rl}+h_{fl})} (\sigma_y)_f z dz + \int_{-(h_c/2+h_{rl})}^{-(h_c/2)} (\sigma_y)_r z dz + \int_{-(h_c/2)}^{(h_c/2)} (\sigma_y)_c z dz + \int_{(h_c/2)}^{(h_c/2+h_{ru})} (\sigma_y)_r z dz + \int_{(h_c/2+h_{ru})}^{(h_c/2+h_{ru}+h_{fu})} (\sigma_y)_f z dz \right) \\ \left(\int_{-(h_c/2+h_{rl})}^{-(h_c/2+h_{rl}+h_{fl})} (\tau_{xy})_f z dz + \int_{-(h_c/2+h_{rl})}^{-(h_c/2)} (\tau_{xy})_r z dz + \int_{-(h_c/2)}^{(h_c/2)} (\tau_{xy})_c z dz + \int_{(h_c/2)}^{(h_c/2+h_{ru})} (\tau_{xy})_r z dz + \int_{(h_c/2+h_{ru})}^{(h_c/2+h_{ru}+h_{fu})} (\tau_{xy})_f z dz \right) \end{Bmatrix} \quad (12)$$

and,

$$\rho h = \begin{Bmatrix} \int_{-(h_c/2+h_{rl})}^{-(h_c/2+h_{rl}+h_{fl})} \rho_f dz + \int_{-(h_c/2+h_{rl})}^{-(h_c/2)} \rho_r dz \\ + \int_{-(h_c/2)}^{(h_c/2)} \rho_c dz + \int_{(h_c/2)}^{(h_c/2+h_{ru})} \rho_r dz \\ + \int_{(h_c/2+h_{ru})}^{(h_c/2+h_{ru}+h_{fu})} \rho_f dz \end{Bmatrix} \quad (13)$$

The inertia coefficient of the sandwich plate can be expressed as:

$$\rho h = [\rho_{fl}h_{fl} + \rho_{fu}h_{fu} + \rho_{rl}h_{rl} + \rho_{ru}h_{ru} + \rho_c h_c] \quad (14)$$

Therefore, by using many mathematical formulations, the general equation of motion for plate structure with rubber faces effect can be represented as,

$$\left(\frac{1}{12} \left(\begin{aligned} & \left(\begin{aligned} & 3h_c^2 + \\ & 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) \\ & + \left(\begin{aligned} & 3h_c^2 + 6h_ch_{rl} \\ & +4h_{rl}^2 \end{aligned} \right) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \\ & \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) + \left(\begin{aligned} & 3h_c^2 + \\ & 6h_ch_{ru} \\ & +4h_{ru}^2 \end{aligned} \right) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \end{aligned} \right) \frac{\partial^4 w}{\partial x^4} \right. \\ \left. + \left(\begin{aligned} & 3h_c^2 + \\ & 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fu}E_{fu}}{1-\nu_{fu}^2} \right) \right. \\ \left. + \frac{1}{12} \left(\begin{aligned} & \left(\begin{aligned} & 3h_c^2 + \\ & 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) + \\ & \left(\begin{aligned} & 3h_c^2 + 6h_ch_{rl} \\ & +4h_{rl}^2 \end{aligned} \right) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \\ & \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) + \left(\begin{aligned} & 3h_c^2 + \\ & 6h_ch_{ru} \\ & +4h_{ru}^2 \end{aligned} \right) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \end{aligned} \right) \frac{\partial^4 w}{\partial y^4} \right. \\ \left. + \frac{1}{12} \left(\begin{aligned} & \left(\begin{aligned} & 3h_c^2 + \\ & 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) \\ & + \left(\begin{aligned} & 3h_c^2 + 6h_ch_{rl} \\ & +4h_{rl}^2 \end{aligned} \right) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \\ & \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) + \left(\begin{aligned} & 3h_c^2 + 6h_ch_{ru} \\ & +4h_{ru}^2 \end{aligned} \right) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \end{aligned} \right) \frac{\partial^4 w}{\partial x^2 \partial y^2} \right. \\ \left. + (\rho_{fl}h_{fl} + \rho_{fu}h_{fu} + \rho_{rl}h_{rl} + \rho_{ru}h_{ru} + \rho_c h_c) \frac{\partial^2 w}{\partial t^2} \right) = 0 \quad (15)$$

