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A calibration-based method for interval reliability analysis of the multi-manipulator system

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Highlights

- Numerical evaluation of reliability for multi-manipulators system from the new perspective of kinematics.
- Efficient conversion from the series system to the parallel system via the base frame calibration.
- Significant improvement of reliability based on the establishing connections among all the manipulators.

Abstract

The multiple manipulators can construct a special multi-agent system with the distinction that the type can be serial or parallel according to their cooperative way. We proposed a comprehensive method to handle the problem of reliability estimation. The wide and narrow bound method are applied to calculate the interval reliability respectively when multiple manipulators work as the series system. Aims to decrease the system complexity and enhance the dynamic adjustment capability, the base frame calibration technique is presented to convert the series system to a parallel one, naturally the reliability can be improved significantly. A system composed by three manipulators is utilized as an example to illustrate the feasibility of the proposed method.

Keywords

multiple manipulators, series/parallel system, reliability estimation, base frame.

1. Introduction

Manipulators have a wide range of applications in industry because of their high efficiency, accuracy and easy operation. They are often used in the field of transportation, gripping and assembling etc. in which end-effector of the manipulator is designed to move from one point to another with the same position and orientation repetitively[1,4].In practice, errors that originated from manufacturing and assembling process of the manipulator cannot be eliminated. The main uncertainties include joint clearance, dimensional deviations, material deformation et al. which can finally result in the erratic shocks, vibration and deterioration of motion capability over its service life [9, 13,22]. Therefore, analyzing the behavior of a manipulator with consideration of parameter uncertainties and evaluating the reliability appropriately can be significant issues.

To calculate the reliability of a manipulator with reference to a particular point during a repetitive work. Rao and Bhatti [20] proposed a probabilistic method to measure the extent of influence caused by the joint clearance on the repeatability of the two-link manipulator. The instability of the behavior from the aspect of dynamics and kinematics are both analyzed. Kim et al. [12] focused on the analysis of impact caused by joint clearance, all the variables were treated as normally distributed and then the first order reliability method (FORM) was applied. With integration of the second order Taylor expansion and an entropy-based optimization approach, Wang et al. [27] analyzed the reliability of a manipulator when confronted with arbitrarily distributed joint clearance.

Though the reproducibility of a manipulator outperforms its ability to reach an expected position [18], there are a lot of demands in which the motion should be controlled in the entire trajectory instead of only a few points, such as tasks of welding, sculpture, spraying etc. [4,29,31]. Pandey and Zhang[17]proposed a fractional moment estimation method, the drawback lies in that two layers of optimization process are required, which makes it complicated. Zhao et al. [30] developed an approximated approach to study the motion reliability of a parallel mechanism during a circle trajectory movement, in their work, the first passage method, Lie group and Lie algebra were utilized to describe the variables with time-variant character. Other popular methods like FOSM (first order second moment method) and MCS (Monte Carlo simulation) are also widely used [26].

The researches mentioned-above exhibit an obvious feature in common that only one failure mode is needed to be concerned. When the multi-agent system consists of many subsystems, the numerical

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evaluation for such a sophisticated system can be a big challenge [11]. In the context of the system reliability analysis. Cornell [5] proposed the first order bound method (also called the wide bound method) to compute the failure probability of the series/parallel system, an interval instead of an exact value was firstly used to measure the stability and safety. However, the range of the result was too wide to satisfy the requirement of accuracy in practical applications. Later on, given the correlation and dependence of failure between the paired subsystems, Ditlevsen[7] developed the second order bound method (also called narrow bound method), the precision of result can be much better enhanced compared to that of the wide bound method. Through introducing a truncation technique to limit the possible value of random variables, Qiu et al.[19] analyzed the truss system and figured out the interval of failure probability. Safaei et al. [21] studied the redundant system mixed by the series and parallel subsystems, in order to reduce the cost caused by the sudden breaking down of machines and keep the manufacturing cellular with high reliability and production efficiency, they established a multi-objective optimization model. Aims at making a better trade-off between calculation accuracy and efficiency, Bichon et al.[3] put forward a surrogate model-based method to handle the problem of reliability analysis of a large system with multiple failure modes. Xie et al. [28] defined a time-domain series system for the gear train that has typical features of time-dependent multiple configuration, then proposed a reliability modeling technique to measure the system reliability. Some improved approaches with the foundation of the bound method have been also proposed to handle the problem of their unique structural systems[23,32].

As well known, with the rapid development of modern equipment technology, the cooperative system made up of several manipulators exhibits the dominant advantages compared to the single manipulator. The reasons can also stem from the fact that the multi-manipulator system has many outstanding abilities, for instance, the wider working space, greater caring capacity, better suitability to manufacture the large-size and complex components etc.[15,24]. Naturally, the research on the multi-manipulator system has drawn intensive attention. In order to realize that the leading manipulator can be followed fast and accurately by the other one. Liu et al.[16] proposed an adaptive impedance control algorithm for two cooperative manipulators. A simple task was planned to demonstrate feasibility of the method in their simulation and experiment. To ensure the qualified transportation of objects, Aldo et al. [2] designed a predefined-time controller with utilization of the hybrid/position sliding model control algorithm. All the subsystems including cooperative manipulators, tools and objects were supposed to be rigid. Korayem et al.[14] put forward an optimal control method based on the state-dependent Riccati equation, which was used to strengthen the capability of dual-arms to carry the heavy loads. Dohmann and Sandra[8] divided the transportation task into two subtasks including object trajectory tracking and grasp maintenance, then they proposed a distributed impedance control scheme to increase the two-manipulators system flexibility and efficiency of reaction to disturbance.

