

## DESIGN OF DECOUPLED PI CONTROLLERS FOR TWO-INPUT TWO-OUTPUT NETWORKED CONTROL SYSTEMS WITH INTRINSIC AND NETWORK-INDUCED TIME DELAYS

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**Abstract:** Proportional integral controller design for two-input two-output (TITO) networked control systems (NCSs) with intrinsic and network-induced time delays is studied in this paper. The TITO NCS consists of two delayed sub-systems coupled in a 1-1/2-2 pairing mode. In order to simplify the controller design, a decoupling method is first applied to obtain a decoupled system. Then, the controllers are designed based on the transfer function matrix of the obtained decoupled system and using the boundary locus method for determining the stability region and the well-known Mikhailov criterion for the stability test. A comparative analysis of the designed controllers and other controllers proposed in previous literature works is thereafter carried out. To demonstrate the validity and efficacy of the proposed method and to show that it achieves better results than other methods proposed in earlier literature works, the implementation in simulation of Wood–Berry distillation column model (methanol–water separation), a well-known benchmark for TITO systems, is carried out.

**Key words:** proportional integral controller, two-input two-output (TITO) systems, networked control systems, stability region boundary locus, Wood–Berry distillation column model, time delay, Mikhailov criterion

### 1. INTRODUCTION

Several processes in industry, such as heat exchange, distillation process, and chemical reactions and so on, require the control of two or more output variables. In turn, the control of output variables requires the manipulation of two or more input variables (Ajayi and Oboh, 2012); such systems are known as multi-input multi-output (MIMO) systems. The most common form of MIMO system is a two-input two-output (TITO) system (Zhuang and Atherton, 1993). The problem of control of TITO systems has attracted the attention of many researchers (see: Hamdy et al., 2018; Baruah et al., 2018; Heris et al., 2019; Li et al., 2019; Ustoglu et al., 2016; Qian et al., 2017; Vargas et al., 2013; Maghade and Patre, 2013; and references therein). In a TITO system, the interaction between loops makes analysis and design of the controller a very difficult task, and this task becomes more difficult and complex when the system to be controlled involves intrinsic time delays. Therefore, many of the methods dealing with TITO system control proposed in the literature have been interested in reducing the interaction loops by using a decoupling technique in a way that a change in each of the two loops does not affect the other (see: Tanaka et al., 2015; Hazarika and Chidambaram, 2014; Mahapatro and Subudhi, 2020; and references therein). Other research works existing in the literature have been interested on the control of TITO systems with time delays (see: Jeng and Jian, 2017; Jin et al., 2016; Liu et al., 2006; Khandekar and Patre, 2017; Hajare et al., 2017; and references therein).

On the other hand, networked control systems (NCSs), which are systems in which a band-limited network is used by the plant,

sensor and controller to share control signals and information among them, have many advantages in terms of reduction of wiring, lower maintenance cost, increased system agility, ease of information sharing and so on (Hong et al., 2017), compared to traditional point to point control systems in which the components and devices are connected via wires. Due to their advantages, NCSs have found application in many fields, such as process control engineering (Sun and El-Farra, 2012; El-Farra and Mhaskar, 2008), teleportation (Liu, 2015), vehicle industry (Jin et al., 2014), power systems (Park, 2015; Yao et al., 2015), transportation systems (Park et al., 2014; Barrero et al., 2014) and so on. One of the major factors that make the control of NCSs a very challenging problem is the presence of inevitable time delay induced by the transmission of control signals and information over a network that may be used by others devices and systems. This justifies why many of research papers related to the control of NCSs with network-induced time delay have been published over the past few decades (see: Li et al., 2016; Zhang et al., 2006; Huang and Nguang, 2009; Liu and Liu, 2020; Mohamed Vall, 2020b; Pang et al., 2016; Mohamed Vall, 2020a; Elahi and Alfi, 2017). However, to the author's best knowledge, in the literature, there are few research works related to the control of TITO NCSs with intrinsic and network-induced time delay (Wang et al., 2000; Sharma and Padhy, 2017; Astrom et al., 2002). Moreover, most of the approaches dealing with this problem are very complex and very time consuming or even not applicable in real-world control problems.

Motivated by the above discussion, we propose, in this paper, a simple and practicable method for the control of TITO NCSs with intrinsic and network-induced time delays. The proposed method comprises two steps: first, a decoupler for the TITO NCS

to be controlled is calculated; second, two decoupled proportional integral (PI) controllers for the augmented system, consisting of the obtained decoupler and the TITO NCS to be controlled, are separately designed using the boundary locus method for determining the stability region (Siljak, 1966; Chao and Han,1998; Wang,2011) and the Mikhailov criterion for stability test as presented by Barker (1979) and Mikhailov (1938).

In summary, the main objectives of this work are as follows:

- To design decoupled PI controllers that guarantee robustness and good set point tracking, as well as good disturbance rejection, for TITO NCSs with intrinsic and network-induced time delays.
- To carry out a comparative analysis of the designed controllers with other controllers proposed in previous literature works (Hajare and Patre, 2015) and show the superiority of the method proposed in this paper.