Now, assume the solution to be,

$$w = \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \quad (16)$$

Then, by doing multiple integrations processes and comparing to,

$$\frac{\partial^2 w}{\partial t^2} + \omega_{mn}^2 w t = 0 \quad (17)$$

The equation of natural frequency of sandwich plate is,

$$\omega_{mn}^2 = \frac{\left(\begin{aligned} & \left(\begin{aligned} & (3h_c^2 + 6h_c(h_{fl} + 2h_{rl})) + \\ & 4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) \\ & + (3h_c^2 + 6h_ch_{rl} + 4h_{rl}^2) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \\ & \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) + (3h_c^2 + 6h_ch_{ru} + 4h_{ru}^2) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \\ & + \left(\begin{aligned} & 3h_c^2 + 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fu}E_{fu}}{1-\nu_{fu}^2} \right) \end{aligned} \right) \left(\frac{m\pi}{a} \right)^4 \\ + \left(\begin{aligned} & \left(\begin{aligned} & 3h_c^2 + 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) + \\ & (3h_c^2 + 6h_ch_{rl} + 4h_{rl}^2) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) \\ & + (3h_c^2 + 6h_ch_{ru} + 4h_{ru}^2) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \\ & + \left(\begin{aligned} & 3h_c^2 + 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fu}E_{fu}}{1-\nu_{fu}^2} \right) \end{aligned} \right) \left(\frac{n\pi}{b} \right)^4 \\ + \left(\begin{aligned} & \left(\begin{aligned} & 3h_c^2 + 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fl}E_{fl}}{1-\nu_{fl}^2} \right) \\ & + (3h_c^2 + 6h_ch_{rl} + 4h_{rl}^2) \left(\frac{h_{rl}E_{rl}}{1-\nu_{rl}^2} \right) + \\ & \left(\frac{h_c^3 E_c}{1-\nu_c^2} \right) + \left(\begin{aligned} & 3h_c^2 + 6h_ch_{ru} \\ & +4h_{ru}^2 \end{aligned} \right) \left(\frac{h_{ru}E_{ru}}{1-\nu_{ru}^2} \right) \\ & + \left(\begin{aligned} & 3h_c^2 + 6h_c(h_{fl} + 2h_{rl}) \\ & +4(h_{fl}^2 + 3h_{fl}h_{rl} + 3h_{rl}^2) \end{aligned} \right) \left(\frac{h_{fu}E_{fu}}{1-\nu_{fu}^2} \right) \end{aligned} \right) \left(\frac{mn\pi^2}{ab} \right)^2 \end{aligned} \right)}{12(\rho_{fl}h_{fl} + \rho_{fu}h_{fu} + \rho_{rl}h_{rl} + \rho_{ru}h_{ru} + \rho_c h_c)} \quad (18)$$

By building a computer program [52], the natural frequency for sandwich plate structure with the influence of nanoparticle reinforcement and mounting rubber skin layer can be evaluated.

3. Numerical investigation

The finite element method (FEM) is an excellent numerical way to analyse the dynamic response and the prediction of total deformation load, at which free vibration natural frequency can be seen in the composite sandwich panel [53-60]. Accordingly, the numerical model of the desired structure with PLA metal core, rubber layers, and aluminium skin at the top and bottom surfaces is carried out using the ANSYS software 2021 R1, as illustrated in Figure 2. The 3D finite element model with dimensions a=b=0.25 m. Poly(lactic acid (PLA), rubber, and aluminium (Al)

material properties are inserted as a new assignment into the model engineering data view. The face sheet thickness is maintained at 3.0 mm for upper and lower surfaces. For the mesh refinement process to work well, further mesh refining and convergence approach was carried out to get high-accuracy numerical results, as shown in Figure 3, which uses 59211 nodes. The generated model requires a thin layer of glue to pass through the connection area and between the layers to attain accurate results [61-68]. The boundary conditions were applied, and the modal analysis was carried out for each composite type. The results of total deformation and natural frequency are obtained based on many parameters of volume fraction and geometrical properties. Figure 4 shows a sample of ANSYS's first mode shape results, representing the fundamental natural frequency that identifies the stability of the sandwich structure. Extra modes for each panel dimension and composite parameter can be obtained using modal analysis [69-75].