As described in previous, we can note that great progress has been made since so many efforts have been devoted to this area. Both the theory and practical applications are abundant in the reliability analysis of a single manipulator as well as the control scheme developing for multi-agent system. From another perspective, if the single manipulator is regarded as a subsystem, hence the manipulators can constitute a kind of special multi-agent system with the distinction that the type and configurations may be varied. The existing research maybe insufficient in terms of the reliability analysis for a multi-manipulator system. Some aspects can be still improved and they are listed as follows.

(1) *Immutable type of the multi-agent system*. In the open literature, the categories of a multi-agent system mainly include the series, parallel and hybrid system [7,18,32]. The failure mode is decided ever since the structure design process has been completed. Therefore, the system type only can belong to one of the three modes and is unchangeable. Whereas cooperative manipulators may differ quite a lot, they can carry out the task together by a cooperative way with the assistant of machine vision, contact force controller etc. [2,4] or just by an independent way with the movement along its own pre-planned path[1]. The failure mode varies with their cooperation way and the type can be even transformed from the series to the parallel, articles in this field are rather limited.

(2) *Insufficient analysis of the kinematic reliability.* Most current approaches aim to develop a kind of control scheme to grasp the rigid object based on the complicated dynamic model [8,16,24], instead of trying to evaluate the reliability numerically. From the perspective of dynamics, the system stability is measured by the changes of the joint torque and the path tracking error. However, since there is the probability of failure both in kinematics and dynamics for a single manipulator[12,17,19,24], it is reasonable to believe that the multi-manipulator system maybe also failed to meet the demand in the kinematics over the execution of tasks from the perspective of probability theory, no matter how robust and excellent the controller is. So the kinematic reliability analysis of the multi-manipulator system can provide a new insight and become an important issue.

Given all, we proposed a novel method that can analyze the reliability of multi-manipulator system comprehensively for the first time. The wide and narrow bound method are applied to obtain the reliability interval that is rigorously limited by the lower and upper bound respectively.

The type of such a special multi-agent system depends on the way that how the single component is connected and cooperated. When each manipulator is just designed to move along its own trajectory planned beforehand independently and no connection is available for the internal communication, multiple manipulators can be treated as a series system. The mission will be failed as long as any one of the manipulators breaks down or becomes unreliable in kinematics. Since the negative influence caused by the failed manipulator can be expanded and propagated through the closed chain of internal force into the rest of the components, due to the lack of the dynamic adjustment capability, the other manipulators may deviate from their desired positions simultaneously.

In order to construct the connections among the multiple manipulators, an efficient base frame calibration method is introduced to figure out the relative position between any pair of cooperative manipulators. Similar to the leader-follower scheme, all the manipulators that play the role of a follower can have the flexibility to track the position of the leader. Furthermore, the actual position of the leading manipulator can be treated as the desired position which means the leader's behavior can be always acceptable and reliable in kinematics with the failure probability dropping to zero. In this situation, the failure mode of the multi-manipulator system can be identical to that of a parallel system, under the condition that all single components are defected, the system can be failed. The significant contributions in our work mainly include three aspects: 1) numerical evaluation of the multi-manipulator system reliability from the novel view of kinematics for the first time, 2) efficient conversion from the series system to a parallel system via the base frame calibration and 3) remarkable optimization of the reliability.

Remainder of this paper is organized as follows: section 2 presents the probabilistic failure model of the manipulator, section 3 describes the details about the proposed method that mainly include the computation and conversion process for the series /parallel system with multi-manipulator. Simulation is conducted in section 4 followed by discussion in section 5. Conclusions are summarized in the last section.

2. Probabilistic modeling of a manipulator

2.1. Forward kinematic with random variables

The common used manipulator is constructed a several of linkages, as shown in Fig.1 (a).

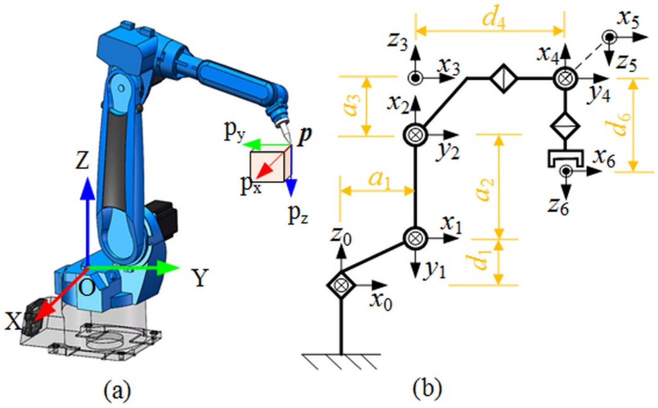


Fig. 1 a schematic diagram of a single manipulator

The relative position between a pair of adjacent joints can be described by a homogeneous matrix, written as[6]:

$$A_i^{i-1} = \begin{bmatrix} Cos\theta_i & -Sin\theta_i Cos\alpha_i & Sin\theta_i Sin\alpha_i & a_i Cos\theta_i \\ Sin\theta_i & Cos\theta_i Cos\alpha_i & -Cos\theta_i Sin\alpha_i & a_i Sin\theta_i \\ 0 & Sin\alpha_i & Cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $(a_i, d_i, \alpha_i, \theta_i)$ are the D-H parameters of the ith link.

The position and orientation of the end-effector can be obtained through a series of transformation from the base frame to the final one, the whole D-H coordinates of such a manipulator is drawn in Fig.1 (b), formulated as:

$$T = \prod_{i=1}^{n} A_i^{i-1} = \begin{bmatrix} \boldsymbol{Rot} & \boldsymbol{Pos} \\ 0 & 1 \end{bmatrix} \quad (2)$$

where $\boldsymbol{Rot}$ denotes the orientation matrix, $\boldsymbol{Pos} = \left[p_x, p_y, p_z\right]$ represents the position of the end-effector and n is the total number of degree of freedoms.