The remainder of the paper is organized as follows. In Section 2, the structure of TITO NCSs with network-induced time delays is presented. The proposed method for the control of TITO NCSs with intrinsic and network-induced time delays is presented in Section 3. In Section 4, a simulation example is given to show the validity and effectiveness of the proposed method. Conclusions are given in Section 5.

## 2. PROBLEM FORMULATION

As mentioned above, this research work deals with the control of TITO-NCSs with intrinsic and network-induced time delays. Fig. 1 shows the structure of a TITO-NCS with network-induced delays. As one can notice in Fig. 1, there are three time delays

affecting each sub-system of the whole system, which are as follows:

- delay sensor-to-controller  $\tau^{(s_i c_i)}$  ( $i = 1,2$ )
- delay controller-to-actuator  $\tau^{(c_i a_i)}$  ( $i = 1,2$ )
- controller computation delay  $\tau^{(c_i)}$  ( $i = 1,2$ )

Although controller computation time delay always exists, it is usually small compared to network-induced time delays and can be neglected.

In this paper, for simplicity and without losing generality, we neglect the controller computation time delay and assume that the network-induced delays affecting each sub-system are lumped together as one control time delay  $\tau_i$  given by the following expression:

$$\tau_i = \tau^{(s_i c_i)} + \tau^{(c_i a_i)} \quad (i = 1,2) \tag{1}$$

Depending on the medium access control (MAC) protocol of the control network, network-induced delay can be constant, time varying, or even random' (Zhang et al., 2001). Here, we are considering constant network-induced delay, 'which can be achieved by using an appropriate network protocol' (Zhang et al., 2001), and our objective is to design PI controllers to compensate for these time delays. The method proposed contains two steps. In the first step, the system is decoupled in order to simplify the controllers' design and to obtain controllers that can guarantee that changes in the reference signal of a sub-system do not affect the output of the other and vice versa. In the second step, for the decoupled system, PI controllers are designed utilizing the stability boundary locus method.

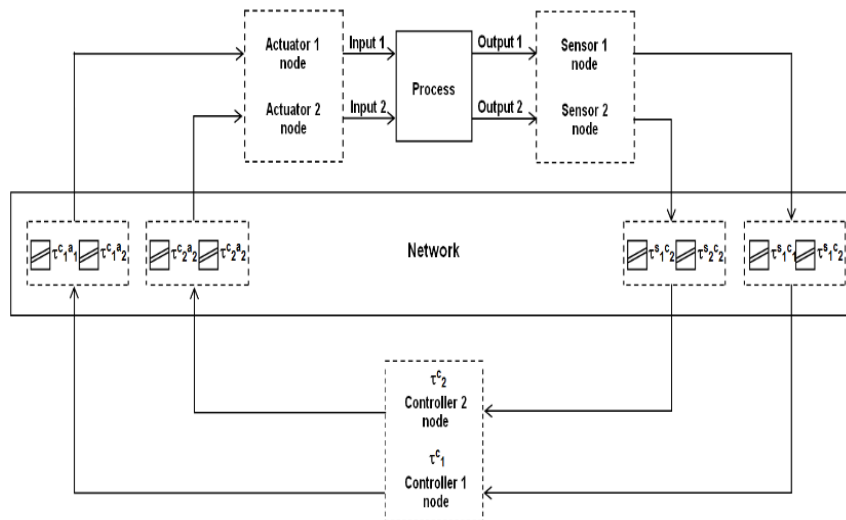


Fig. 1. Structure of TITO-NCS with Network-Induced Time Delays (NCS – networked control system; TITO – two-input two-output)

## 3. PROPOSED METHOD

### 3.1. Decoupling of the TITO system

Consider the closed-loop TITO system shown in Fig.2, where  $r_1$  and  $r_2$  are reference signals;  $C_1(s)$  and  $C_2(s)$  are two

controllers to be designed;  $G_{11}(s)$ ,  $G_{12}(s)$ ,  $G_{21}(s)$  and  $G_{22}(s)$  are the transfer functions of the system;  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  are the time delays; and  $y_1$  and  $y_2$  are the outputs of the system.

The cross-interaction between the input-output pairs makes the design of the controllers very hard and may – in many control problems – lead to undesirable effects. Therefore, to simplify the controller design and eliminate the effects of loop interactions as much as possible, the following decoupling method is used.

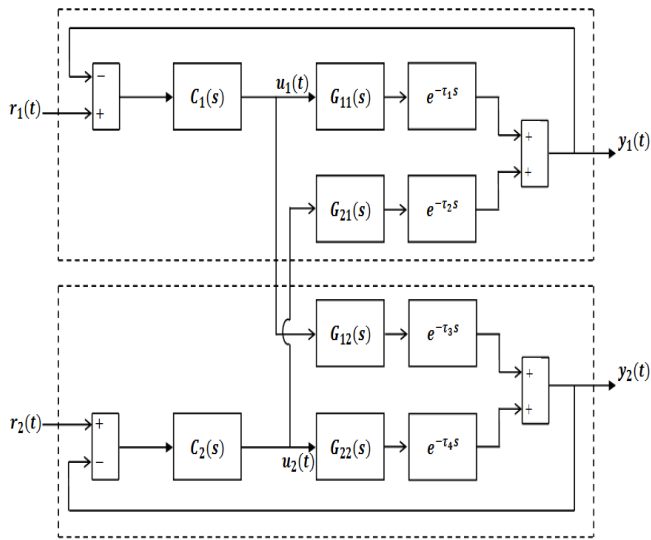


Fig. 2. 1-1/2-2 closed-loop TITO system with time delays (TITO – two-input two-output)

