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# Analysis of Dynamics of Remote-Controlled Artillery-Missile System

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**Abstract.** The paper presents physical and mathematical model of the modified remote-controlled the artillery-missile system (AMS) system of the Wróbel II type that is intended to be located on a warship. The proposed system is used for independent detection, tracking and counteracting the means of air attack at short distances. Moreover, principle of operation of SAR is presented and analysis of the system's dynamics was carried out. The results of investigations are given in a graphical form.
**Keywords:** artillery-missile system, artillery-missile defence, dynamics, control

## 1. INTRODUCTION

The naval set Wróbel II (ZU-23-2MR) is, first of all, intended for fight against manoeuvring aerial targets and being in spot hovering ones. However, efficiency of the set decreases with increase in sea disturbance.

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A warship at sea waves is subjected to angular and linear extortions that through a deck are transferred to the set [1, 2].

As a result, line of sight moves along the complex trajectory that has to be manually compensated by an operator. Sometimes it is not possible because of too dynamic changes of displacements, what negatively influences on the set accuracy during combat missions [3]. One of the method of counteracting this phenomenon is automation of detection, guidance, and tracking of targets [4,5]. Moreover, additional advantage is the possibility of relocation of an operator work, e.g., under a deck, where he will not be exposed to direct attacker fire.

Essential modification of the proposed system would be the possibility of the set operation in an automatic mode. The target would be detected and tracked with the use of a scanning-tracking optoelectronic head [6]. After the object identification, the information on its coordinates goes to a control system [7] where in real time the control signals are determined for the set drives: rotation (azimuth) and lift (elevation). An operator function will be limited only to confirmation of the shot fire, after receiving an adequate prompt through the system's interface.

## 2. PHYSICAL MODEL OF THE SET

It was assumed that disturbances from angular and linear displacements of a warship will be compensated by the existing rotation and elevation drives, i.e., apart the tracking movements, these drives will make also compensating movements. To investigate such a control method, 3D model of the set, being the base for further considerations, has been built.

The developed model is shown in Fig. 1. As it was mentioned previously, a basis for this model was the existing naval set Wróbel II. Using available data, pictures, and technical descriptions, the authors tried very precisely to reflect the set's geometry.

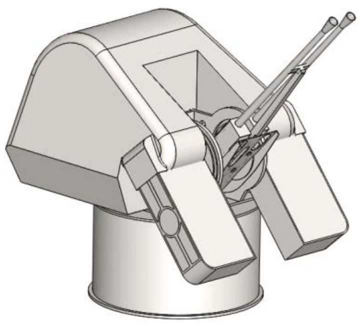

Fig. 1. 3D model of the Wróbel II set – general view

Figure 2 shows main parts of the set consisting of: an immobile cylindrical base, on which a rotating upper gun carriage is mounted, decisive for azimuth angle of the set; a cradle, with a pair of 2A14 air cannons, setting an elevation angle, situated in pivots of the upper gun carriage, ammunition boxes that are attached to the sides of the upper gun carriage (bed); a shield and mass element balancing an interior equipment. Modelling has been performed with SolidWorks program that, on the basis of geometry and physical parameters of the applied materials, can determine the mass and moments of inertia of the given element or a group of elements in regard of any defined axis.

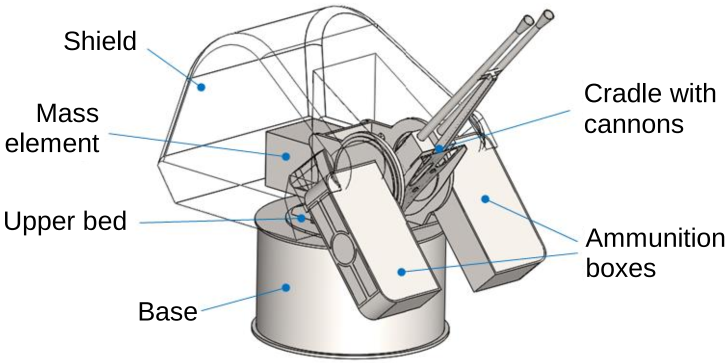


Fig. 2. Modelled elements of the set

The 3D model, presented in Fig. 2, was used for a mathematical model determination. It is a system of two degrees of freedom what is presented in Fig. 3. It consists of body 1, corresponding to the upper gun carriage with the attached to it elements and body 2 corresponding to the cradle with cannons.