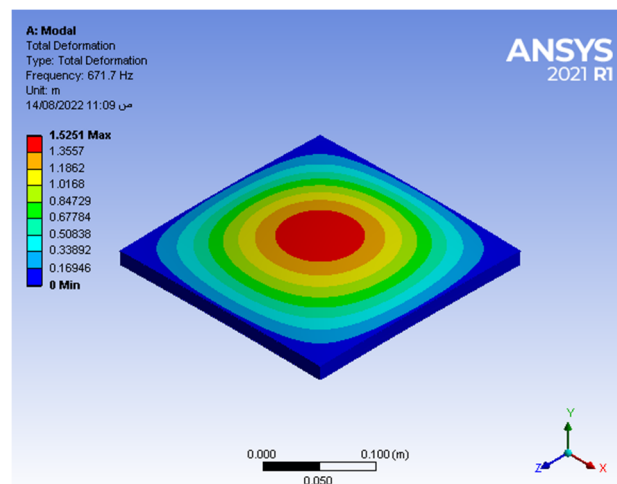


Fig. 4. 1st mode shape results

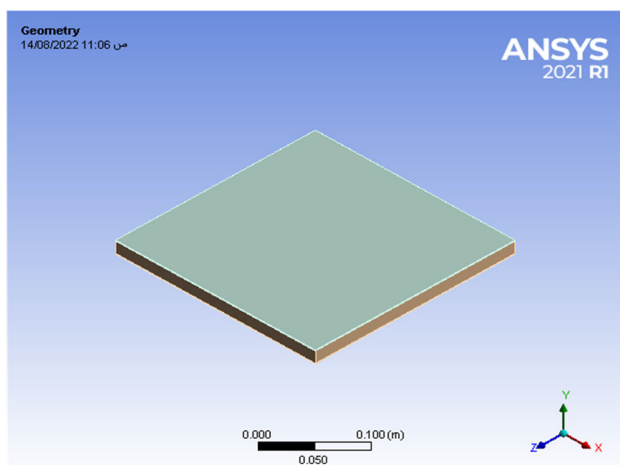


Fig. 2. The geometry of the generated model

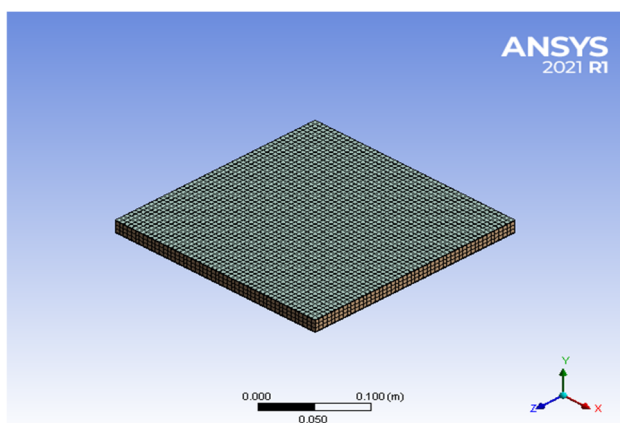


Fig. 3. The model with mesh

4. Results and discussion

The free vibration behaviour of sandwich construction with a PLA core and composite face sheets strengthened by nanoparticles of Al_2O_3 and SiO_2 with two layers of rubber is investigated in this paper. The differential equation of motion that regulates the sandwich structure's vibrational behaviour in the rubber layers must be created first and then calculating the natural frequency using various parameters. The acquired results, which can be expressed using Microsoft Excel 2021, are the normal frequencies of sandwich constructions with composite faces and rubber layers exposed to bending forces.

