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ANALYTICAL METHOD OF A CALCULATION OF A TORQUE RIPPLE OF A TWO-PHASE ASYNCHRONOUS MOTOR SUPPLIED BY A PWM CONTROLLED INVERTER

Abstract: The paper deals with steady state calculation of torque and currents ripples of a two-phase asynchronous machine, which is supplied by an IGBT transistors half-bridge connected inverter. The inverter output voltage is controlled by a PWM of the input DC voltage. The complex Fourier series analysis of the inverter output voltage was made, to obtain a spectrum of the harmonic supply voltages. The different voltage harmonic was applied to the two-phase induction machine model to obtain the electromagnetic torque and currents waveforms for various operation conditions.
Keywords: *asynchronous motor, PWM controlled inverter, modeling, analytical method*

1. Introduction

Electrical low-power drives (around 100W) which are supplied by a single-phase voltage, used in different industrial and domestic devices are presently increasingly deployed by a two-phase motors.

Two-phase motors by their characteristics no differ from three-phase motors. Their advantage is easier winding, which is of great importance for automated production. The two-phase motors are manufactured as either squirrel cage asynchronous or permanent magnets synchronous motors. They are deployed as drive of pumps in a washing machines and dishwashers, but also as the circulating pumps for central domestic heating. A permanent magnet is in this case, water and lye resistant, allows making a pump with an absolute waterproof. Two-phase voltage is produced from the single-phase network supplied by converters.

Use two-phase motor has any several advantages. The stator winding has most simple form. Three shifted coil winding form one phase. The stator windings can be configured in either a serial or parallel two-phase system.

Normally, the windings are identical. The windings which form one phase are connected to induce opposite magnetic polarity.

2. Mathematical model of the supply converter

For inverter's operation study at steady state we consider following idealized conditions:

- Power switch, that means the switch can handle unlimited current and blocks unlimited voltage.
- The voltage drop across the switch and leakage current through switch are zero.
- The switch is turned on and off with no rise and fall times.
- Sufficiently good size capacity of the input voltage capacitors divider, to can suppose converter input DC voltage to by constant.

This assumption helps us to analyze a power circuit and helps us to build a mathematical model for the inverter at steady state. Figure 1 shows two-phase converter circuit layout.

An improvement to the notched waveform is to vary the on and off periods such that the on-periods are longest at the peak of the wave. This form of control is known as pulse-width modulation (PWM).

If the desired reference voltage is sine-wave, two parameters define the control:

- *Coefficient of the modulation* m - equal to the ratio of the modulation and reference frequency.
- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the DC supply voltage.

Most the synchronic modulation is used. In synchronic modulation the modulation frequency is an integer multiple of the reference sine-wave.

Generally to control the inverter numeric control device is used. The turn on (α) and turn off (β) angles are calculated by the discredit of the reference sine-wave. That means the reference sine-wave is by a values discreet replaced.

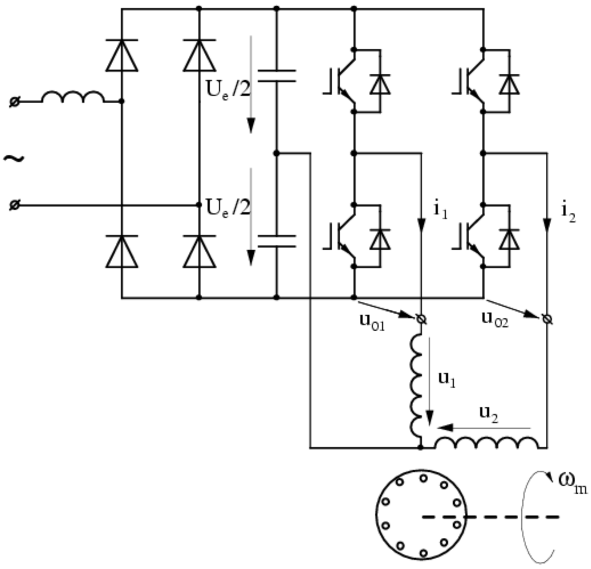

Fig.1 Supply converter circuit layout

If the coefficient of modulation m is sufficiently great, the difference between real values and discrete values is negligible.