The kinematics is studied based on the 6-DOFs manipulator, the corresponding structure parameters are listed in Table 1.

Table 1. The standard D-H parameters

No.	$a_i\,(mm)$	$d_i\,(mm)$	$\alpha_i\,(°)$	$\theta_i\,(°)$	Initial angle (°)
1	40	330	−90	θ_1	0
2	315	0	0	θ_2	−90
3	70	0	−90	θ_3	0
4	0	310	90	θ_4	0
5	0	0	−90	θ_5	90
6	0	70	0	θ_6	0

As presented in section 1, due to the unavoidable errors that originated from defects of manufacturing, assembling and material deformation etc., the manipulator can be seriously affected by those uncertainties. In this work, the dimensional deviations and joint clearances are mainly concerned and they are treated as variables because of the random nature [23], which are listed in Table 2.

Table 2. Distribution of the random variables

variable	distribution	μ (mm)	σ(mm)
a_1	normal	40	0.04
a_2	normal	315	0.32
a_3	normal	70	0.07
d_1	normal	330	0.33
d_4	normal	310	0.31
d_6	normal	70	0.07

Under the influence of the joint clearance, the actual angle of ith link θ_i can be modeled as:

$$\theta_i = \tilde{\theta}_i + \zeta \quad (3)$$

where $\tilde{\theta}_i$ is the desired value of θ_i and ζ is a small normally distributed variable with its $\mu_\zeta = 0$ and $\sigma_\zeta = 0.5°$.

With consideration of the impact caused by dimensional deviations and joint clearance, the forward kinematic model is:

$$\mathrm{T} = f(\boldsymbol{X}) \quad (4)$$

where $\boldsymbol{X}$ represents the vector of the random variables (including dimensional parameters $\boldsymbol{a,d}$ and joint angle $\boldsymbol{\theta}$).

2.2. Failure probability of a manipulator

Due to the effect of joint clearance and link dimension deviations, the actual position of the end-effector may deviate from the desired position, the possible location with reference to the ideal position can be plotted in Fig.2.

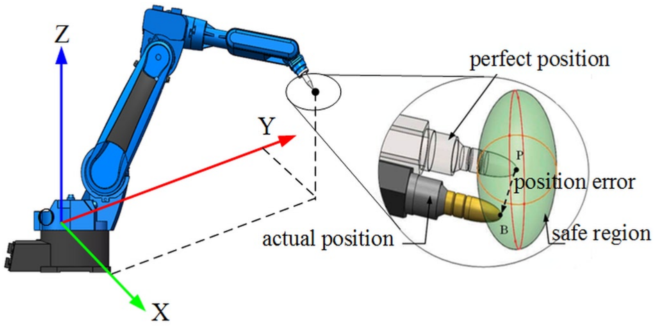


Fig. 2 the relative position between actual and desired location

This deviation is defined as the position error ε, thus [12]:

$$\varepsilon(\mathbf{X}) = \sqrt{(x_d - p_x)^2 + (y_d - p_y)^2 + (z_d - p_z)^2} \quad (5)$$

where (x_d, y_d, z_d) denote the desired position and (p_x, p_y, p_z) represent the actual position.

The unaccepted performance of a manipulator means the end-effector falls outside a permissible region under the influence of random variables, suppose the size of such a safe area is δ, the performance function can be expressed as:

$$\rho = g(X) = \delta - (X) \tag{6}$$

FOSM can provide a convenient way to evaluate the reliability for a system with the normally distributed output. As it linearizes the performance function $g(X)$ with the first order Taylor expansion at the mean value of variables μ_X, we have [23]:

$$\rho \approx g(\mu_X) + \sum \frac{\partial g(\mu_X)}{\partial X}(X - \mu_X) \tag{7}$$

Define the reliability index β as:

$$\beta = \mu_\rho / \sigma_\rho \tag{8}$$

where μ_ρ and σ_ρ are the mean value and the standard deviation of ρ.

So the probability of failure defined in Eq. (6) can be computed by:

$$P_f = 1 - \varnothing(\beta) \tag{9}$$

where $\varnothing(\cdot)$ is the standard cumulative distribution function of a Gaussian variable.

3. Interval reliability estimation of the system with multiple manipulators

Though the kinematic reliability analysis of the multi-manipulator system can be extended from the single manipulator with much similarity, the dependent component can introduce mutual impact on the each other and finally result in multiple potential failure modes instead of only one, which makes it rather complicated. A practical attempt is to work out an appropriate interval of failure probability in theoretic [5]. For brevity, the *i*th manipulator is denoted as R_i for a multi-agent system with a number of *m*.

3.1. Reliability analysis of the series system

When every component is isolated to each other and is controlled independently to move along its own trajectory, in which the relative position between any cooperative manipulators cannot be obtained, the object is hold and remains relative static to all the end-effectors by the contact force provided by the manipulators. Because of the error originated from the failed one, the others can be negatively influenced by the disturbance propagated via the closed-chain of internal force, therefore, the task is terminated and failed. Such the number of *m* manipulators can be regarded as a series system, the failure mode can be shown in Fig. 3.

