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# BEHAVIOUR OF HOT-ROLLED PURLINS CONNECTED WITH SANDWICH PANELS

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The influence of sheeting made by sandwich panels on the lateral-torsional buckling resistance of hot rolled purlin was studied in this paper. The actual shear and torsional stiffness as well as resistance of connections between sandwich panel and purlins were considered in analysis. Parameters of these factors were determined using the finite element method, as well as by own experimental tests. Simple models with beam elements were used in LBA analysis to calculate the critical moments of the purlins. Advanced models with GMNIA analysis using shell elements was performed to simulate the behaviour of the purlins stabilized by sandwich panels. The results show that the stiffness of sheeting made by sandwich panels is insufficient to ensure the full protection of purlin against lateral-torsional buckling. The connections resistance also limited the ability of purlins stabilisation. Nevertheless including sandwich panels in purlin stability analysis results in a significant increase in their LTB resistance.

*Keywords**:** purlin, lateral-torsional buckling, sandwich panels, FEM analysis*

## 1. INTRODUCTION

Purlins are one of the structure members exposed to instability phenomena. Steel purlins are considered simple structural members supporting the roof covering and usually designed as hot-rolled or cold-formed elements. In both cases, the most essential parameter to be considered for their safety

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is proper assessment of their lateral torsional buckling (LTB) resistance. In addition to the cross-section geometry, the various boundary conditions of purlin have a influence on its stability, e.g. purlin-to-girder connection or contribution made by diaphragm action in sheeting.

The last factor may especially have a crucial influence on the stability of the purlins. As a result, the Eurocode [8] specifies 3 structural classes differing from each other in the way of including the cooperation between structure and sheeting in design:

Structural Class 1: Sheeting is included in the calculations of the overall strength and stability of a structure;

Structural Class 2: Sheeting is included in the calculations of strength and stability of individual structural members;

Structural Class 3: Sheeting is considered as an element that only transfers loads to the structure.

The ability of purlins to be stabilized by a trapezoid sheet is well known and commonly used in design practice by considering it Structural Class 2 ([4], [11]). However, the attempts at using the sandwich panels in purlin stability analysis ([2], [12], [17], [23]) were lately conducted, but influence of this factor is still not fully recognized. The connections of purlin with girders are by default assumed to be a fork support, which is usually sufficient approximation [19]. The influence of sandwich panels on stability of hot-rolled purlins is presented in this paper.

## 2. Lateral Torsional Buckling Resistance

The LTB resistance analysis can be performed in several ways. The modern advanced method based on the geometrical and material nonlinear analysis with imperfections (GMNIA) uses the finite element method (FEM). This method allows simulating the behaviour of the structure including wide spectrum of factors and phenomena (e.g., [1], [6], [15], [25], [32]). However, the application of GMNIA requires sophisticated software and an advanced model of the analysed problem, which results in an increase in the computational time and costs.

A more classic method originally introduced by Timoshenko [30] is based on linear buckling analysis (LBA), which results in the critical moment $M_{cr}$. In design practice, the LTB resistance of the beam, e.g., according to code [7], is considered by multiplying the beam sectional resistance by the reduction factor $\chi_{LT}$, which is given by the formula (2.1):

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \tag{2.1}$$

where:

$$\Phi_{LT} = 0{,}5\,[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2]$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

$W_y$ – section modulus, $f_y$ – yield strength, $\alpha_{LT}$ – imperfection factor depended on the type of element

Critical moment can be obtained using theoretical formulas from the theory of elasticity. In the code [6], there are no formulas for critical moment but the formulas can be taken from pre-code [10] or from the literature, e.g., [27], [28], [31]. The basic formula has the following form (2.2):

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_\omega}{I_z} + \frac{(kL)^2 G I_T}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2} - C_2 z_g - C_3 z_j \tag{2.2}$$

where:

$C_1$, $C_2$, $C_3$ – factors depending on the loading and boundary conditions, $k$, $k_w$ – effective length factors, $I_z$, $I_\omega$ $I_T$ – geometric characteristics of the beam cross-section, $G$, $E$ – material characteristics, $z_g$ – distance between the point of load application and shear centre of the cross-section, $z_j$ – value depending on the position of the shear centre and the centroid of the cross-section.

Although this formula is widely used in engineering calculations, it has a limited applicability because it covers only elementary support and load cases. Some research has been done (e.g. [3], [20], [26], [33]) to enlarge the scope of usability of this formula. Nevertheless, the number of limitations is still significant. As a result, using the basic formula can lead to underestimation of the value of the critical moment.

Avaliable "engineering" software using beam elements in FEA has a more versatile character, giving more exact estimation of the critical moment and allowing the calculation of more complex cases. Several types of software can be applied, differing in modelling geometry and loading, solvers used and general capabilities. The most popular free available computer software to calculate the critical moment by FEM is LTBeam [24] which allows the consideration of support flexibility, various bending moment distributions and additional discrete or continuous lateral supports.