The inverter's output voltage of the first branch can be mathematically expressed as a complex Fourier series of the form:

$$u_{01} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} \tag{1}$$

$$\begin{cases} c_{01n} = \dfrac{1}{j2k\pi}\left(e^{-jk\alpha_{01n}} - e^{-jk\beta_{01n}}\right) & for \quad k \neq 0 \\ c_{01n} = \dfrac{\beta_{01n} - \alpha_{01n}}{2\pi} & for \quad k = 0 \end{cases}$$

Similarly for the second branch:

$$u_{02} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} \tag{2}$$

$$\begin{cases} c_{02n} = \dfrac{1}{j2k\pi}\left(e^{-jk\alpha_{02n}} - e^{-jk\beta_{02n}}\right) & for \quad k \neq 0 \\ c_{02n} = \dfrac{\beta_{02n} - \alpha_{02n}}{2\pi} & for \quad k = 0 \end{cases}$$

Calculated waveforms of the branch's voltages are unipolar. Based on the voltage equation, the phase voltages are given as a different between branch voltages and voltage of the capacitor divider:

$$\begin{aligned} u_1 &= u_{01} - \frac{U_e}{2} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} - \frac{U_e}{2}; \\ u_2 &= u_{02} - \frac{U_e}{2} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} - \frac{U_e}{2} \end{aligned} \tag{3}$$

In the Fig. 2 are shown the phase voltages waveforms. The voltages are bi-polar with amplitude equal to half of DC input voltage.

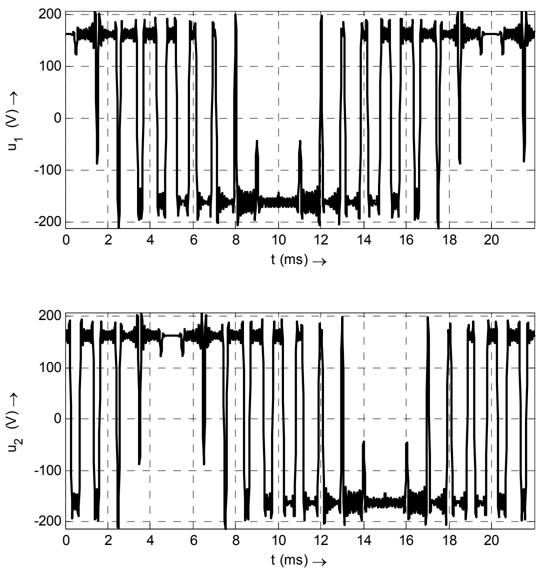

Fig.2 Waveforms of the phase voltages

3. Harmonic analysis of the supply voltages

On the base of Fourier series formulas of the supply voltages, can be made a harmonic analysis of the supply waveforms.

The amplitude of each harmonic is calculated on the base of equations (3). Amplitude of k^th harmonic is given:

$$A_k = \sum_{n=1}^{m} \left(c_{01n}^{k} + c_{01n}^{-k} \right) \tag{4}$$

The Fig.3 depicts a harmonic analysis of the PWM output voltage for frequency of $50\,Hz$ and modulation frequency of $1\,kHz$.