To determine the effects of different layer arrangements of the composite panel constitutes on the natural frequency results, several examples were carried out using an analytical solution by considering the following mechanical characteristics for sandwich layers [60].

1. Polylactic acid (PLA) core $E_c = 1.2$ Gpa, $\rho_c = 1360$ kg/m^3 , $\nu_c = 0.38$ and $h_c = 8$ mm
2. Rubber $E_r = 8$ Gpa, $\rho_c = 1480$ kg/m^3 , $\nu_c = 0.3$ and $h_r = 3$ mm
3. Nanocomposite faces and $h_f = 3$ mm, the nanoparticle volume fraction go-between (0% to 2.5%).

This study investigates the influence of rubber on the free vibration of sandwich plates with Nanocomposite faces. Kirchhoff's theory is the governing equation for modest deflections. As previously mentioned, assumptions were made to simplify the equation solution by applying different volume fractions (0 to 2.5%) of SiO_2 and Al_2O_3 nanoparticles. The findings show that sandwich plates benefit from the rubber study's key illustrations.

Table 1 presents the natural frequency results when using alumina nanoparticles as a reinforcement material for the composite faces of a sandwich plate ($\text{Al}_2\text{O}_3/\text{epoxy}$) at two densities (i.e., 1180 and 1210 kg/m^3). It is found that with an increase in volume fraction, the young modulus rises with increasing densities, yielding to grow in natural frequency. This might be due to the increased stiffness coefficients in the final equation of natural frequency. Also, the same behaviour can be seen with increased density from 1180 to 1210 kg/m^3 . The difference between the natural frequency of sandwich structures with and without rubber layers has been recorded. It is noticed that the ideal parentage reduction for rubber and SiO_2 nanoparticles at a density of 1180 kg/m^3 is 38.8%, whereas the ideal value is 40.2% with a density of 1210 kg/m^3 .

Furthermore, Table 2 gives results of the natural frequency of sandwich plates reinforced with SiO_2 nanoparticles. The increase in natural frequency becomes 3.3% if it is compared with Al_2O_3 nanoparticles. Additionally, it is

found that Al_2O_3 nanoparticles with a density of 1180 kg/m^3 and rubber are seen; the percentage drop is 36.8% at the density of 1210 kg/m^3 , while for $V_f=2.5$ per cent, it is only 38.4%. Hence, adding Al_2O_3 and SiO_2 nanoparticles to the sandwich panel increases the natural frequency. This may be due to nanoparticles sitting through voids in the composite material; as a result, composite materials can form stronger bonds between particles and grains.

Figure 5 shows the relationship between natural frequency, six values of volume fractions, and two skin densities by adding rubber by Al_2O_3 and SiO_2 . A linear trend was observed from the curves drawn, indicating that increasing the volumetric ratios with the two different densities and various nanomaterials leads to an increase in the natural frequency. As for rubber presence influence on the natural frequency, it is noticed that the natural frequency of sandwich plates rises as density and young's modulus increase. Composite faces have a greater natural frequency than the core.

Table 1.

Results of adding rubber layer on a sandwich plate with composite faces ($\text{Al}_2\text{O}_3/\text{Epoxy}$) at two densities

S	$\rho_f, \text{kg}/\text{m}^3$	$V_f, \%$	E_f, GPa	ν_f	E_c, GPa	ν_c	E, GPa	ν_r	ω, Hz	ω_r, Hz	Variation, %
1	1180	0.00	15.85	0.22	1.20	0.38	0.008	0.3	646.67	501.14	22.50
2		0.50	17.34	0.22	1.20	0.38	0.008	0.3	675.86	523.98	22.47
3		1.00	18.42	0.22	1.20	0.38	0.008	0.3	696.26	539.93	22.45
4		1.50	19.83	0.22	1.20	0.38	0.008	0.3	722.027	560.07	22.43
5		2.00	20.88	0.22	1.20	0.38	0.008	0.3	740.62	574.61	22.42
6		2.50	22.47	0.22	1.20	0.38	0.008	0.3	767.93	595.95	22.40
7	1210	0.00	16.56	0.18	1.20	0.38	0.008	0.3	655.88	506.24	22.82
8		0.50	18.23	0.18	1.20	0.38	0.008	0.3	687.61	530.95	22.78
9		1.00	19.47	0.18	1.20	0.38	0.008	0.3	710.26	548.58	22.76
10		1.50	20.44	0.18	1.20	0.38	0.008	0.3	727.48	561.99	22.75
11		2.00	21.89	0.18	1.20	0.38	0.008	0.3	752.49	581.46	22.73
12		2.50	24.13	0.18	1.20	0.38	0.008	0.3	789.57	610.31	22.70