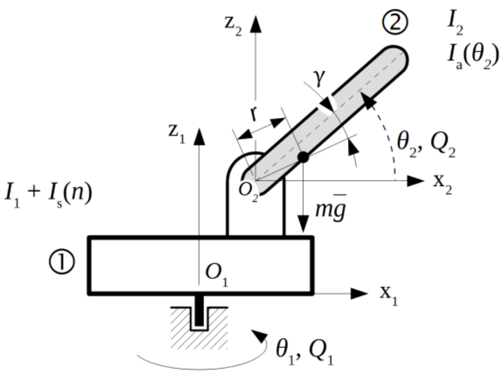


Fig. 3. Scheme of a physical model of the considered system

Body 1 revolves around the axis $z_1$ about the azimuth angle $\theta_1$, and body 2 around the axis $y_2$ (directed perpendicularly deep into the figure) about the elevation angle $\theta_2$. Both angles are directed angles.

Mathematical symbols in Fig. 3:

| | | |
|---|---|---|
| $\theta_1$ | – | angle of rotation (the set rotation) |
| $\theta_2$ | – | angle of elevation |
| $Q_i = M_i - T_i$ | – | generalised torque acting on the $i$-th body |
| $M_i$ | – | driving torque acting on the $i$-th body |
| $T_i$ | – | torque of friction acting on the $i$-th body |
| $I_1$ | – | constant mass moment of inertia of body 1 vs. the $z_1$ axis |
| $I_s(n)$ | – | variable mass moment of inertia of body 1 vs. the $z_1$ axis, dependent on a number of cartridges in the boxes $n$ |
| $I_2$ | – | constant mass moment of inertia of body 2 vs. the $y_2$ axis |
| $I_a(\theta_2)$ | – | variable mass moment of inertia of body 2 vs. the $z_1$ axis, dependent on the elevation angle $\theta_2$ |
| $m$ | – | mass of body 2 |
| $g$ | – | gravitational acceleration |
| $r$ | – | distance between centre of gravity of body 2 and the rotation axis $y_2$ |
| $\gamma$ | – | angular displacement of the centre of gravity of body 2 vs. barrels' axes |

## 3. MATHEMATICAL MODEL OF THE SET

### 3.1. Determination of mass moments of inertia

The mass moments of inertia $I_1$ = 160 kg·m/s$^2$ and $I_2$ = 60 kg·m/s$^2$ are constant and they can be determined numerically using SolidWorks program. The variable moments $I_s(n)$ and $I_a(\theta_2)$ should be described with some function. The following activities have been done. First, the mass moment of inertia of the $I_s(n)$ boxes was determined for five various cartridges contents what is marked with points on the chart shown in Fig. 4a. It should be noticed that these points are along some straight line thus, it is possible to match some linear function and to find its coefficients. The above mentioned function is described by Eq. 1.

$$I_s(n) = pn + q = 0.93n + 14 \tag{1}$$

Analogously, it was done with the moment of inertia $I_a(\theta_2)$, determining a dozen or so of its values dependent on an elevation angle of cannons. Third-order polynomial was matched to the determined values and its coefficients were found. This function is described by Eq. 2 and its course is shown in Fig. 4b.

$$I_a(\theta_2) = a\theta_2^3 + b\theta_2^2 + c\theta_2 + d = 28.82\,\theta_2^3 - 77.65\,\theta_2^2 + 10.41\,\theta_2 + 112.8 \qquad (2)$$