Modelling using beam elements is usually completely adequate to achieve satisfactory compatibility of results from analysis and reality. Nevertheless, sometimes additional factors must be considered, e.g., corrugated web [21] or cellular beam [13]. In this situation, the geometry should be modelled

using shell or even brick elements, and more advanced software is needed. One of the types of software that meets these requirements is Autodesk Simulation Mechanical (ASM) [29].
This paper is focussed mainly on the calculation of the critical moment based on the beam model of purlin available in LTBeam (LBA analysis). However, the 3D models of purlins made of shell elements were also analysed using ASM to verify the accuracy of formulas for sandwich panels given in the literature (GMNIA analysis).

## 3. Influence of cooperation between purlin and sandwich panels

### 3.1. Stiffness and resistance of sandwich panel and connection

In the conventional designing of steel roofs, sandwich panels are considered as non-cooperating sheeting, mainly because of rather thin steel plates used (ca. 0.5 mm) and connection by very long screws. In addition, lack of seam fasteners between each of the sandwich panels induces a low-effective way of transferring load in the sheeting plane, because each panel must act independently, as shown in Fig. 1 [2].

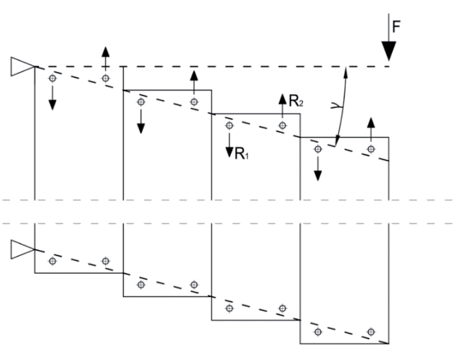


Fig. 1. Assumption of transferring load *F* acting in sheeting plane by sandwich panels

Nevertheless, some research is currently available (e.g., [14], [22], [23]) showing noticeable stiffness of sheeting made of sandwich panels and the possibility of including them in the purlins stability.
According to Eurocode 3 [7], the beam may be regarded as protected against lateral-torsional buckling if its compressed flange is connected to sheeting which shear stiffness S fulfils the requirement given by formula (3.1) as follows:

(3.1)
$$S \geq \left(EI_w \frac{\pi^2}{L^2} + GI_t + EI_z \frac{\pi^2}{L^2} 0{,}25h^2\right)\frac{70}{h^2}$$

where:

$E$, $G$ – elastic and shear modulus, $I_w$, $I_z$, $I_t$ – characteristics of cross-section, $h$ – height of cross sections, $L$ – length of the beam between lateral supports.

For purlins made of IPE profiles, the required stiffness of sheeting according to (3.1) is vary in range between 7000 – 16 000 kN/m/m, depending on the cross-section and span length.

According to [7], beside of shear stiffness, the beam stability is proven if its torsional restraint fulfilled the requirement (3.2).

(3.2)
$$C_{\vartheta,k} > \frac{M_{pl,k}^2}{EI_z} K_\theta K_v$$

Where:

$K_v$ = 0.35 for elastic analysis, $K_v$ = 1.0 – for plastic analysis, $K_\theta$ – factor considering the moment distribution and boundary conditions, $M_{pl,k}$ – characteristic value of the plastic moment of the beam

If condition (3.1) or (3.2) is met, purlin can be treated as connected to rigid support. If condition (3.1) or (3.2) is not met, actual roof sheeting flexibility must be included in the design.

According to [12] shear stiffness of sheeting made by sandwich panels may be calculated by the following formula (3.3):

(3.3)
$$S_i = \frac{k_v}{2B} \sum_{k=1}^{n_k} c_k{}^2$$

where:

$k_v$ – the stiffness of purlin-to-sandwich panel connection, calculated according to [12] or experimentally determined, $B$ – width of sandwich panel, $c_k$ – distance between two fastenings of a pair, $n_k$ – number of pairs of fasteners per panel and support (usually $n_k$ = 1).

Considering the behaviour of sandwich panels in purlin stabilization, the most adequate approach is including them in analysis as a continuous support preventing rotation against the minor axis of the purlins cross-section with stiffness given by formula (3.4) as follows:

(3.4) $$S_{v'} = \frac{k_v}{2}\sum_{k=1}^{n_k} {c_k}^2$$

According to [8] the stiffness of the torsional support for elements where distortional deformations of the cross sections may be neglected can be calculated using formula (3.5):

(3.5) $$C_D = \frac{1}{\frac{1}{C_{D,A}}+\frac{1}{C_{D,C}}}$$

where:

$C_{D,A}$ – rotational stiffness of the connection between purlin and sheeting, $C_{D,C}$ – rotational stiffness provided by the sheeting to the purlin

The conservative value of rotational stiffness $C_{D,C}$ can be obtained from (3.6):

(3.6) $$C_{D,C} = \frac{kEI_{eff}}{s}$$

where:

$k$ – numerical coefficient depended on location of the purlin (side or middle) and configuration of purlins rotations (opposite or similar) based on [8], $I_{eff}$ – moment of inertia of sheeting per width unit, $s$ – spacing between purlins.