Table 2.

Results of adding rubber layer on a sandwich plate with composite faces ($\text{SiO}_2/\text{Epoxy}$) at two densities

S	$\rho_f, \text{kg}/\text{m}^3$	$V_f, \%$	E_f, GPa	ν_f	E_c, GPa	ν_c	E, GPa	ν_r	ω, Hz	ω_r, Hz	Variation, %
1	1180	0.00	15.85	0.22	1.20	0.38	0.008	0.3	646.67	501.14	22.50
2		0.50	17.92	0.22	1.20	0.38	0.008	0.3	686.89	532.60	22.46
3		1.00	19.12	0.22	1.20	0.38	0.008	0.3	709.16	550.02	22.44
4		1.50	20.33	0.22	1.20	0.38	0.008	0.3	730.93	567.04	22.42
5		2.00	21.74	0.22	1.20	0.38	0.008	0.3	755.51	586.25	22.40
6		2.50	23.87	0.22	1.20	0.38	0.008	0.3	791.20	614.13	22.38
7	1210	0.00	16.56	0.18	1.20	0.38	0.008	0.3	655.88	506.24	22.82
8		0.50	19.46	0.18	1.20	0.38	0.008	0.3	710.08	548.44	22.76
9		1.00	20.74	0.18	1.20	0.38	0.008	0.3	732.73	566.07	22.75
10		1.50	22.31	0.18	1.20	0.38	0.008	0.3	759.58	586.97	22.72
11		2.00	23.85	0.18	1.20	0.38	0.008	0.3	785.03	606.78	22.71
12		2.50	25.78	0.18	1.20	0.38	0.008	0.3	815.81	630.71	22.69

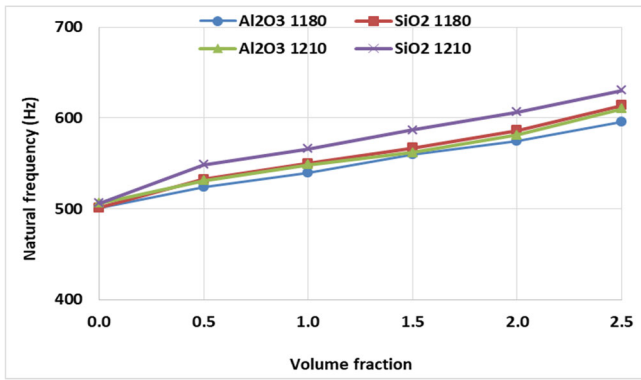


Fig. 5. The relationship between natural frequency and volume fraction using two skin densities with adding rubber by Al₂O₃ and SiO₂

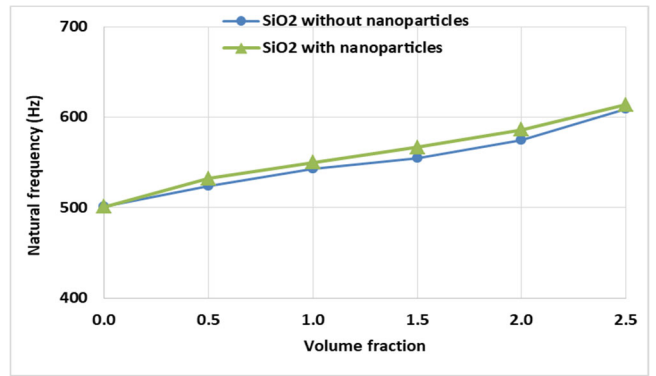


Fig. 8. The natural frequency results with and without SiO₂ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1180 kg/m³