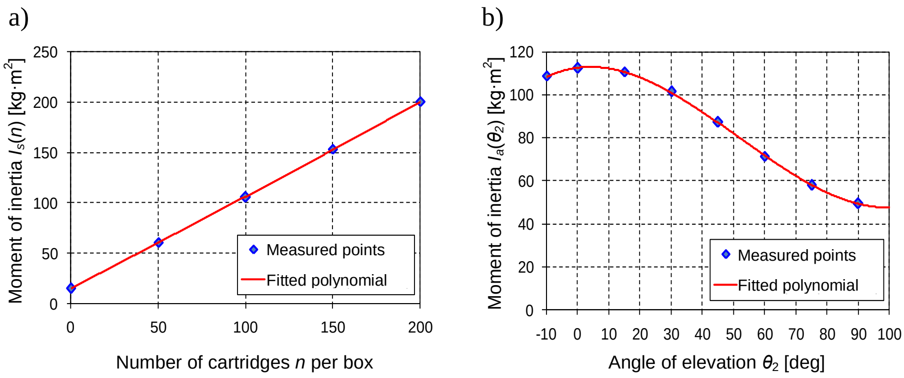


Fig. 4. Courses of functions of mass moments of inertia:
a) boxes and b) body 2 vs. the vertical rotation axis $z_1$

A relative error of the value of the matched polynomial at the measurement points is small and it does not exceed +/- 0.5%.

## 3.2. Equations of the system movement

Equations of the system movements were formulated on the basis of the Lagrange's equation of the second kind. Thus, energies of both bodies, shown in Fig. 3, have been described. The first body has the kinetic energy $E_1$ resulting from its rotation around the axis $z_1$ and the second body has the kinetic energy $E_2$ that is the sum of rotation energy of this body around the axis $y_1$ and the energy of its rotation around the axis $z_1$. Kinetic energies of particular bodies are described by the following relations:

$$E_1 = \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}I_s\dot{\theta}_1^2 = \frac{1}{2}(I_1 + I_s)\dot{\theta}_1^2 = \frac{1}{2}(I_1 + pn + q)\dot{\theta}_1^2 \qquad (3)$$

$$\begin{aligned} E_2 &= \frac{1}{2}I_2\dot{\theta}_2^2 + \frac{1}{2}I_a\dot{\theta}_1^2 = \frac{1}{2}\left(I_2\dot{\theta}_2^2 + I_a\dot{\theta}_1^2\right) \\ &= \frac{1}{2}\left[I_2\dot{\theta}_2^2 + (a\theta_2^3 + b\theta_2^2 + c\theta_2 + d)\dot{\theta}_1^2\right] \end{aligned} \qquad (4)$$

For the first body, the potential energy $V_1$ is constant and equal to zero and the second body has the potential energy $V_2$, that is dependent on a vertical component of mass centre position of this body in regard of the axis $y_2$.

Potential energies of particular bodies are described by the following relations:

$$V_1 = 0 \tag{5}$$

$$V_2 = mgh = mgr\sin(\theta_2) \tag{6}$$

Next, after calculation of Lagrangian and derivatives, equations of the generalized torques $Q_1$ and $Q_2$ were formulated:

$$(3a\theta_2^2 + 2b\theta_2 + c)\dot{\theta}_1\dot{\theta}_2 + (I_1 + pn + q + a\theta_2^3 + b\theta_2^2 + c\theta_2 + d)\ddot{\theta}_1 = Q_1 \tag{7}$$

$$I_2\ddot{\theta}_2 - \frac{1}{2}(3a\theta_2^2 + 2b\theta_2 + c)\dot{\theta}_1^2 + mgr\cos(\theta_2 + \gamma) = Q_2 \tag{8}$$

The generalised torque $Q_i$ is a driving torque at the body $M_i$, reduced by the friction torque $T_i$. The assumed model of a friction torque [8] consists of two elements: the element $T_{i0}$, dependent on a rotation velocity and the constant element $T_{i1}$, dependent on the load. Particular friction torques describe the following relations:

$$T_i = T_{i0} + T_{i1} \tag{9}$$

$$T_{i0} = f_0 \cdot 10^{-7} \cdot (v \cdot n)^{2/3} d_i^3 \tag{10}$$

$$T_{i1} = u_1 \cdot f_1 \cdot P_{i0} \cdot \frac{d_i}{2} \tag{11}$$

where:
$f_0$, $f_1$, $u_1$– coefficient dependent on a bearing type, [3-4],
$v$ – kinematic viscosity of lubricant, mm$^2$/s,
$n$ – rotational velocity, rot/min,
$d_i$ – pitch diameter of a bearing, mm,
$P_{i0}$ – loading force, N.