The transfer function matrix of the system shown in Fig.2 is

$$G(s) = \begin{bmatrix} G_{11}(s)e^{-\tau_1 s} & G_{12}(s)e^{-\tau_2 s} \\ G_{21}(s)e^{-\tau_3 s} & G_{22}(s)e^{-\tau_4 s} \end{bmatrix}. \quad (2)$$

Let the transfer function matrix of the decoupler be

$$D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}. \quad (3)$$

The matrix D(s) should be chosen so that the matrix

$$P(s) = G(s)D(s) \quad (4)$$

is diagonal.

As mentioned above, many decoupling methods for TITO systems have been proposed in the literature. In this paper, we propose to use the decoupler whose transfer function matrix is given in previous studies (Koo et al., 2017; de A. Aguiar and Barros, 2020; Naik et al., 2020).

$$D(s) = \begin{bmatrix} 1 & \frac{-G_{12}(s)e^{-(\tau_2-\tau_1)s}}{G_{11}(s)} \\ \frac{-G_{21}(s)e^{-(\tau_3-\tau_4)s}}{G_{22}(s)} & 1 \end{bmatrix} \quad (5)$$

Fig. 3 shows the structure of a TITO system with the proposed decoupler, where  $W_{12}(s) = \frac{-G_{12}(s)e^{-(\tau_2-\tau_1)s}}{G_{11}(s)}$  and  $W_{21}(s) = \frac{-G_{21}(s)e^{-(\tau_3-\tau_4)s}}{G_{22}(s)}$ . Notice that the decoupler is not causal if  $\tau_2 - \tau_1 < 0$  or/and  $\tau_3 - \tau_4 < 0$  (Wanget al., 2000). However, even if the decoupler is not causal, one can introduce simple modifications in its structure so that it becomes causal without affecting the decoupled system's structure (Sharma and Padhy, 2017).

Inserting Eqs. (2) and (5) in Eq. (4) gives the transfer function matrix of the decoupled system as

$$P(s) = \begin{bmatrix} P_{11}(s) & 0 \\ 0 & P_{22}(s) \end{bmatrix}, \quad (6)$$

where  $P_{11}(s) = G_{11}(s)e^{-\tau_1 s} - \frac{G_{12}(s)e^{-\tau_2 s}G_{21}(s)e^{-(\tau_3-\tau_4)s}}{G_{22}(s)}$  and

$$P_{22}(s) = G_{22}(s)e^{-\tau_4 s} - \frac{G_{21}(s)e^{-\tau_3 s}G_{12}(s)e^{-(\tau_2-\tau_1)s}}{G_{11}(s)}.$$

Based on the above decoupled system, PI controllers can be designed independently using the stability locus method, as presented in the following sub-sections.

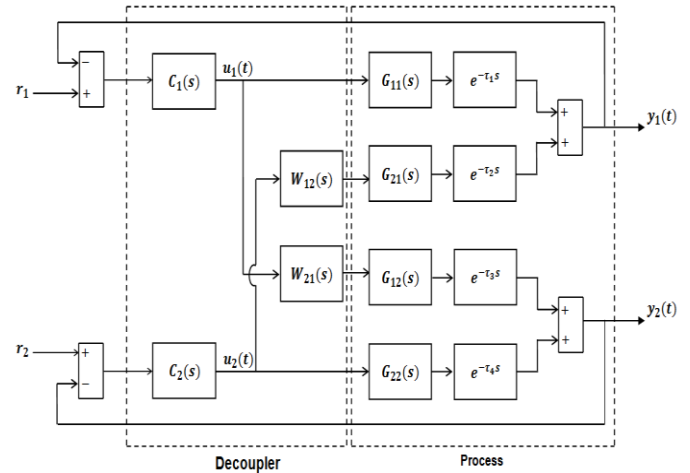


Fig. 3. Decoupling structure

### 3.2. Calculation of decoupled PI controllers

Suppose that network-induced time delays affect the decoupled system whose transfer function matrix is defined in Eq. (6). The closed-loop transfer functions of the sub-systems constituting the whole system are, thus, given by the following expressions:

$$T_1(s) = \frac{C_1(s)\{G_M(s)e^{(-\tau_1)s} - G_D(s)e^{-(\tau_2+\tau_3-\tau_4)s}\}e^{-\tau_{Ni11}s}}{G_{22}(s) + C_1(s)\{G_M(s)e^{(-\tau_1)s} - G_D(s)e^{-(\tau_2+\tau_3-\tau_4)s}\}e^{-\tau_{Ni11}s}} \quad (7)$$