Define the failure event for *i*th manipulator as E_i, the reliable event can be denoted as $\bar{E}_i$,the failure probability of the series system can be formulated by[20]:

$$P(E) = P(E_1 \cup E_2 \cdots \cup E_m) \tag{10}$$

If the failure event is independent, we can have:

$$P(E) = 1 - \prod_{i=1}^{m} P(\bar{E}_i) \tag{11}$$

In practice, the single manipulator can hardly avoid to bring about the impact on the others since all they touch the same object, which means the assumption of independent component is spurious. Suppose the correlation coefficient between the failure modes of *i*th and

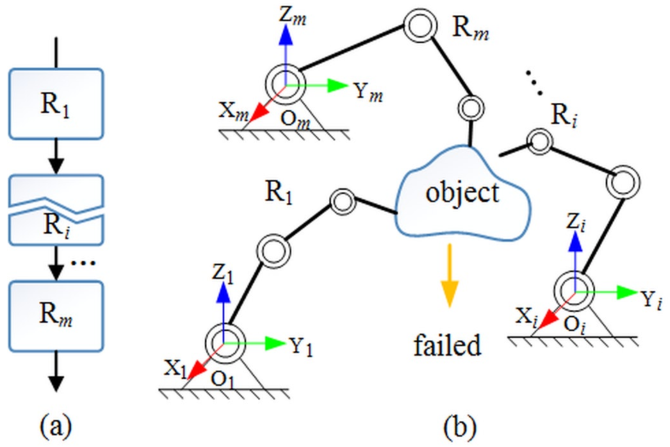


Fig. 3. the series system with multiple manipulators

*j*th manipulator is $\rho_{ij}\ (1 \le i, j \le m)$ which can be easily deduced that the index is positive (ρ_{ij}>0).

According to the conditional probability law, we can obtain:

$$P(E_i E_j) = P(E_i) P(E_j | E_i) \tag{12}$$

$P(E_j|E_i) \ge P(E_j)$ because of (ρ_{ij}>0), therefore, we have $P(E_i E_j) \ge P(E_i)P(E_j)$,which leads to the following result:

$$P(E) \le 1 - \prod_{i=1}^{m} P(\bar{E}_i) \tag{13}$$

Due to $P(E_i) \in [0,1]$, we can derive the lower bound as:

$$P(E) \ge \max\{P(E_1), P(E_2), \cdots, P(E_m)\} \tag{14}$$

The wide reliability interval for the series multi-manipulator system can be formulated by:

$$\max_{1 \le i \le m}(P_{fi}) \le P(E)_W \le 1 - \prod_{i=1}^{m} P(\bar{E}_i) \tag{15}$$

The narrow bound theory is frequently applied because of its outstanding trade-off between accuracy and efficiency[7]. To acquire a more precise result of the reliability interval, Eq. (10)can be represented as:

$$P(E) = \sum_{i=1}^{m} P(E_i) - \sum_{1 \le i \le j \le m}^{m} P(E_i E_j) + \sum_{1 \le i \le j \le k \le m}^{m} P(E_i E_j E_k) + \cdots + (-1)^{m-1} P(E_1 E_2 \cdots E_m) \tag{16}$$

Since $P(E_i) \ge P(E_i E_j) \ge P(E_i E_j \cdots E_m)\ (1 \le i \ne j \le m)$, the lower bound of the narrow reliability interval is:

$$P(E)_N \ge P(E_1) + \sum_{i=2}^{m} \max\left\{P(E_i) - \sum_{j=1}^{i-1} P(E_i E_j), 0\right\} \tag{17}$$

Furthermore,

$P(E_1 E_3) + P(E_2 E_3) - P(E_1 E_2 E_3) = P[(E_1 E_3) \cup (E_2 E_3)]$ and

$P\left[(E_1E_3)\cup(E_2E_3)\right]\geq \max\{P(E_1E_3),P(E_2E_3)\}$, so the upper bound for the narrow reliability interval is:

$$P(E)_N \leq \sum_{i=1}^{m} P_{fi} - \sum_{i=2}^{m} \max_{i<j} P_{fi,j} \tag{18}$$

where $P_{fi,j}$ denotes the joint failure probability that the ith and jth manipulator can fail simultaneously.

3.2. Base frame calibration

The lack of communications among the components makes the multiple manipulators become the series system and result in the poor dynamic adjustment capability, the behavior of all the manipulators have to be acceptable to ensure the system safety. In order to overcome this defect and enhance the system reliability. A feasible way is to figure out the relative position for the cooperative manipulators which can be realized by the base frame calibration [10].

The projection-based method is a simple but quite efficient calibration approach for manipulators under the typical installation，the requirement of only two points and straightforward calculation process make it attractive when compared with the matrix-equation methods[25].

As shown in Fig. 4. The calibration procedure can be briefly described as follows:

- Step1 Select one point in two manipulators' common working space and denote as P_1.
- Step2 Drive manipulators and make their measuring tips touch the calibration block respectively.
- Step3 Record down the coordinate values.
- Step4 Repeat step1 to step3 once more in another point denoted as P_2.

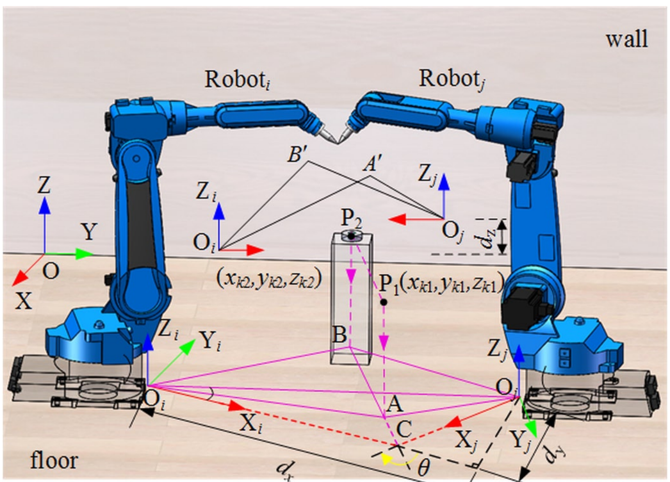


Fig. 4. Two floor-mounted manipulators

As we can see from the Fig. 4, the transformation from O_i-$X_iY_iZ_i$ to O_j-$X_jY_jZ_j$ can be obtained by the rotation around axis-Z_i and translation along the axis-X_i,Y_i and Z_i. ${}^{i}T_j = [dx, dy, dz]$ and θ are represented as the translation vector and rotation angle respectively. The rotation matrix can be formulated as[6]:

$$Rot(\theta,Z)=\begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

where Rot (θ, Z) means a rotation matrix with angle θ around axis-Z.