## 4. REVERSE ANALYSIS OF THE SYSTEM's DYNAMICS

### 4.1. Movement with rotation mechanism

Figure 5 presents simulation of movement of the first body, i.e., change in azimuth at the constant angle of elevation of 45°. Angular acceleration, shown in Fig. 5a, was set. Figures 5b and 5c show the velocity and position value of the azimuth angle. In Fig. 5d, a solid line shows a course of the driving torque which should be put in order to make the determined rotation of the set and broken line shows the torque which should be put to the second body.

It should be noticed that despite constant angle of elevation of 45$^o$, the torque is not constant and from 1 s to 1.5 s it reaches its maximum value. It results from acting the centrifugal force on the whole system during the change of the azimuth angle. The simulation has been carried out for full ammunition boxes.

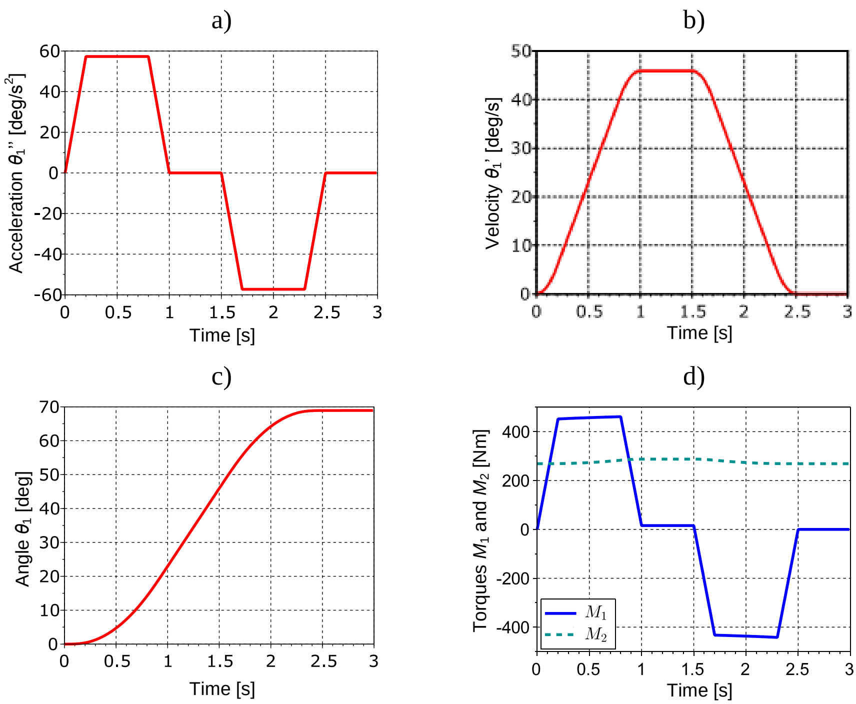


Fig. 5. Courses of model simulation for body 1 movement: a) acceleration, b) velocity, c) position, d) resulting driving torques on bodies 1 and 2

Figure 6 presents the components of the torque of friction in a joint of body 1 during the above described movement.