$$T_2(s) = \frac{C_2(s)\{G_M(s)e^{(-\tau_4)s} - G_D(s)e^{-(\tau_3+\tau_2-\tau_1)s}\}e^{-\tau_{Ni22}s}}{G_{11}(s) + C_2(s)\{G_M(s)e^{(-\tau_4)s} - G_D(s)e^{-(\tau_3+\tau_2-\tau_1)s}\}e^{-\tau_{Ni22}s}} \quad (8)$$

where  $G_M(s) = G_{11}(s)G_{22}(s)$ ,  $G_D(s) = G_{12}(s)G_{21}(s)$ ,  $C_1(s)$  and  $C_2(s)$  are the two PI controllers to be designed and  $\tau_{Ni11}$  and  $\tau_{Ni22}$  are the network-induced time delays that affect Loops 1–1 and 2–2, respectively.

To determine the parameters of the controllers, the stability region locus method is used as follows.

#### 3.2.1. Determination of the parameters of $C_1(s)$

Let:

$$C_1(s) = \alpha_1 + \frac{\beta_1}{s}; \quad (9)$$

$$\tau_{11} = \tau_{Ni11} + \tau_1; \quad (10)$$

$$\tau_{12} = \tau_{Ni11} + (\tau_2 + \tau_3 - \tau_4). \quad (11)$$

By inserting Eqs. (9), (10) and (11) in Eq. (7), we can express the transfer function  $T_1(s)$  as follows:

$$T_1(s) = \frac{(\alpha_1 + \frac{\beta_1}{s})\{G_M(s)e^{-\tau_{11}s} - G_D(s)e^{-\tau_{12}s}\}}{G_{22}(s) + (\alpha_1 + \frac{\beta_1}{s})\{G_M(s)e^{-\tau_{11}s} - G_D(s)e^{-\tau_{12}s}\}}. \quad (12)$$

By substituting  $s = j\omega$ , decomposition of the numerators and denominators of  $G_M(s)$ ,  $G_D(s)$  and  $G_{22}(s)$  into their even and odd parts and using Euler's identity, one can express the system characteristic equation as follows:

$$R_{Q_1}(\omega, \alpha_1, \beta_1) + jI_{Q_1}(\omega, \alpha_1, \beta_1) = 0. \quad (13)$$

where  $R_{Q_1}$  and  $I_{Q_1}$  are two functions in  $\omega$ ,  $\alpha_1$ ,  $\beta_1$  with constant real coefficients.

Dropping  $\omega$  for ease and equating  $R_{Q_1}(\omega, \alpha_1, \beta_1)$  and  $I_{Q_1}(\omega, \alpha_1, \beta_1)$  to zero result in the following system of equations:

$$\begin{cases} M_{11}\alpha_1 + M_{12}\beta_1 = X_1 \\ M_{21}\alpha_1 + M_{22}\beta_1 = X_2 \end{cases} \quad (14)$$

where  $M_{11}$ ,  $M_{12}$ ,  $M_{21}$ ,  $M_{22}$ ,  $X_1$  and  $X_2$  are functions in  $\omega$  with constant real coefficients.

Solving the equation system (14) for  $\alpha_1$  and  $\beta_1$  gives

$$\begin{cases} \alpha_1 = \frac{N_{\alpha_1}}{D_{\alpha_1}} \\ \beta_1 = \frac{N_{\beta_1}}{D_{\beta_1}} \end{cases} \quad (15)$$

where  $N_{\alpha_1}$ ,  $D_{\alpha_1}$ ,  $N_{\beta_1}$  and  $D_{\beta_1}$  are functions in  $\omega$  with constant real coefficients.

Plotting the dependency relation between  $\alpha_1$  and  $\beta_1$  and the axis  $\beta_1 = 0$  on the  $(\alpha_1, \beta_1)$ -plane splits the plane into two regions. One of these regions is a stability region of the system. To determine which region is a stability region, the Mikhailov criterion for stability test is used. Next, once the stability region is determined, the values of the parameters of  $C_1(s)$  that stabilise the system can be thus determined by choosing a point within the stability region (Chao and Han, 1998).

### 3.2.2. Determination of the parameters of $C_2(s)$

The parameters of  $C_2(s)$  can be determined in a way similar to that for  $C_1(s)$ .

Let:

$$C_2(s) = \alpha_2 + \frac{\beta_2}{s}; \quad (16)$$

$$\tau_{22} = \tau_{N_{i22}} + \tau_4; \quad (17)$$

$$\tau_{21} = \tau_{N_{i22}} + (\tau_3 + \tau_2 - \tau_1). \quad (18)$$

By inserting Eqs. (16), (17) and (18) in Eq. (8), we can express the transfer function  $T_2(s)$  as follows:

$$T_2(s) = \frac{(\alpha_2 + \frac{\beta_2}{s})\{G_M(s)e^{-\tau_{22}s} - G_D(s)e^{-\tau_{21}s}\}}{G_{11}(s) + (\alpha_2 + \frac{\beta_2}{s})\{G_M(s)e^{-\tau_{22}s} - G_D(s)e^{-\tau_{21}s}\}}. \quad (19)$$

By substituting  $s = j\omega$ , decomposition of the numerators and denominators of  $G_M(s)$ ,  $G_D(s)$  and  $G_{11}(s)$  into their even and odd parts and using Euler's identity, one can express the system characteristic equation as follows:

$$R_{Q_2}(\omega, \alpha_2, \beta_2) + jI_{Q_2}(\omega, \alpha_2, \beta_2) = 0. \quad (20)$$

where  $R_{Q_2}$  and  $I_{Q_2}$  are two functions in  $\omega$ ,  $\alpha_2$ ,  $\beta_2$  with constant real coefficients.