The coordinate values of P_1 and P_2 relative to its own base frame are denoted as $P_{k1}(x_{k1}, y_{k1}, z_{k1})$ and $P_{k2}(x_{k2}, y_{k2}, y_{k2})$(k=i,j). The relative position between two base frames in X-Y plane and Y-Z plane are obtained by the projection, when cooperative manipulators shake hands with each other at P_1 and P_2.

The translation parameters dx and dy can be computed by:

$$\begin{cases} d_x = |O_iO_j| \times cos\angle O_jO_iC \\ d_y = |O_iO_j| \times sin\angle O_jO_iC \end{cases} \tag{20}$$

In a similar way, the rotation angle θ can be computed by:

$$\theta = \angle ACO_j + \angle ACO_i \tag{21}$$

As for the parameter d_z that measures the distance between two base frames along axis-Z_i. The topological structure is projected into Y-Z plane (wall) in the world frame. The formulation can be written as:

$$d_z = \frac{(z_{i1}-z_{j1})+(z_{i2}-z_{j2})}{2} \tag{22}$$

For a better illustration of the calibration procedure, more specific details about the computation of middle parameters can be found in appendix A.

Since all the paired manipulators are calibrated, the whole system can be naturally calibrated according to the coordinate transformation theory.

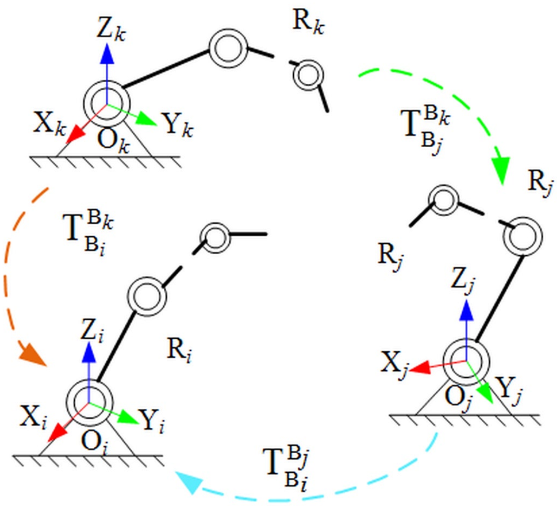


Fig. 5. Coordinate transformation for the multi-manipulator system

The relative transformation among base frames of the manipulators is plotted in Fig. 5, if the transformation matrix $T_{B_i}^{B_j}$ and $T_{B_j}^{B_k}$ have been calibrated by the projection-based method, then the $T_{B_i}^{B_k}$ can be determined by the following Eq. (23), formulated as:

$$T_{B_i}^{B_k} = T_{B_i}^{B_j} \cdot T_{B_j}^{B_k} \tag{23}$$

For the multi-agent system consists of m manipulators, we just need to calibrate m-1 times and the relative transformation matrix among all the manipulators can be computed.

3.3. Reliability analysis of the parallel system converted from a series system

Since the connections among the manipulators have been set up by the base frame calibration, the movement of each manipulator can be coordinated rather than independent. We use the leader-follower scheme to describe the role that each manipulator can play in the system. One of the manipulators is designed as the leader and the others become follower that can adjust themselves' position dynamically and timely according to the leader's position. One of the manipulators' failure cannot result in the final mission failure, therefore, the series system can be converted into a parallel system, as shown in Fig. 6.

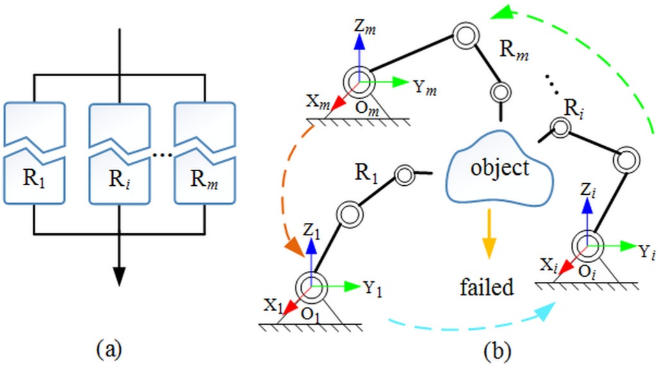


Fig. 6. The parallel system with multiple manipulators

The failure of a parallel system can be modeled as:

$$P(E)=P(E_1 \cap E_2 \cdots \cap E_m) \tag{24}$$

According to Eq. (12), the above equation can be rewritten as:

$$P(E)=P(E_1)P(E_2|E_1)\cdots P(E_m|E_1E_2\cdots E_{m-1}) \tag{25}$$

Similarly, because of $\rho_{ij}>0$, $P(E_i|E_1E_2\cdots E_{i-1}) \ge P(E_i)$,the wide bound of the probability of failure is:

$$\prod_{i=1}^{m} P_{fi} \le P(E)_W \le \min(P_{f1},P_{f2},\cdots,P_{fm}) \tag{26}$$