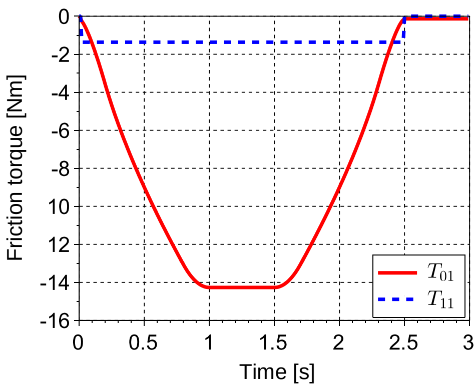


Fig. 6. Components of the torque of friction in a joint of body 1

## 4.2. Movement with elevation mechanism

The second case concerns the movement in elevation. The simulation results are given in Fig. 7. The angle of azimuth is zero and remains constant. Analogously as in the previous section, the course of angular acceleration has been set for body 2 what is shown in Fig. 7a. Angular velocity increases up to the maximum value of about 45°/s what is illustrated in Fig. 7b. The cannons elevate from 0 to 70 degrees for 2.5 s what can be seen in Fig. 7c. The torque which should be set for the second body is shown in Fig. 7d. The driving torque for the first body is constant and equals 0.

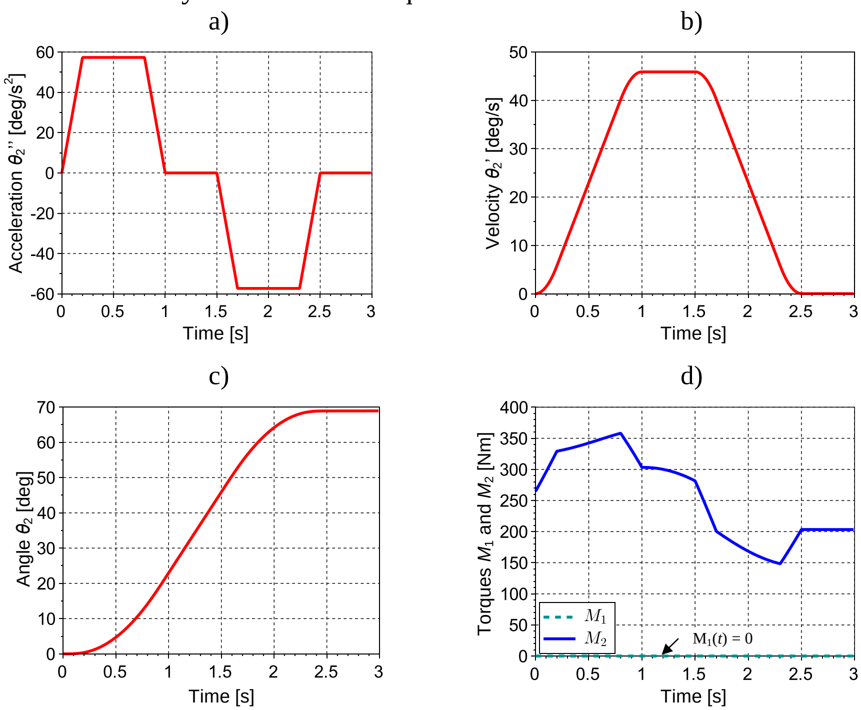


Fig. 7. Courses of model simulations for movement of body 2: a) acceleration, b) velocity, c) position, d) resulting driving torques on bodies 1 and 2

## 4.3. Assumption for rotation and elevation

The third, from the presented cases, concerns simultaneous movement of both bodies. Simultaneously, the set is rotated in azimuth with the constant velocity of 57°/s and the cannons are elevated as it was shown in the previous point. Figures 8a, 8b, and 8c present the values of acceleration, velocity, and angular position of both bodies. Interesting are the courses of driving torques of bodies, especially the first body (blue line).

We rotate the set with a constant velocity, and at some torques, the driving torque reaches a negative value, i.e., it acts in opposite direction to the movement direction. This phenomenon results from decrease in the torque of inertia of the whole set in regard of vertical axis during the cannons elevation.