According to [22] the rotational stiffness of connections between purlin and sandwich panel can be calculated by formula (3.7):

(3.7) $$C_{D,A} = C_{\vartheta A} = \frac{m_K}{\vartheta(m_K)} = \frac{3}{2}\cdot\frac{C_{\vartheta 1}}{\left(\frac{C_{\vartheta 1}}{C_{\vartheta 1}+C_{\vartheta 2}}+1\right)}$$

where:

$$C_{\vartheta 1} = c_1 E_{c,t,\theta} b^2$$

$$C_{\vartheta 2} = c_2\, n_f E_{c,t,\theta} b_k^2$$

$$E_{c,t,\theta} = \frac{E_C}{1+\varphi_{\vartheta,t}}\sqrt{k_1^3}$$

$c_1$, $c_2$, – parameters depending on core material and geometry of outer face, $\varphi_{\vartheta,t}$– parameter depending on core material and duration of loading, $b$ – width of purlins flange, $b_k$ – distance between the fastener and the edge of flange, $n_f$ – number of fasteners per meter length of the purlin, $E_c$ – elastic modulus of the core = mean value of compressive modulus $E_{Cc}$ and tensile modulus $E_{Ct}$

$$k_1 = \frac{\sigma_{w,\,+80°C}}{\sigma_{w,\,+20°C}}$$

where values $\sigma_{w,+80°}$ and $\sigma_{w,+20°}$ should be taken from [8]. For a temperature of 0°C during the time of loading $k_1$ can be assumed as 1.0.

The second limitation of cooperation between sandwich panels and purlins is associated with the ultimate resistance of connections between these elements. In the case of the initial bow imperfection, the greatest stabilization forces occur at the ends of the member (Fig. 2).

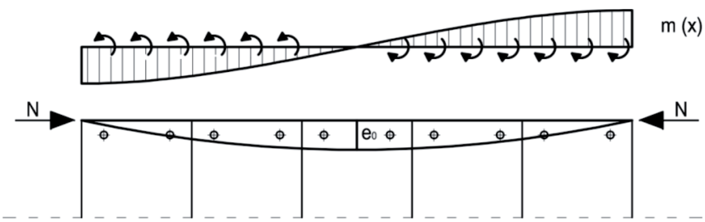


Fig. 2. Stabilization moment distribution along the purlin induced by bow imperfection of compressed flange.

According to [12], the maximum value of these forces may be calculated using (3.8) as follows:

$$V_{s,max}^{M} = \frac{M_k \pi e_0 \alpha B}{Lh \sum \frac{c_k^{\,2}}{c_1}} \tag{3.8}$$

where:

$M_k$ – bending moment in purlin, $e_0$ – initial imperfection of purlin, $L$ – purlin's span, $h$ – height of purlin, $\alpha$ – the amplification factor due to second order effects

### 3.2. Experimental tests

The stiffness $k_v$ and resistance $F_R$ of the connection between the sandwich panel and the purlin by a single fastener was experimentally determined. Two types of sandwich panels were used for this purpose [16]:

- BalexMetal BALEXTERM-PU-W-PLUS, polyurethane core with thickness 60 mm, face thickness 0.4 mm, S250GD, and

- BalexMetal PWS, polystyrene core with thickness 100 mm, face thickness 0.5 mm, S250GD.

The sandwich panels were connected to supporting elements simulating purlins – fixed part of IPE 160 and movable flat bar with thickness 8 mm. Self-drilling screws 5.5*6.3 mm with the seals of the washers made of EPDM were used in connections. The horizontal force was applied by pre-loading threaded rod screwed to a fixed support.

In addition to the applied force (F), 4 horizontal displacements were measured during the tests (Fig. 3):

$\Delta_1$ –displacement of the sandwich panel in the place of connection with movable support,

$\Delta_2$ – displacement of the sandwich panel in the place of connection with fixed support,

$\Delta_3$ – displacement of fixed support,

$\Delta_4$ – displacement of movable support,

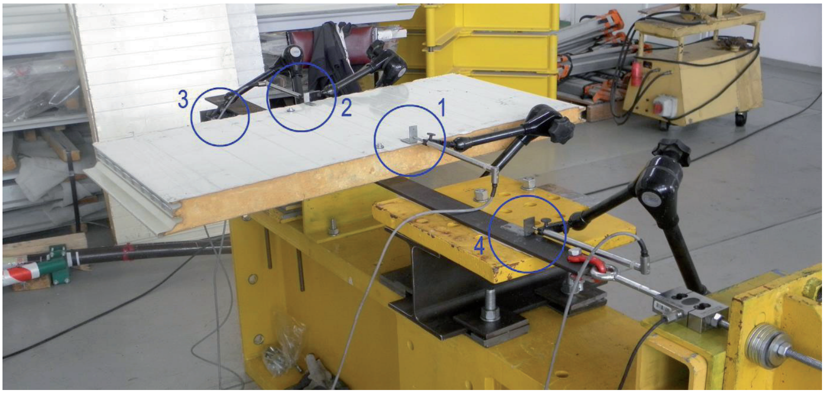


Fig. 3. Arrangement of displacement sensors (see text)