Dropping  $\omega$  for ease and equating  $R_{Q_2}(\omega, \alpha_2, \beta_2)$  and  $I_{Q_2}(\omega, \alpha_2, \beta_2)$  to zero result in the following equation system:

$$\begin{cases} H_{11}\alpha_2 + H_{12}\beta_2 = Z_1 \\ H_{21}\alpha_2 + H_{22}\beta_2 = Z_2 \end{cases} \quad (21)$$

where  $H_{11}$ ,  $H_{12}$ ,  $H_{21}$ ,  $H_{22}$ ,  $Z_1$  and  $Z_2$  are functions in  $\omega$  with constant real coefficients.

Solving the equation system (21) for  $\alpha_2$  and  $\beta_2$  gives

$$\begin{cases} \alpha_2 = \frac{N_{\alpha_2}}{D_{\alpha_2}} \\ \beta_2 = \frac{N_{\beta_2}}{D_{\beta_2}} \end{cases} \quad (22)$$

where  $N_{\alpha_2}$ ,  $D_{\alpha_2}$ ,  $N_{\beta_2}$  and  $D_{\beta_2}$  are functions in  $\omega$  with constant real coefficients.

Plotting the dependency relation between  $\alpha_2$  and  $\beta_2$ ; and the axis  $\beta_2 = 0$  on the  $(\alpha_2, \beta_2)$ -plane splits the plane into two regions. One of them is a stability region of the system. To determine which region is a stability region, the Mikhailov criterion for stability test is used. Then, the values of the parameters of  $C_2(s)$  that stabilise the system can be thus determined by choosing a point within the stability region (Chao and Han, 1998).

## 4. SIMULATION EXAMPLE

In this section, a simulation example is given to show the validity and efficacy of the proposed method.

The simulation is carried out in Simulink and TrueTime in the environment of MatLab. The network communication mode is carrier-sense multiple access with collision detection (CSMA/CD; Ethernet), the transmission rate is 80 Kbit/s and the network-induced time delays that affect Loops 1–1 and 2–2 are  $\tau_{N_{i11}} = 0.13$  second and  $\tau_{N_{i22}} = 0.17$  sec, respectively. The sampling period is 0.02 sec. The process to be controlled is a Wood–Berry distillation column model (methanol–water separation), which has the following transfer function matrix (Astrom et al., 2002):

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \quad (23)$$

The transfer function matrix of the decoupler calculated according to Eq. (5) is

$$D(s) = \begin{bmatrix} 1 & \frac{1.48(16.7s+1)e^{-2s}}{21.0s+1} \\ \frac{0.34(14.4s+1)e^{-4s}}{10.9s+1} & 1 \end{bmatrix}. \quad (24)$$

The transfer function matrix of the decoupled system calculated according to Eq. (6) is

$$P(s) = \begin{bmatrix} P_{11}(s) & 0 \\ 0 & P_{22}(s) \end{bmatrix}, \quad (25)$$

$$\text{where } P_{11}(s) = \frac{12.8e^{-s}}{16.7s+1} - \frac{6.43(14.4s+1)e^{-7s}}{(21.0s+1)(10.9s+1)} \text{ and } P_{22}(s) = \frac{-19.4e^{-3s}}{14.4s+1} + \frac{9.77(16.7s+1)e^{-9s}}{(10.9s+1)(21.0s+1)}.$$

Consequently, based on the decoupled system in Eq. (25) and taking into account that Loops 1–1 and 2–2 are affected by the network-induced time delays  $\tau_{N_{i11}} = 0.13$  sec and  $\tau_{N_{i22}} = 0.17$  sec, respectively, and using Eqs (7) and (8), the closed-loop transfer functions of the sub-systems are obtained as follows:

$$T_1(s) = \frac{C_1(s)(A(s)e^{-s}+B(s)e^{-7s})e^{-0.13s}}{\frac{-19.4}{14.4s+1}+C_1(s)(A(s)e^{-s}+B(s)e^{-7s})e^{-0.13s}}; \quad (26)$$

$$T_2(s) = \frac{C_2(s)(A(s)e^{-3s}+B(s)e^{-9s})e^{-0.17s}}{\frac{12.8}{16.7s+1}+C_2(s)(A(s)e^{-3s}+B(s)e^{-9s})e^{-0.17s}}; \quad (27)$$

where  $A(s) = \frac{-248.32}{(16.7s+1)(14.4s+1)}$ ,  $B(s) = \frac{124.74}{(21.0s+1)(10.9s+1)}$ , and

$C_1(s) = \alpha_1 + \frac{\beta_1}{s}$  and  $C_2(s) = \alpha_2 + \frac{\beta_2}{s}$  are the two PI controllers.