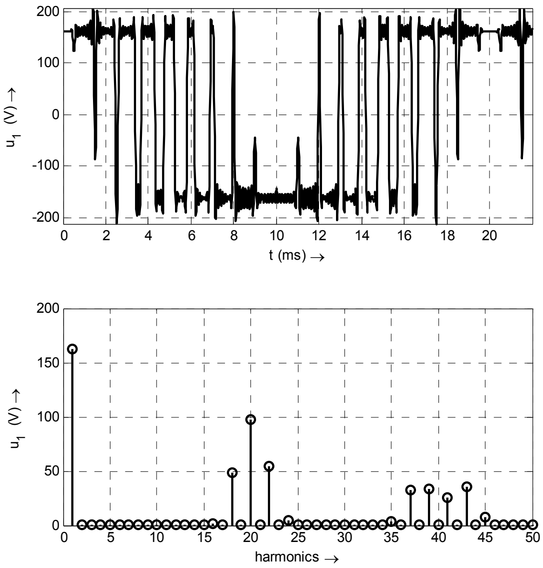

Fig.3 Harmonic analysis

Tab.1 The main harmonics

harmonics	Amplitude	1. phase	2.phase
1	162,3 V	cos	sin
18	48,4 V	-cos	cos
20	97,8 V	cos	cos
22	54,2 V	-cos	cos
37	32,6 V	cos	sin
39	33,3 V	-cos	sin
41	25,7 V	-cos	-sin
43	25,4 V	-cos	-sin

In the Tab.1 are given the parameters of the main harmonics.

4. Steady-state mathematical model of a two-phase asynchronous motor.

For the system which is associated with the rotating magnetic field, following equation of a two-phase asynchronous machine are valid.

$$\begin{aligned} u_{ds} &= R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_s \psi_{qs} \\ u_{qs} &= R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_s \psi_{ds} \\ u_{dr} &= R_r i_{dr} + \frac{d\psi_{dr}}{dt} - \left(\omega_s - \omega_m\right) \psi_{qr} \\ u_{qr} &= R_r i_{qr} + \frac{d\psi_{qr}}{dt} + \left(\omega_s - \omega_m\right) \psi_{dr} \end{aligned} \tag{5}$$

Since homopolar components of the system are zero, the equation of the machine can be transformed into single axis. Quantities $d-q$ are replaced by a single complex variable with real axis d and imagine axis q .

$$\begin{aligned} \mathbf{u_s} &= u_{ds} + j u_{qs} \\ \mathbf{u}_r &= u_{dr} + j u_{qr} \\ \boldsymbol{\psi}_s &= \psi_{ds} + j \psi_{qs} \\ \boldsymbol{\psi}_r &= \psi_{dr} + j \psi_{qr} \\ \mathbf{i}_s &= i_{ds} + j i_{qs} \\ \mathbf{i}_r &= i_{dr} + j i_{qr} \end{aligned}$$

Equations *(5)* take a form.

$$\begin{aligned} \mathbf{u}_s &= R_s \mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt} + j\omega_s \boldsymbol{\psi}_s \\ \mathbf{u}_r &= R_r \mathbf{i}_r + \frac{d\boldsymbol{\psi}_r}{dt} + j\left(\omega_s - \omega_m\right) \boldsymbol{\psi}_r \end{aligned} \tag{6}$$

With the flux linking components defined as.

$$\begin{aligned} \boldsymbol{\psi}_s &= L_s \mathbf{i}_s + L_m \mathbf{i}_r \\ \boldsymbol{\psi}_r &= L_m \mathbf{i}_s + L_r \mathbf{i}_r \end{aligned} \tag{7}$$

After the solving (6) and (7), we obtain for stator and rotor current phasors.

$$\begin{aligned} \mathbf{i}_s &= \frac{\mathbf{u}_s \left(R_r / s + j\omega L_r \right)}{\left(R_s + j\omega L_s \right).\left(R_r / s + j\omega L_r \right) + \omega^2 L_m^2} \\ \mathbf{i}_r &= -\frac{j\omega L_m}{\left(R_s + j\omega L_s \right).\left(R_r / s + j\omega L_r \right) + \omega^2 L_m^2} \end{aligned} \tag{8}$$

The phase currents $i_{s1}; i_{s2}; i_{r1}; i_{r2}$ are obtained on the base of current phasors by Clarke transformation.
For the electromagnetic torque the following relation is valid.

$$M_{em} = p L_m \left(i_{s2} i_{r1} - i_{s1} i_{r2} \right) \tag{9}$$

5. Examples of calculus

The following pictures show waveforms of stator currents, rotor currents and electromagnetic torque in a steady state. The parameters of a prototype of a two-pole induction motor were used. The motor has the parameters:

$R_s = 31\Omega;$

$R_r = 51\Omega;$

$L_{1h} = 1{,}181H;$

$L_{1\sigma} = L'_{2\sigma} = 0{,}15H;$

The stator and rotor inductance are defined.