The stiffness of connection was calculated from formula (3.9):

(3.9) $$k_v = \frac{F}{\Delta_4 - \Delta_2}$$

The average stiffness of connections was obtained using trend lines with assumption of a starting point in coordinates (0,0). The resistances were set as the highest values of force recorded in tests. The relationship between applied force and the displacement difference ($\Delta_4$ - $\Delta_2$) for tested sandwich panels is shown in Fig. 4. The results are shown in Table 1.

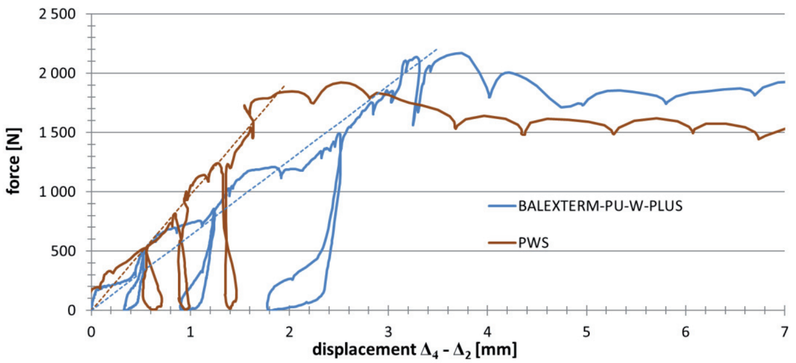


Fig. 4. The relationship between applied force $F$ and the displacement difference ($\Delta_4$ - $\Delta_2$) for tested sandwich panels

Table 1. Characteristics of tested connections between sandwich panels and purlin

| Sandwich panel | Average stiffness $k_v$ [N/mm] | Resistance $F_R$ [N] |
|---|---|---|
| BALEXTERM-PU-W-PLUS | 632 | 2150 |
| PWS | 972 | 1920 |

The values of stiffness that were obtained from tests are noticeably smaller than recommended in [12] where the values starts from 1600 N/mm and depending on the steel grade and the thickness of inner face of sandwich panel. For further analysis, the average stiffness $k_v$ and resistance of the connections between the sandwich panel and the purlin $F_R$ have been assumed to be, respectively, 700 N/mm and 1500 N.

### 3.3. Ability of stabilization by sandwich panels – limitation due to stiffness of connections

Basing of most frequently used in practice solutions for sheeting made of sandwich panels, the following assumptions were made:

$B = 1.0$ m, $c = 0.9$ m, $n_k = 1$, $b_k = ¼\, b$, $EI_{eff} = 200$ kN·m$^2$

$c_1 = 0.18$, $c_2 = 0.052$, $\varphi = 1.83$ for polyurethane core and profiled outer face (based on [12])

$k_v = 700$ N/mm = 700 kN/m, $F_R = 1500$ N (based on experimental tests described in chapter 3.2),

$E_{c,t,\theta} = 5$ MPa (based on [18])

In addition, numerical coefficient k = 4 and spacing between purlins s = 3.0 m were assumed.

The shear stiffness according to (3.3) equals:

$$S_i = \frac{k_v}{2B}\sum_{k=1}^{n_k} {c_k}^2 = \frac{700}{2 \cdot 1} \cdot 0.9^2 = 283.5\, kN/m/m$$

The value of shear stiffness for analysed sheeting made of sandwich panels obtained from (3.3) is far too small to ensure the full stabilization of the purlin according to values given by (3.1). To determine its actual influence on purlin LTB resistance, the critical moment using the LTBeam software was calculated and the reduction factor according to (2.1) was determined. Instead of continuous lateral support, the continuous support preventing rotation against vertical axis of beam, with stiffness according to (3.4) was considered. Therefore:

$$S_{v'} = \frac{700}{2} \cdot 0.9^2 = 283.5 kNm$$

The torsional stiffness of the analysed sheeting according to (3.6) equals:

$$C_{D,C} = \frac{kEI_{eff}}{s} = \frac{4 \cdot 200}{3} = 266.6 \frac{kNm}{rad}$$