Let us determine the parameters of  $C_1(s)$  and  $C_2(s)$  based on the theory presented in Sub-section 3.2 of this paper.

By substituting Eq. (26) into Eq. (12), solving Eqs (13)–(14) for  $\alpha_1$  and  $\beta_1$ , and plotting the dependency between the obtained values of  $\alpha_1$  and  $\beta_1$  and the axis  $\beta_1 = 0$  on the  $(\alpha_1, \beta_1)$ -plane, we obtain Fig. 4.

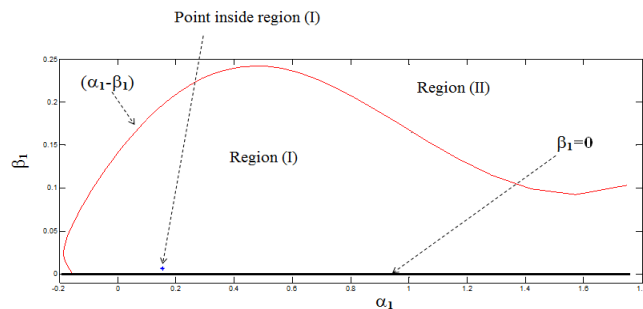


Fig. 4.  $\alpha_1$  versus  $\beta_1$

As one can see from Fig. 4, the curve  $(\alpha_1, \beta_1)$  and the axis  $\beta_1 = 0$  split the  $(\alpha_1, \beta_1)$ -plane into two regions: Region (I) and Region (II).

Now, let us choose the point (0.1532, 0.0067) of Region (I) in Fig.4 and check whether the system in Eq. (26) is stable when one chooses the parameters of the controller  $C_1(s)$  as  $\alpha_1 = 0.1532$  and  $\beta_1 = 0.0067$ . To check the stability, the Mikhailov criterion for stability test can be used as mentioned earlier in this paper.

By determining the polynomial function  $Q_1(j\omega)$  for the transfer function  $T_1(s)$  given in Eq. (26),  $\alpha_1 = 0.1532$  and  $\beta_1 = 0.0067$ , as explained by Barker (1979) and Mikhailov (1938), we obtain the Mikhailov curve shown in Fig. 5. As one can see from Fig. 5, the system is stable since the Mikhailov stability criterion is fulfilled, as also reported by Barker (1979) and Mikhailov (1938).

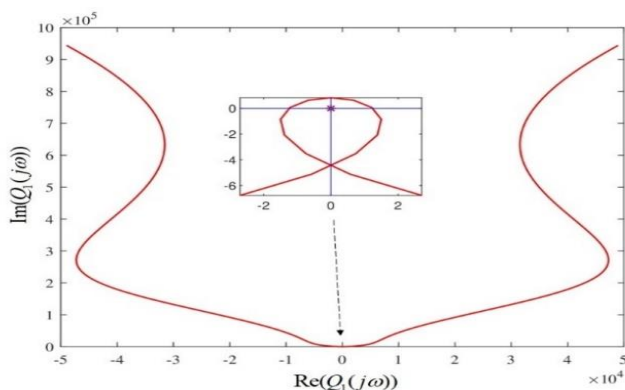


Fig. 5. Loop 1-1 Mikhailov curve for  $\alpha_1 = 0.1532$ ,  $\beta_1 = 0.0067$

In a similar way, one can determine the parameters of the controller  $C_2(s)$  as those of the controller  $C_1(s)$  were determined. By substituting (27) into (19), solving the equations (20)-(21) for  $\alpha_2$  and  $\beta_2$ , and plotting the dependency between the obtained values of  $\alpha_2$  and  $\beta_2$  and the axis  $\beta_2 = 0$  on the  $(\alpha_2, \beta_2)$ - plan, we obtain Fig. 6.

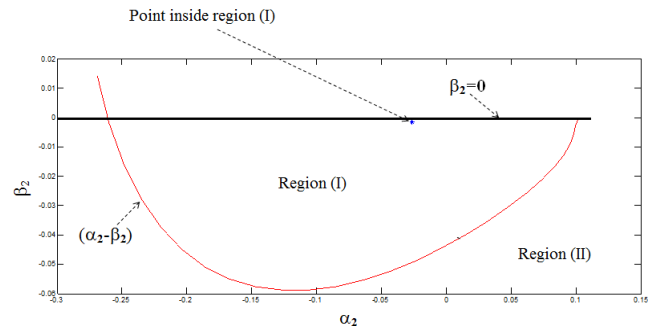


Fig. 6.  $\alpha_2$  versus  $\beta_2$

As one can see, from Fig. 6., the curve  $(\alpha_2, \beta_2)$  and the axis  $\beta_2 = 0$  split the  $(\alpha_2, \beta_2)$ - plan into two regions: Region (I) and Region (II). To determine which of them is a stability region of the system, let's choose the point (-0.0266, -0.0015) of the region (I) and use Mikhailov stability criterion to check whether the system is stable when one chooses the parameters of the controller  $C_2(s)$  as  $\alpha_2 = -0.0266$ ,  $\beta_2 = -0.0015$ .