$L_s = L_m + L_{1\sigma} = 1{,}331H$

$L_r = L_m + L'_{2\sigma} = 1{,}331H$

The stator, rotor currents and torque were calculated for each of harmonic. For the waveform the following equations are valid.

$$i_{1s} = \sum_{k=1}^{8} i_{1s}^{k}; i_{2s} = \sum_{k=1}^{8} i_{2s}^{k}; i_{1r} = \sum_{k=1}^{8} i_{1r}^{k}; i_{2r} = \sum_{k=1}^{8} i_{2r}^{k}; \quad (10)$$

Similarly we can write for the torque waveform.

$$M_{em} = \sum_{k=1}^{8} M_{em}^{k} \quad (11)$$

A figure (4) and (5) shows stator and rotor current waveforms for no loaded machine. Figure (6) depicts the electromagnetic torque waveform at constant speed and no loaded machine.

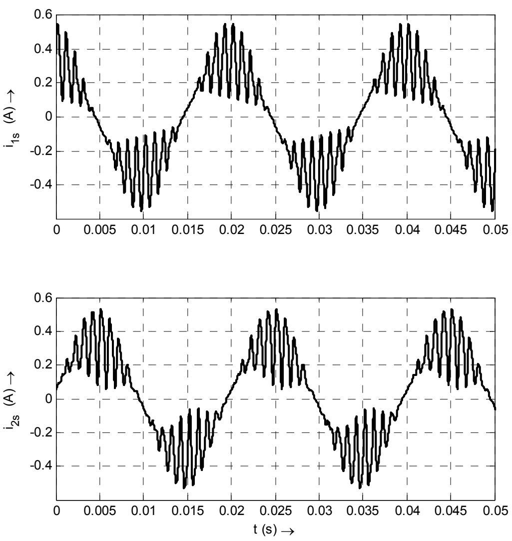

Fig.4 Stator current waveforms for no loaded machine

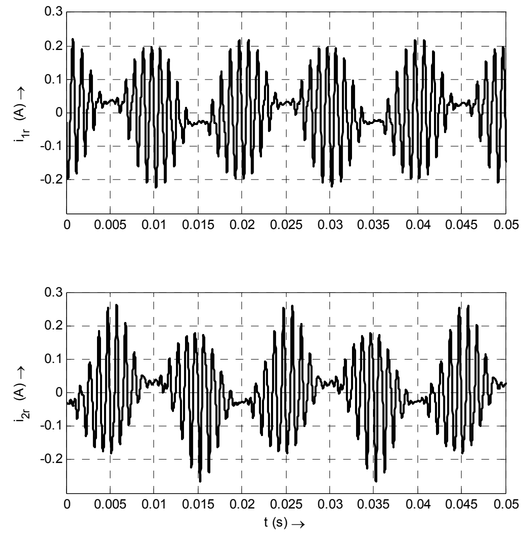

Fig.5 Rotor current waveforms for no loaded machine

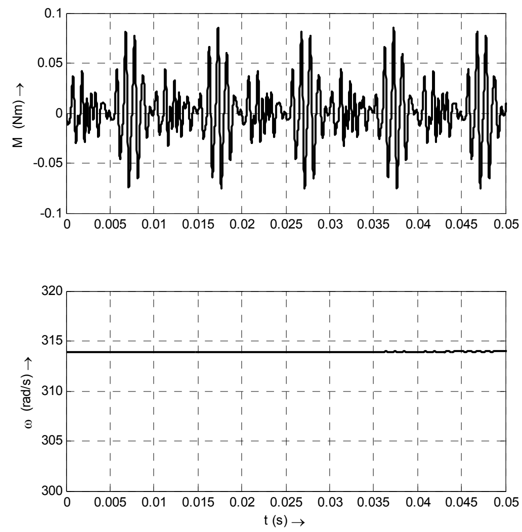

Fig.6 Torque waveform and speed for no loaded machine

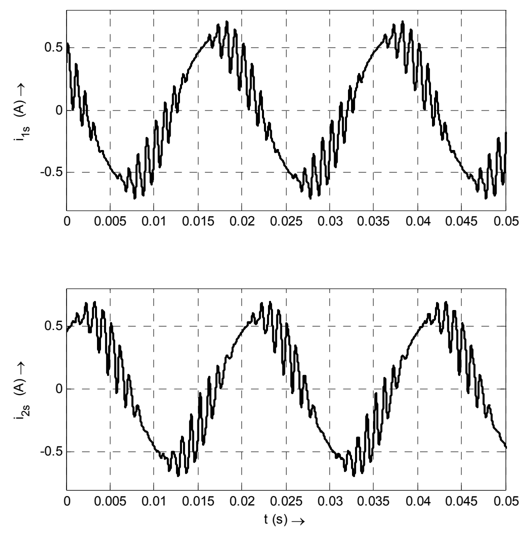

Fig.7 Stator current waveforms for loaded machine