The torsional stiffness of connection between purlin and sandwich panels $C_{D,A}$ given by formula (3.7) depends on coefficients $C_{\vartheta 1}$ and $C_{\vartheta 2}$ . For the adopted assumptions it can be presented as:

$$E_{c,t,\theta} = \frac{E_C}{1+\varphi_{\vartheta,t}} \cdot \sqrt{k_1^3} = \frac{5000}{1+1.83} \cdot \sqrt{1^3} = 1766\, kPa$$

$$C_{\vartheta 1} = c_1 E_{c,t,\theta} b^2 = 0.18 \cdot 1766 \cdot \mathrm{b}^2 = 318 b^2$$

$$C_{\vartheta 2} = c_2\, n_f E_{c,t,\theta} b_k^2 = 0.052 \cdot 1 \cdot 1766 \cdot \left(\frac{b}{4}\right)^2 = 5.7 b^2$$

$$C_{D,A} = \frac{3}{2} \cdot \frac{C_{\vartheta 1}}{\left(\frac{C_{\vartheta 1}}{C_{\vartheta 1}+C_{\vartheta 2}}+1\right)} = \frac{3}{2} \cdot \frac{318 b^2}{\left(\frac{318 b^2}{318 b^2 + 5.7 b^2}+1\right)} = 240.9 b^2$$

Table 2 shows the values of the torsional stiffness $C_D$ given by formula (3.5) for selected cases of IPE purlins.

Table 2. The values of torsional stiffness of the support $C_D$ for selected cases of IPE purlins

| **Cross section of purlin** | **Stiffness of connection $C_{D,A}$ [kNm/rad]** | **Stiffness of sheeting $C_{D,C}$ [kNm/rad]** | **Total stiffness $C_D$ [kNm/rad]** | **Stiffness required by formula (3.2) [kNm/rad]** |
|---|---|---|---|---|
| IPE 100 | 0.7 | 266.6 | 0.7 | 3.6 |
| IPE 120 | 1.0 | 266.6 | 1.0 | 4.9 |
| IPE 140 | 1.3 | 266.6 | 1.3 | 6.4 |
| IPE 160 | 1.6 | 266.6 | 1.6 | 8.3 |
| IPE 180 | 2.0 | 266.6 | 2.0 | 10.0 |
| IPE 200 | 2.4 | 266.6 | 2.4 | 12.7 |
| IPE 220 | 3.0 | 266.6 | 3.0 | 14.6 |
| IPE 240 | 3.5 | 266.6 | 3.4 | 17.5 |
| IPE 270 | 4.4 | 266.6 | 4.3 | 20.5 |

The component related to stiffness of sheeting has neglected influence on the final value of stiffness of torsional support. None of the analysed case meet the requirements given by [7], so the sheeting made of sandwich panels cannot be considered as full protection against lateral-torsional buckling in terms of torsional supporting.

None of the separately considered supports had enough stiffness to gives full protection against lateral-torsional buckling. Table 3 shows the comparison of influence of each type of support on purlins resistance. In addition, the values of reduction factor $\chi_{LT}$ *(S+C)* obtained from LBA analysis including both type of supports were included. Static schema as a simple supported beam and steel grade S235 were assumed. For comparison, the case with no cooperation was also considered.

Table 3. Influence of including each type of support stiffness on reduction factor $\chi_{LT}$ for simple beams

| Cross section of purlin | Span length [m] | Stabilization by sandwich panels – reduction factor $\chi_{LT}$ | | | | Resistance increasing ratio (S+C) |
|---|---|---|---|---|---|---|
| | | Not included | Only shear stiffness (S) | Only torsional stiffness (C) | Combined effect (S + C) | |
| IPE 100 | 4.0 | 0.41 | 0.89 | 0.56 | 0.92 | 2.24 |
| IPE 120 | 5.0 | 0.35 | 0.86 | 0.54 | 0.90 | 2.58 |
| IPE 140 | 6.0 | 0.31 | 0.84 | 0.52 | 0.89 | 2.89 |
| IPE 160 | 6.5 | 0.30 | 0.80 | 0.51 | 0.87 | 2.88 |
| IPE 180 | 7.0 | 0.29 | 0.78 | 0.51 | 0.86 | 2.92 |
| IPE 200 | 7.5 | 0.29 | 0.74 | 0.50 | 0.84 | 2.85 |
| IPE 220 | 8.0 | 0.29 | 0.74 | 0.51 | 0.82 | 2.81 |
| IPE 240 | 9.0 | 0.28 | 0.73 | 0.51 | 0.81 | 2.85 |
| IPE 270 | 10.0 | 0.26 | 0.71 | 0.51 | 0.79 | 3.01 |

The results show that although neither of the analysed types of support fully prevent lateral-torsional buckling, the resistance of the purlin increased significantly. The combined effect of the supports results in further increasing of the reduction lateral-torsional factor, but still it was not enough to ensure full stabilisation of purlin. Actual stiffness of the connections between sandwich panels and purlins do not allow to reach full hot-rolled purlins resistance, but the benefits of including sandwich panels in calculations of purlins stability are worth to consider.

### 3.4. THE ABILITY OF STABILIZATION – LIMITATION DUE TO CONNECTION RESISTANCE

When stabilizing the purlin, each sandwich panel acts independently (Fig. 1), resulting in a small arm of forces in fasteners and inducing significant force on each fastener even in the case of a small external load. Simultaneously, small thickness of panel's face results in small values of connection resistance. Therefore, in the analysis of stabilization of the purlin by sandwich panels, the limitation due to connection resistance should be considered.