By determining the polynomial function  $Q_2(j\omega)$  for the transfer function  $T_2(s)$  given in (27),  $\alpha_2 = -0.0266$ ,  $\beta_2 = -0.0015$ , as explained in in (Barker, 1979) and (Mikhailov, 1938), we obtain the Mikhailov curve shown in Fig. 7. As one can see, from Fig. 7., the Mikhailov stability criterion holds, as also reported in (Barker, 1979) and (Mikhailov, 1938), the system is, therefore stable.

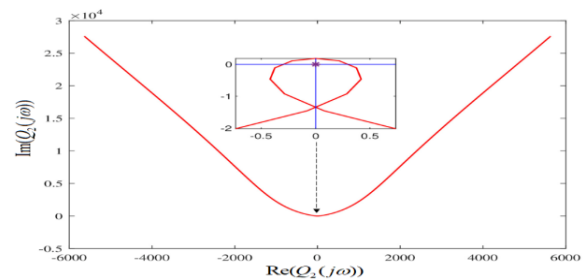
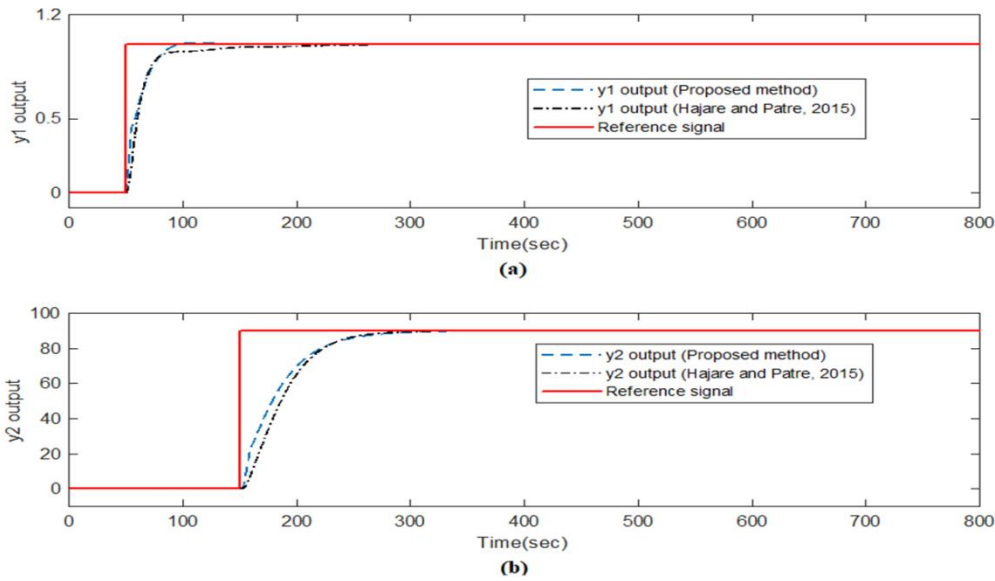


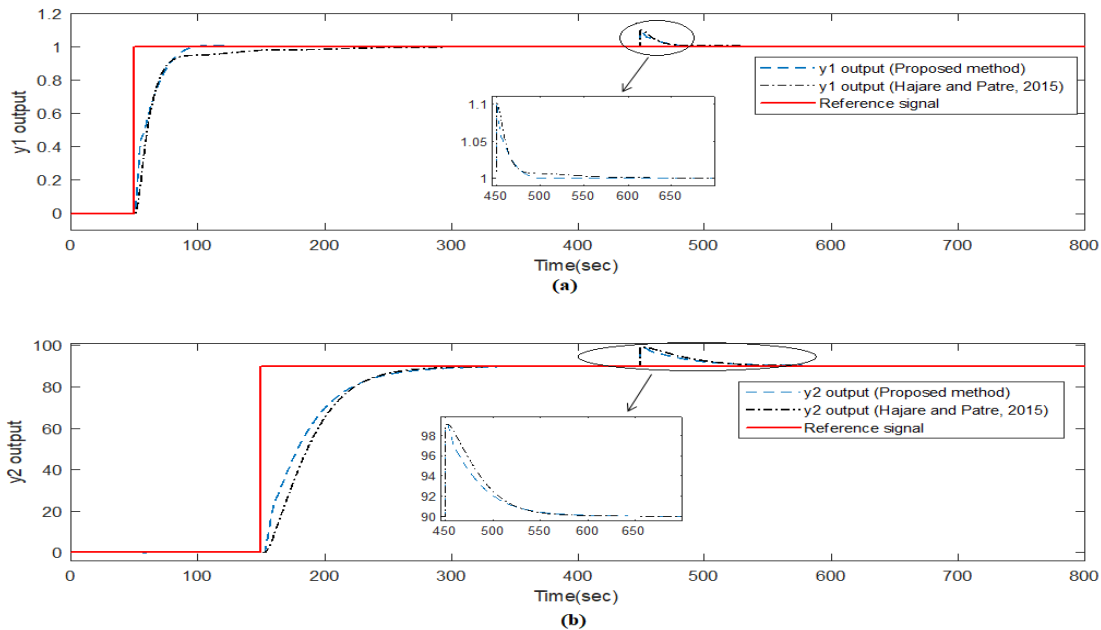
Fig. 7. Loop 2-2 Mikhailov curve for  $\alpha_2 = -0.0266$ ,  $\beta_2 = -0.0015$