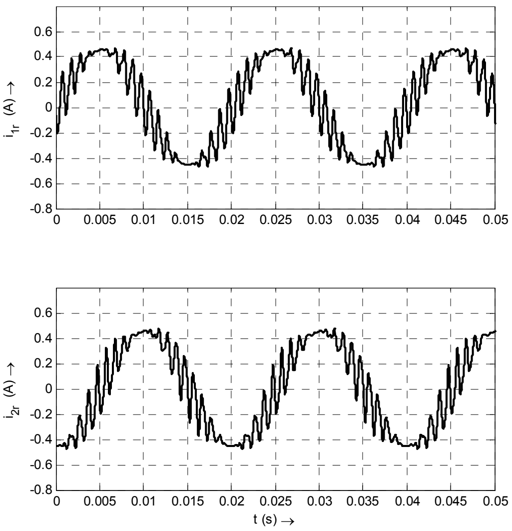

Fig.8 Rotor current waveforms for loaded machine

A figure (7) and (8) shows stator and rotor current waveforms for loaded machine. Suppose machine is loaded by a constant load of $0,2Nm$.

Figure (9) depicts the electromagnetic torque waveform at constant speed and loaded machine.

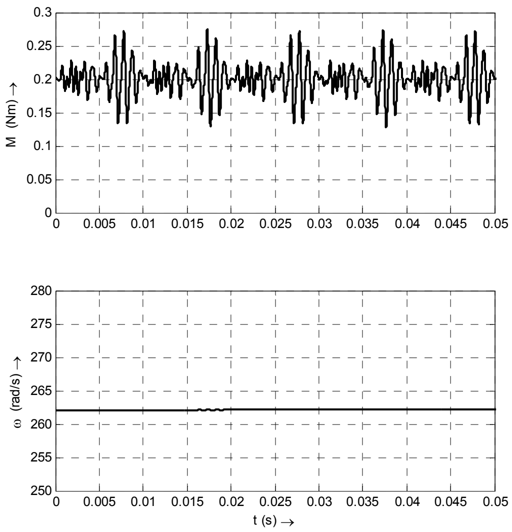

Fig.9 Torque waveform and speed for loaded machine

Conclusion

In presented paper analytical method of current and torque waveform calculus is presented. Waveform of currents and electromagnetic torque were calculated on the basis of harmonic analysis of supply PWM voltage. Only main harmonic components were taken in consideration. To currents and torque ripple have the greatest impact harmonic components from around modulation frequency. These harmonics produce in the machine pulsating magnetic fields that are source of current and torque deformations. It is therefore advantageous to have the modulation frequency of the highest possible.

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