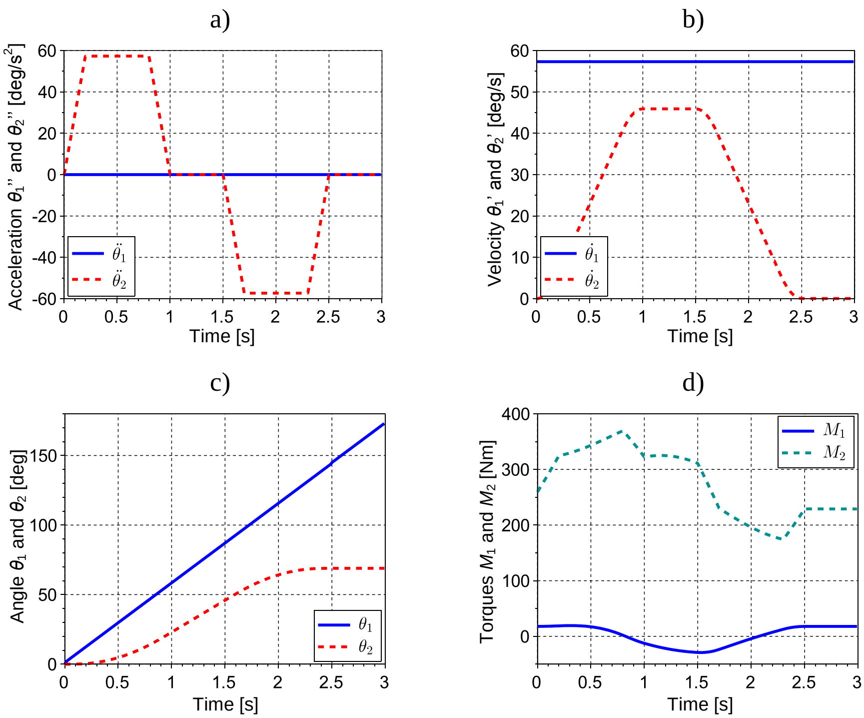


Fig. 8. Courses of model simulations for movement of bodies 1 and 2:
a) acceleration, b) velocity, c) position, d) resulting driving torques on bodies 1 and 2

## 5. SUMMARY AND CONCLUSIONS

3D model of artillery-missile set has been built on the basis of the Wróbel II set. This model was used for numerical determination of masses and moments of inertia of the bodies in regard of the defined rotation axes. Next, the mathematical model of the set was formulated using the Lagrange's equations of motion of the second kind. The obtained mathematical model was applied for reverse analysis of the system's dynamics.

The driving torques, that should be put to particular bodies for making the given movement were investigated. Also, the friction torques in bearings were taken into account. The carried-out analysis showed the dynamic dependence between movements of both bodies.

Movement of one body, in determined situations influences the driving torque that should be put to the second body.

It means that the considered system is not a stationary one from the point of view of a control process, Thus, it should be expected that typical controllers of PID and PD type will not ensure optimal control for various configurations of the system, However, the courses and values of the driving torques show that application of adequate gear ratios provides the possibility of physical implementation of the given movements and remote control of the system. Moreover, it was noticed that the torque controlling the elevation can be decreased by a shift of the mass centre of the cradle with cannons towards the horizontal axis of rotation.

Further investigations will concern the extension of the mathematical model by means of adding the influence of displacements of a mobile object on which the set is located. Such a set will be next tested in a closed control system using various controllers. Control investigations of the set will be carried out also in the conditions of influence of random disturbances, i.e., disturbances originating from a mobile object motion, at which the set is mounted, as well as from the process and measurement noises.

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# Analiza dynamiki zdalnie sterowanego systemu artyleryjsko-rakietowego

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**Streszczenie.** W pracy przedstawiono model fizyczny i matematyczny zmodyfikowanego, zdalnie sterowanego systemu artyleryjsko-rakietowego (SAR) typu Wróbel II przeznaczonego do posadowienia na okręcie. Proponowany system przeznaczony jest do autonomicznego wykrywania, śledzenia oraz zwalczania środków napadu powietrznego na bliskich odległościach. Ponadto w artykule przedstawiono zasadę działania SAR oraz przeprowadzono analizę jego dynamiki. Wyniki badań przedstawiono w postaci graficznej.
**Słowa kluczowe:** system artyleryjsko-rakietowy, obrona przeciwlotnicza, dynamika, sterowanie