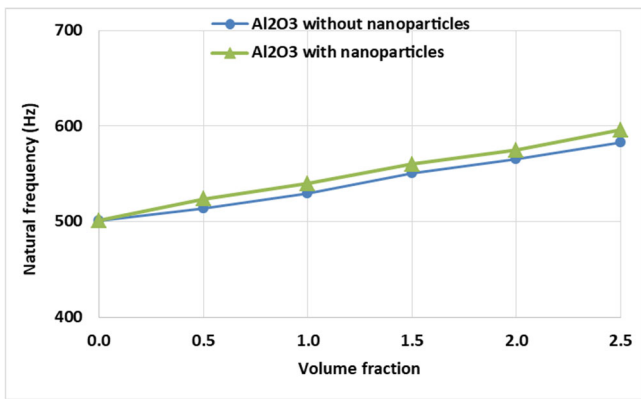


Fig. 6. The results of the natural frequency with and without Al₂O₃ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1180 kg/m³

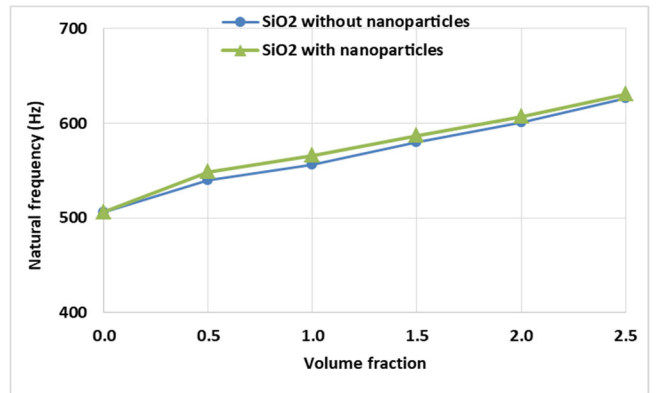


Fig. 9. The natural frequency results with and without SiO₂ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1210 kg/m³

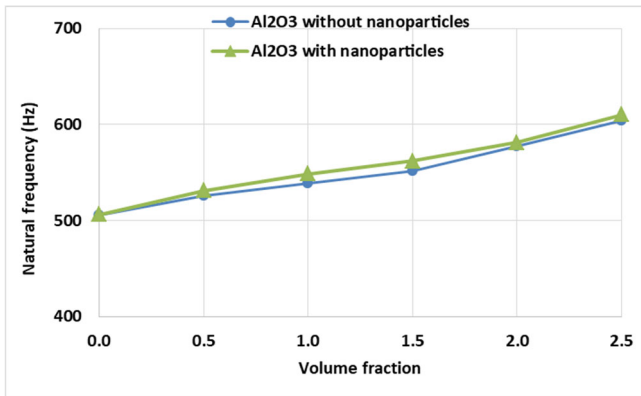


Fig. 7. The natural frequency results with and without Al₂O₃ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1210 kg/m³

The addition of a rubber layer between the faces and core has a significant effect on reducing vibration. Figures 6 and 7 show the natural frequency results with and without Al₂O₃ nanoparticles for the sandwich panel using a rubber cushion layer at skin density (1180 and 1210 kg/m³), respectively. It is seen that the natural frequency value is almost constant at ($V_f = 0$); as it starts to rise steeply, the natural frequency develops due to an increase in the elastic modulus of structural parts. The results show a clear improvement in natural frequency results with increasing mass density of skins due to an increase in overhaul stiffness of the sandwich panel; for example, by increasing mass density at a volume fraction of 2.5%, the enhancement of natural frequency is 2.35%. On the other hand, this phenomenon occurs in Figures 8 and 9 for sandwich panels reinforced with SiO₂ nanoparticles with increasing natural frequency by 2.63% at a volume fraction of 2.5% when changing mass density from

1180 to 1210 kg/m³. In the sandwich panels, the increasing skin density results in less panel deflection and hence increase stiffness, leading to the increased natural frequency.