To check the influence of connection resistance on the ability to stabilize the purlin, the GMNIA analysis of the 9-metre simple beam made of IPE 240 was performed in Autodesk Simulation Mechanical software. The mesh consists of 20 mm shell elements. Additional truss elements in the top flange plane representing the behaviour of connections between sandwich panels and purlin has been assumed in the model. Their properties (stiffness $k_v$ and resistance $F_R$) were assumed based on chapter 3.2. Truss elements were connected in pairs with one common node at one end and 2 separate nodes at opposed ends, in the location of fasteners. It results in opposite forces in each pair of truss elements according to assumptions shown in Fig. 1.

The initial imperfection according to the form of global instability derived from LBA analysis under bending was assumed. The amplitude of imperfection was set as 1/1000 of beam length. Ideal elastic-plastic material model of steel S235 was used in analysis.

Fig. 5 shows the dependence between the displacement of the top flange and the applied load. Two cases were considered. In the first case (linear model of connections), only the stiffness of connections $k_v$ equals 700 N/mm was considered. In the second case (nonlinear model of connections), the actual resistance of connection was additionally included by ideal elastic-plastic model of truss elements with maximum resistance $F_R$ = 1500 N (according to chapter 3.2). The deformed shape of purlin is shown in Fig. 6.

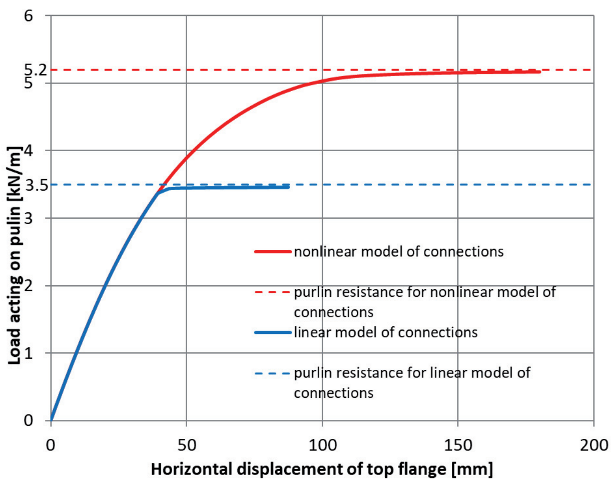


Fig. 5. Relationship between the displacement of the top flange and the applied load of the analysed purlin.

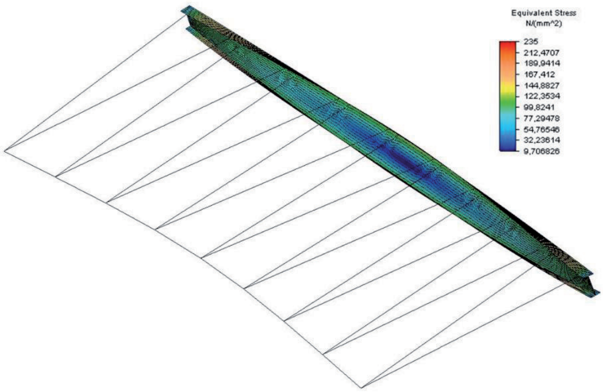


Fig. 6. Deformed shape of the analysed purlin

Including the limitation of connection resistance in analysis resulted in a significant decrease in the LTB resistance of the purlin. Reaching the resistance of connections between sandwich panels and purlins resulted in abrupt instability of the whole beam. The comparison between forces in connections obtained from FEM analysis and derived by formula (3.8) are shown in Fig. 7.

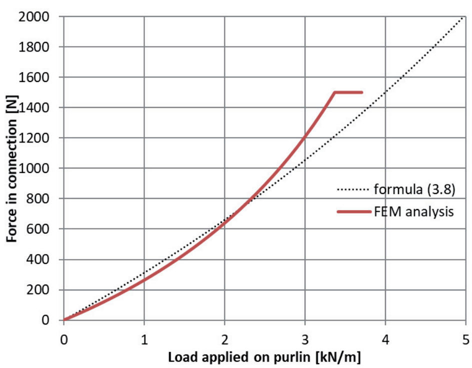


Fig. 7. Comparison of forces in connections obtained from FEM analysis and formula (4.8)

The formula (3.8) gives results very similar to FEM analysis. Although FEM analysis characterizes stronger nonlinearity, the connection resistance has been achieved at similar load values. Therefore, in other analysed cases, formula (3.8) was used.

The load caused reaching the connection resistance may be interpreted as the biggest load that the beam can withstand. Therefore, the bending moment induced by this load is the beam resistance due to the limitation of the connection resistance. The reduction factor limited due to connection resistance $\chi_{LT}(F_R)$ can be calculated as a limited bending moment versus the cross-section resistance ratio. Table 6 shows the values of the reduction factor due to the connection resistance for selected cases of IPE purlins. Values for no stabilization are also included.