Since $P(E_1E_2\cdots E_m) \le P(E_iE_j)(1 \le i \ne j \le m)$,the narrow bound of the probability of failure is:

$$\prod_{i=1}^{m} P_{fi} \le P(E)_N \le \min_{1\le i<j\le m} P_{fij} \tag{27}$$

Based on Eq.(9), we can derive that P_{fi} is $\varnothing(-\beta_i)$. The P_{fij} of two manipulators can be calculated by [5]:

$$P_{fij}=\int_{-\infty}^{-\beta_i}\int_{-\infty}^{-\beta_j}\frac{1}{2\pi\sqrt{1-\rho_{ij}^2}}\exp\left\{\frac{2\rho_{ij}\beta_i\beta_j-\beta_i^2-\beta_j^2}{2\left(1-\rho_{ij}^2\right)}\right\}d\beta_j d\beta_i \tag{28}$$

mhere the Matlab functions *intergral* and *prod* are utilized to figure out the P_{fij}.

4. Numerical example

To better illustrate the problem, a handing system consists of three 6-DOFs manipulators, as shown in Fig. 7, is used as an example.

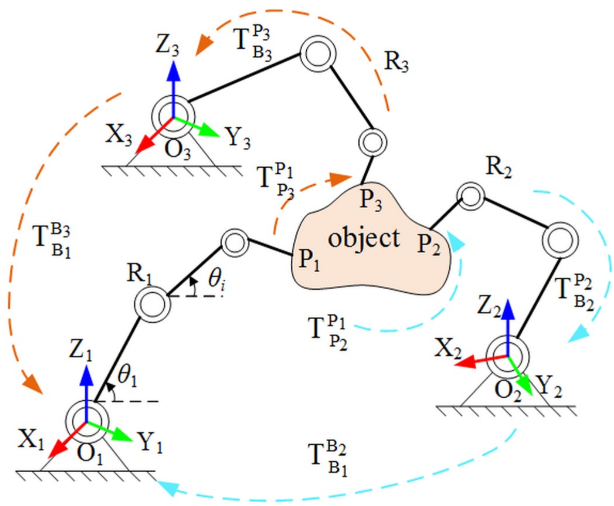


Fig. 7. The three-manipulators system

Each manipulator forms its own configuration and the corresponding desired joint angles are $\bar{\theta}_{R1}$= [6.17°, 22.25°, 5.62°, 2.31°, 46.12°, 21.68°], $\bar{\theta}_{R2}$= [−18.78°, 12.22°, −162.57°, 113.46°, −113.72°, −138.07°] and $\bar{\theta}_{R3}$= [75.52°, −37.28°, 122.43°, 75.88°, −121.79°, 28.46°] respectively, the radius of the permissible region is δ=2.5mm,the correlation coefficients are $\rho_{12}=\rho_{23}=\rho_{13}=0.8$. The distributions of all random variables are normal. For simplicity, the homogeneous matrix with the size of 4×4 is represented by T=[p_x,p_y,p_z,α,β,γ], in which α,β and γ denotes the series of rotation angles around axis-X, axis-Y, and axis-Z respectively, they can also be called as the RPY angle [6].

With utilization of $\boldsymbol{\mu}_{\mathbf{x}}$ and $\boldsymbol{\sigma}_{\mathbf{x}}$ listed in Table 2, the FOSM presented in section 2.2 can be adopted to compute $\boldsymbol{\mu}_\rho$ and $\boldsymbol{\sigma}_\rho$ of the performance function ρ= g($\boldsymbol{X}$) for the three manipulators. They are $\boldsymbol{\mu}_\rho$ = [1.8652, 1.9326, 1.8956] and $\boldsymbol{\mu}_\rho$= [0.6103, 0.6186, 0.6293]. The point in which the manipulator's end-effector contact with the object is P_i (i=1,2,3), assume that the relative position between P_1 and P_2 is T_{p1}^{p2} = [17.36, 22.28, ,21.32, 8.27°, 5.15°, 5.26°], P_1 and P_3 is T_{p1}^{p3} = [29.55,27.42, 28.73, 5.85°, 6.38°, 9.11°],the tool frames are given as Tol_i = [0,0, 20, 0°, 0°, 0°] (i=1,2,3).

4.1. The P_f of the series system

As the relative position is unknown and no information can be transmitted among the three manipulators. They have to move along their own trajectory that have been planned beforehand. Every manipulator can't adjust the position with reference to the others' automatically.

According to Eq.(8), the reliability index can be obtained and they are $\beta = \boldsymbol{\mu}_\rho/\boldsymbol{\sigma}_\rho$= [3.0564, 3.1241, 3.0124]. Then, we can figure out the failure probability of the three manipulators, they are $P_{f1}=1.120\times10^{-3}$, $P_{f2}=0.8917\times10^{-3}$ and $P_{f3}=1.2959\times10^{-3}$ respectively. Through Eq.(28), the joint probability of failure can be calculated as P_{f12}=2.6255×10^{-4}, P_{f13}=3.2606×10^{-4} and P_{f23}=2.8413×10^{-4}.

Based on Eq.(15), the wide interval of $P(E)_W$ for the series system can be estimated as:

$$\begin{cases} P(E)_W \ge \max(P_{f1},P_{f2},P_{f3}) \\ P(E)_W \le 1-(1-P_{f1})(1-P_{f2})(1-P_{f3}) \end{cases} \tag{29}$$

Therefore, we can finally compute the result which is presented as follows:

$$1.2959\times10^{-3} \le P(E)_W \le 3.3042\times10^{-3}.$$

As mentioned above, because of the positive correlation relationship among the manipulators, a more accurate estimation process can be carried out and the corresponding narrow bound of the failure probability for the series system can be obtained.