This study found that Nano SiO₂ improves mechanical performance, modifies vibration characterization, and increases composite panel strength better than Nano Al₂O₃.

To validate the analytical solution results, Figure 10 and Figure 11 illustrate the analytical and numerical solution comparison of the effect of adding nanoparticles Al₂O₃ and SiO₂ to the composite structure on the free vibration characteristics. The findings show good agreement between the numerical and analytical natural frequencies with a maximum percentage of no more than 5%. Furthermore, it is seen that reinforced nanomaterials play a major role in the behaviour of structural stability. The presence of nanoparticles SiO₂ improves the natural frequency better than the

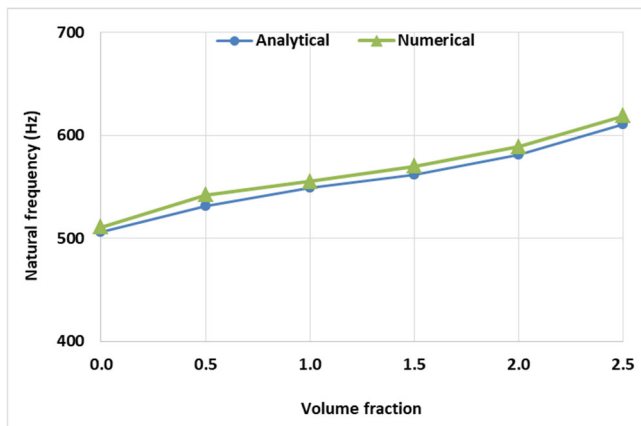


Fig. 10. Analytical and numerical natural frequency results with Al₂O₃ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1210 kg/m³

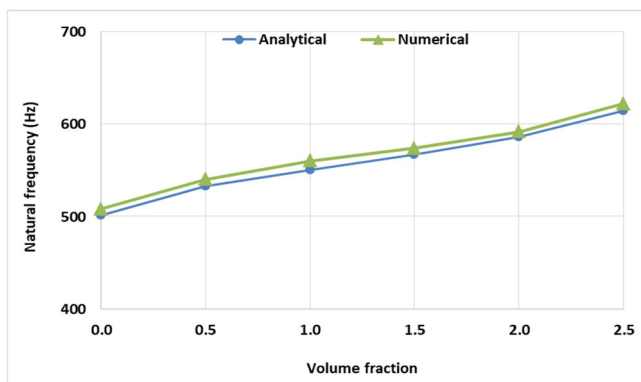


Fig. 11. Analytical and numerical natural frequency results with SiO₂ nanoparticles for the sandwich panel using a rubber cushion layer at skin density 1180 kg/m³

effect of nanoparticles Al₂O₃. However, Figure 11 recorded the highest natural frequency of 614 Hz at a volume fraction of 2.5%. When nano volume fractions are added to composite panels, modulus of elasticity values are improved, natural frequencies are increased, and a composite panel's strength is enhanced. The results show that the panels with an increasing skin density have more natural frequency values due to the greater resistance of the core to penetration.

5. Conclusions

The new technique for determining free vibration characteristics of the multi-layered panel with viscoelastic layers is investigated. The formulations used to describe the behaviour of layered elements of the composite panel is based on the Kirchhoff plate theory. However, some expected behaviours, such as free vibration, are presented to understand the influence of rubber layers and nanomaterials on the performance of composite panels. Results of this investigation show that the vibration characteristics of the composite structure are influenced by the following:

1. Using nano-reinforced composite materials led to an increase in the natural frequency of the sandwich structure.
2. Rubber layers have been added to reduce vibration response and thus reduce the natural frequency.
3. The addition of rubber has a positive effect; hence the mechanical performance is improved.
4. Imperceptible weight gain in contrast to an improvement in performance.

Using the results of this investigation, it seems that sandwich plate vibration patterns and their correlations may be explained. The findings of this research might be used in the future to develop and produce intelligent materials.

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