Table 4. Values of reduction factor $\chi_{LT}(F_R)$ limited due to connection resistance for selected IPE purlins

| Cross section of purlin | Span length [m] | Reduction factor $\chi_{LT}(F_R)$ | Reduction factor for no stabilization | Resistance increasing ratio |
|---|---|---|---|---|
| IPE 100 | 4.0 | 1.0 | 0.41 | 2.44 |
| IPE 120 | 5.0 | 1.0 | 0.35 | 2.86 |
| IPE 140 | 6.0 | 1.0 | 0.31 | 3.23 |
| IPE 160 | 6.5 | 0.89 | 0.30 | 2.97 |
| IPE 180 | 7.0 | 0.75 | 0.29 | 2.59 |
| IPE 200 | 7.5 | 0.64 | 0.29 | 2.21 |
| IPE 220 | 8.0 | 0.55 | 0.29 | 1.90 |
| IPE 240 | 9.0 | 0.47 | 0.28 | 1.68 |
| IPE 270 | 10.0 | 0.40 | 0.26 | 1.54 |

Results of analysis show that the bigger bending moment in purlin, the bigger reduction of its resistance due to insufficient resistance of connections. The analysis that was performed assumed only one pair of fasteners per sandwich panels but increasing numbers of connections bring only a slight increase in the resistance due to smaller and smaller lever arms for further pairs of fasteners. Table 5 shows the final values of the reduction factor for single-span purlins that have been analysed, limited by both actual stiffness and resistance of connections.

Table 5. Values of reduction factor $\chi_{LT}$ for single-span purlins stabilized by sandwich panels

| IPE | Span length [m] | $\chi_{LT}$ | |
|---|---|---|---|
| | | No cooperation | Limited cooperation |
| **100** | 4.0 | 0.41 | 0.92 |
| **120** | 5.0 | 0.35 | 0.90 |
| **140** | 6.0 | 0.31 | 0.89 |
| **160** | 6.5 | 0.30 | 0.87 |
| **180** | 7.0 | 0.29 | 0.75 |
| **200** | 7.5 | 0.29 | 0.64 |
| **220** | 8.0 | 0.29 | 0.55 |
| **240** | 9.0 | 0.28 | 0.47 |
| **270** | 10.0 | 0.26 | 0.40 |

- limitation due to stiffness
- limitation due to connection resistance

Sheeting made by sandwich panels cannot provide full restraint against lateral-torsional buckling of a single-span purlin. In the case of small beams, the limitation is associated with insufficient stiffness of sheeting connections, while for bigger beams, the crucial factor is connections resistance. Nevertheless, including sandwich panels in purlin stabilization results in noticeable profits.
Despite the initial stiffness and resistance of the connections, rheological factors should also be considered. The long-term parameters depends on the durability of connections subjected to repeatable climatic loads. Although some research has been already done [5] [17], and conclusions are promising, nevertheless in case of appearance forces with significant magnitude, the risk of hole ovalization still exists. Therefore, great care should be taken when designing in Structural Class II using sandwich panels.

## 4. Summary

Considering the cooperation of sandwich panels with purlins can lead to improved economy in the design of purlins. Even low values of purlin-to-panel connections stiffness and resistance results in evident benefits in purlins resistance. Analysis shows that although such connections cannot provide full purlin stability, as with trapezoid sheets, they are worth being considered. Depending on factor considered in analysis, the lateral-torsional buckling resistance of hot rolled purlin connected with sandwich panel can increase up to 3 times in compare with no cooperation case. Including only shear stiffness in calculations results in better purlin resistance than including only torsional one, but the best results are achieved for including both types of support in analysis. Simultaneously the bigger bending moment in purlin, the more significant limitation of purlins resistance due to insufficient resistance of connections between sandwich panels and purlins.

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## ZACHOWANIE PŁATWI WALCOWANEJ NA GORĄCO POŁĄCZONEJ Z PŁYTAMI WARSTWOWYMI

*Słowa kluczowe*: płatew, zwichrzenie, płyty warstwowe, ściągi, analiza MES

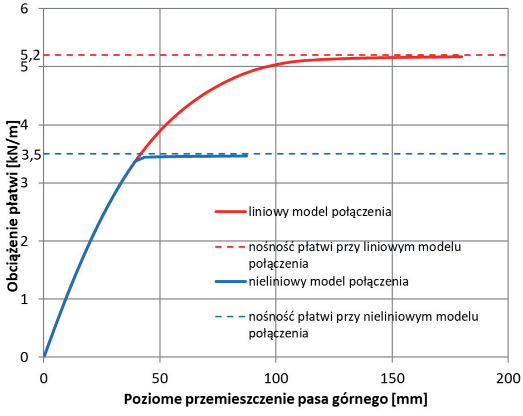
**STRESZCZENIE:**

W artykule przedstawiono możliwości i ograniczenia współpracy pomiędzy stalowymi płatwiami z kształtowników IPE a poszyciem z płyt warstwowych. Określono wartości sztywności postaciowej oraz przeciwskrętnej połączeń pomiędzy płatwiami a płytami warstwowymi. Przeprowadzone analizy wykazały, że sztywności te są niewystarczające do zapewnienia pełnej stablizacji płatwi. Niemniej jednak uwzględnienie w/w współpracy pozwala na bardziej ekonomiczne projektowanie płatwi, ponieważ nawet niewielkie wartości sztywności połączeń pomiędzy płatwią a płytami warstwowymi skutkują wyraźną poprawą nośności płatwi na zwichrzenie, co przedstawiono w Tablicy 1.