The part $\sum_{i=2}^{m} \max\left\{ P(E_i) - \sum_{j=1}^{i-1} P\left(E_i E_j\right), 0 \right\}$ can be equivalent to $\max\left\{P_{f2} - P_{f12}, 0\right\} + \max\left\{P_{f3} - P_{f13} - P_{f23}, 0\right\}$, so we can have the lower bound that is 2.4350×10^{-3}. By the same way, according to Eq.(18), we can calculate the upper bound that is 2.7192×10^{-3}. At the end, the narrow interval value of failure probability for the series manipulator is:

$$2.4350\times10^{-3} \le P(E)_N \le 2.7192\times10^{-3}.$$

4.3. Base frame calibration

As shown in Fig. 7, each manipulator has its own coordinate $O_i - X_i Y_i Z_i \left(i = 1,2,3\right)$ attached on the base. The aim of the base frame calibration is to figure out the transformation matrix T_{B1}^{B2} , T_{B1}^{B3} and T_{B2}^{B3} .

For example, relative to the base frame $O_1 - X_1 Y_1 Z_1$,the two calibration points Pnt_1 and Pnt_2 are recorded as $\mathrm{P}_{11}\left(x_{11}, y_{11}, z_{11}\right)$ and $\mathrm{P}_{12}\left(x_{12}, y_{12}, z_{12}\right)$ respectively. In a similar way, relative to base frame $O_2 - X_2 Y_2 Z_2$, the corresponding values are $\mathrm{P}_{21}\left(x_{21}, y_{21}, z_{21}\right)$ and $\mathrm{P}_{22}\left(x_{22}, y_{22}, z_{22}\right)$. The data are listed in Table 3.

Table 3. Calibration points for base frame1 and base frame2

point	X(mm)	Y(mm)	Z(mm)
$Pnt1$	$x_{11} = 393.44$	$y_{11} = 95.84$	$z_{11} = 591.07$
	$x_{21} = 136.80$	$y_{21} = -168.35$	$z_{21} = 588.15$
$Pnt2$	$x_{12} = 99.06$	$y_{12} = 398.79$	$z_{12} = 598.17$
	$x_{22} = 79.31$	$y_{22} = -586.84$	$z_{22} = 595.25$

According to the computation process detailed in Table 6, we can get the translation parameters (d_x = 610.33, d_y = 100.00, and d_z = 2.92) and rotation angle (θ=−128.0°). So the transformation matrix from O_1-$X_1Y_1Z_1$ to O_2-$X_2Y_2Z_2$ can be represents as T_{B1}^{B2} = [610.33, 100. 00, 2.92, 0°, 0°, −128°]. More specific computation details can be found in appendix A.

By the same way, to figure out T_{B1}^{B3} , another two calibration points ($Pnt1'$ and $Pnt2'$) are selected, they are listed as follows.

Table 4. Calibration points for base frame1 and base frame3

point	X(mm)	Y(mm)	Z(mm)
$Pnt1'$	$x_{11} = 67.35$	$y_{11} = 113.54$	$z_{11} = 578.24$
	$x_{31} = 410.46$	$y_{31} = 149.57$	$z_{31} = 576.68$
$Pnt2'$	$x_{12} = 35.59$	$y_{12} = 265.81$	$z_{12} = 585.25$
	$x_{32} = 378.70$	$y_{32} = 301.84$	$z_{32} = 583.69$

Naturally, through repetition of the calibration process, we can obtain the T_{B1}^{B3} = [95.12, 331.62, 1.56, 0°, 0°, −112°]. Based on the Eq.(23), $T_{B2}^{B3} = \left(T_{B1}^{B2}\right)^{-1} \cdot T_{B1}^{B3}$ = [134.68, −548.59, −1.36, 0°, 0°, 16°].

4.3. The P_f of the parallel system

Since the relative position between any couple of manipulators has been computed. The three manipulators can be converted as a parallel system. The manipulator R_1 is regarded as the leader and the rest R_2 and R_3 are followers. The expected angles of R_1 are $\bar{\theta}_{R1}$, under the influence of joint clearance that is normally distributed. The actual angles may be different from the desired values, and they are $\bar{\theta}_{R1}$ = [4.12°, 21.80°, 8.01°, 3.98°, 48.75°, 22.57°]. Therefore, through forward kinematic model formulated as Eq.(2) We can compute the actual location T_{B1}^{P1} = [168.95, 30.88, 511.40, 172.86°, −78.55°, 11.76°]. To keep contact with the object simultaneously, R_2 and R_3 are desired to arrive the position that is computed according to the leader R_1 position, the formulation is:

$$\begin{cases} T_{B_2}^{P_2} = \left(T_{B_1}^{B_2}\right)^{-1} \cdot T_{B_1}^{P_1} \cdot \left(T_{P_2}^{P_1}\right)^{-1} \\ T_{B_3}^{P_3} = \left(T_{B_1}^{B_3}\right)^{-1} \cdot T_{B_1}^{P_1} \cdot \left(T_{P_3}^{P_1}\right)^{-1} \end{cases} \qquad (30)$$

Thus we can have T_{B2}^{P2} = [168.95, −150.34, 521.36, 96.29°, 46.10°, −80.19°] and T_{B3}^{P3} = [134.22, 377.37, 536.01, 94.60°, 28.04°, −81.92°].

By the invers kinematic, we can further obtain the actual joint angles for the followers R_2 and R_3 that are the result of the dynamic adjustment process, the equation can be written as:

$$\theta_{Ri} = f^{-1}\left[\left(T_{B_1}^{B_i}\right)^{-1} \cdot T_{B_1}^{P_1} \cdot \left(T_{P_i}^{P_1}\right)^{-1} \cdot \left(T\mathrm{ol}_i\right)^{-1}\right] \left(i = 2,3\right) \qquad (31)$$

They are θ_{R2} = [−20.02°, 15.03°, −165.24°, 116.21°, −111.31°, −140.02°] and θ_{R3} = [74.36°, −36.19°, 119.73°, 178.68°,−121.43°, 29.18°] respectively.