To verify the validity and efficacy of the proposed method, the obtained PI controllers:  $C_1(s) = 0.1532 + \frac{0.0067}{s}$ ,  $C_2(s) = -0.0266 - \frac{0.0015}{s}$  are applied to the both loops of the decoupled system. In the simulation, a step change in the reference signal  $r_1$  is made at  $t = 50$  second and a step change in the reference signal  $r_2$  is made at  $t = 150$  sec, respectively. Furthermore, to show the disturbance rejection performance of the obtained controllers, steps disturbance  $d_1(t) = 0.1$  at  $t = 450$  sec and  $d_2(t) = 9$  at  $t = 450$  sec are introduced to loop 1 and loop 2, respectively. The simulation results of both set-point tracking and disturbance rejection are shown in Fig. 8 and Fig. 9, respectively. The performance metrics: Settling time ( $T_s$ ), Integral Absolute Error (IAE), RMS tracking error (RMSE) and Total Variance (TV) are reported in Tab. 1.





**Fig. 8.** System outputs: (a) Reference signal (red solid line),  $y_1$  (loop1) output (blue dashed line) obtained using the proposed method and  $y_1$  (loop1) output (black dot-dash lines) obtained using the method proposed in (Hajare and Patre, 2015) (b) reference signal (red solid line),  $y_2$  (loop2) output (blue dashed line) obtained using the proposed method and  $y_2$  (loop2) output (black dot-dash lines) obtained using the method proposed in (Hajare and Patre, 2015)



**Fig. 9.** System outputs with disturbance rejection: (a) Reference signal (red solid line),  $y_1$  (loop1) output (blue dashed line) obtained using the proposed method and  $y_1$  (loop1) output (black dot-dash lines) obtained using the method proposed in (Hajare and Patre, 2015) (b) reference signal (red solid line),  $y_2$  (loop2) output (blue dashed line) obtained using the proposed method and  $y_2$  (loop2) output (black dot-dash lines) obtained using the method proposed in (Hajare and Patre, 2015)

**Tab.1.** Performance metrics

| Controllers   | Input-output  | $T_s$ (sec) | RMSE  | IAE        | TV         |
|---|---------------|-------------|-------|------------|------------|
| Proposed decoupled PI controllers                     | $u_1$ - $y_1$ | 41          | 0.09  | 12.1131091 | 0.008174.6 |
|   | $u_2$ - $y_2$ | 130         | 13.81 |            |            |
| (PID) controllers proposed in Hajare and Patre (2015) | $u_1$ - $y_1$ | 103.2       | 0.12  | 17.553586  | 0.011236.7 |
|   | $u_2$ - $y_2$ | 119.2       | 16.1  |            |            |

IAE – integral absolute error; PI – proportional–integral; PID – proportional–integral–derivative; RMSE – root mean square tracking error; TV – total variance.

From Fig. 8, it can be seen that the proposed controllers guarantee excellent tracking of the system outputs to the set points and maintain the system stable in both loops. It can be also noticed that the change in the set point of Loop 1 does not affect the response output in the Loop 2 and vice-versa, the change in the set point of the Loop 2 does not affect the response output in the Loop 1. Additionally, the simulation results in Fig. 9 show that the obtained controllers also offer excellent disturbance rejection performance. Moreover, it is evident from the simulation results (shown in Figs. 8 and 9) and the performance metrics (reported in Tab. 1) that the proposed controllers achieve better results than the proportional–integral–derivative (PID) controllers proposed in a previous paper (Hajare and Patre, 2015). For example, it can be seen from Fig. 9 that the proposed controllers take less time and control action to achieve the steady state and zero steady-state error in case of load disturbance.

## 5. CONCLUSIONS

This paper proposes an approach for the design of decoupled PI controllers for TITO NCSs with intrinsic and network-induced time delays. A decoupler that can reduce interaction between loops and simplify the design of controllers is defined. Moreover, a method based on stability region locus and the Mikhailov criterion for stability test is proposed to determine the parameters of PI controllers to control a TITO NCS with intrinsic and network-induced time delays. A comparative analysis of the designed controllers with other controllers proposed in previous research works has been carried out. The validity and efficacy of the proposed approach are shown through a simulation example, in which the well-known benchmark for TITO systems – the Wood–Berry distillation column model (methanol–water separation) – is built in Simulink and TrueTime in the environment of MatLab; network-induced time delays are included and PI controllers are designed based on the approach proposed. The simulation results show that the designed PI controllers guarantee very good tracking of the system outputs to the set points and maintain the system stable in both loops with excellent disturbance rejection performance. Moreover, the simulation results show that the proposed method achieves better results than other methods proposed in earlier literature works.

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