Tablica 1. Wpływ uwzględnienia sztywności podparcia na współczynnik zwichrzenia $\chi_{LT}$ dla belek wolnopodpartych

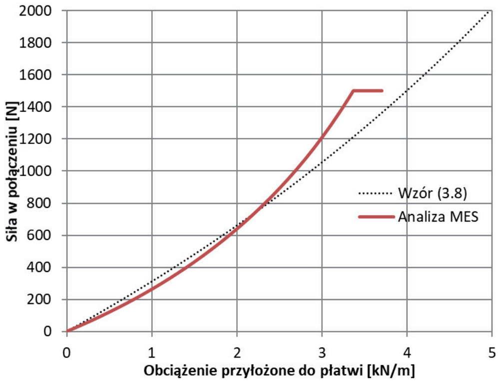
| Przekrój płatwi | Rozp. przęsła [m] | Stabilizacja płytami warstwowymi – współczynnik $\chi_{LT}$ | | | | Stosunek wzrostu nośności (S+C) |
|---|---|---|---|---|---|---|
| | | Brak | Tylko sztywność postaciowa (S) | Tylko sztywność skrętna (C) | Łączny efekt (S + C) | |
| IPE 100 | 4,0 | 0,41 | 0,89 | 0,56 | 0,92 | 2,24 |
| IPE 120 | 5,0 | 0,35 | 0,86 | 0,54 | 0,90 | 2,58 |
| IPE 140 | 6,0 | 0,31 | 0,84 | 0,52 | 0,89 | 2,89 |
| IPE 160 | 6,5 | 0,30 | 0,80 | 0,51 | 0,87 | 2,88 |
| IPE 180 | 7,0 | 0,29 | 0,78 | 0,51 | 0,86 | 2,92 |
| IPE 200 | 7,5 | 0,29 | 0,74 | 0,50 | 0,84 | 2,85 |
| IPE 220 | 8,0 | 0,29 | 0,74 | 0,51 | 0,82 | 2,81 |
| IPE 240 | 9,0 | 0,28 | 0,73 | 0,51 | 0,81 | 2,85 |
| IPE 270 | 10,0 | 0,26 | 0,71 | 0,51 | 0,79 | 3,01 |

Dodatkowo uwzględniono również ograniczenie z uwagi na niewystarczającą nośność połączeń. Przeprowadzone analizy MES wykazały znaczący spadek nośności płatwi po uwzględnieniu rzeczywistej nośności połączeń (Rys. 1).




Rys. 1. Zależność pomiędzy przemieszczeniem pasa górnego a obciążeniem przyłożonym do płatwi

Wyniki analizy numerycznej zostały również porównane ze wzorem zaczerpniętym z literatury. Stwierdzono dobrą zgodność wyników pomiędzy tymi metodami (Rys. 2).




Rys. 1. Porównanie sił w połączeniach otrzymanych z analizy MES oraz ze wzoru z literatury

W przypadku niewielkich belek ograniczenie nośności na zwichrzenie związane jest z niewystarczającą sztywnością ich połączeń z poszyciem dachowym, podczas gdy w odniesieniu do belek obciążonych większym momentem zginającym, głównym czynnikiem jest nośność tych połączeń, co przedstawiono w Tablicy 2.

Tablica 2. Wartości współczynnika zwichrzenia dla płatwi wolnopodpartych stabilizowanych płytami warstwowymi

| **IPE** | **Rozpiętość przęsła [m]** | $\chi_{LT}$ | |
|---|---|---|---|
| | | **Brak współpracy** | **Ograniczona współpraca** |
| **100** | 4,0 | 0,41 | 0,92 |
| **120** | 5,0 | 0,35 | 0,90 |
| **140** | 6,0 | 0,31 | 0,89 |
| **160** | 6,5 | 0,30 | 0,87 |
| **180** | 7,0 | 0,29 | 0,75 |
| **200** | 7,5 | 0,29 | 0,64 |
| **220** | 8,0 | 0,29 | 0,55 |
| **240** | 9,0 | 0,28 | 0,47 |
| **270** | 10,0 | 0,26 | 0,40 |

- ograniczenie z uwagi na sztywność połączenia (niebieskie pola)
- ograniczenie z uwagi na nośność połączenia (zielone pola)

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