The actual location of the leading manipulator R_1 can be regarded as its ideal values since it is the benchmark to the followers. The possible system failure can be only from the failure of the followers R_2 and R_3. The new reliability index for R_2 and R_3 are computed again by the FOSM, they are β = [3.1534, 3.0426]. The actual failure probability are P'_{f2} = 0.8068×10^{-3} and P'_{f3} = 1.1727×10^{-3} respectively. The joint failure probability is P'_{f23} = 2.5329×10^{-4}. According to Eq.(26), the wide interval of the parallel manipulators system can be obtained as:

$$9.4621\times10^{-7} \le P(E)'_W \le 8.0685\times10^{-4}$$

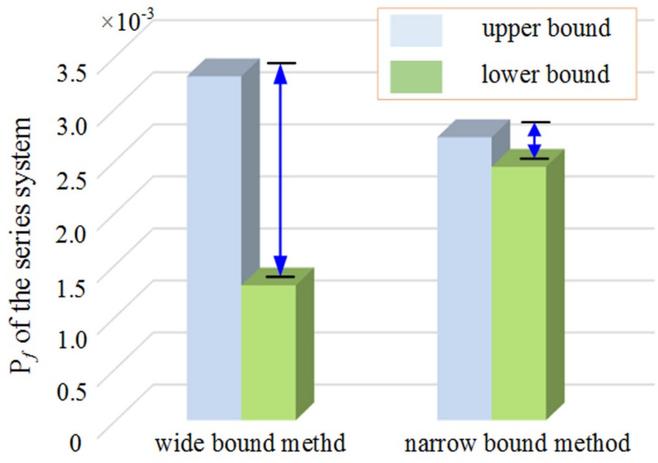
According to Eq.(27), the narrow interval with much more accuracy can be obtained as:

$$9.4621\times10^{-7} \le P(E)'_N \le 2.5329\times10^{-4}$$

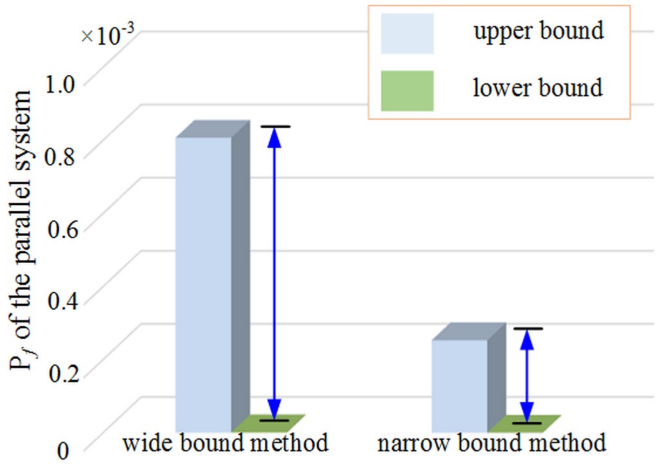
To provide a straightforward insight to the result, the lower bound and upper bound for the interval of failure probability in the series system computed by different methods are plotted in Fig. 8(a), as for the parallel system, the results can be drawn in Fig. 8(b).

For measuring the extent of optimization numerically when we use the narrow bound method to overcome the defect of the unsatisfied accuracy existing in the wide bound method. The following equation is presented, written as:

$$\eta = \frac{\left(P_{f\_\mathrm{upp}}^{W} - P_{f\_\mathrm{low}}^{W}\right) - \left(P_{f\_\mathrm{upp}}^{N} - P_{f\_\mathrm{low}}^{N}\right)}{\left(P_{f\_\mathrm{upp}}^{W} - P_{f\_\mathrm{low}}^{W}\right)} \times 100\% \qquad (32)$$



(a) P_f of the serial multi-manipulator system



(b) P_f of the parallel multi-manipulator system

Fig. 8. Failure probability interval of different system

where the superscripts W and N denote the wide bound and narrow bound method respectively, the subscripts *upp* and *low* represent the upper bound and lower bound of the failure probability interval respectively.

Other things equal, when the system type of multiple manipulators is mainly concerned, the percentage of reliability enhancement realized by conversion from the series into the parallel system can be evaluated by the equation, given as:

$$\lambda = \frac{\left(P_{f\_upp}^{s} - P_{f\_low}^{s}\right) - \left(P_{f\_upp}^{p} - P_{f\_low}^{p}\right)}{\left(P_{f\_upp}^{s} - P_{f\_low}^{s}\right)} \times 100\% \tag{33}$$

where the superscripts S and P denote the series and parallel system respectively.

So we can further obtain the optimization result listed in the Table 5.

Table 5. Percentage of optimization

system type	method	interval length	η	λ
series	wide bound	0.201×10^{-2}	–	–
	narrow bound	0.284×10^{-3}	85.86%	–
parallel	wide bound	0.806×10^{-3}	–	59.86%
	narrow bound	0.253×10^{-3}	68.61%	10.92%

It can be easily observed from Fig. 8 that the upper bound (maximum value) computed by the narrow bound method is much smaller than that of the wide bound method while the lower bound (minimum value) is contrary. In the series system, the interval length of failure probability are 0.201×10^{-2} (wide bound method) and 0.284×10^{-3} (narrow bound method) respectively. The narrower interval means the higher approximation accuracy. The optimization percentage provided by the narrow bound method is 85.86%, as for the parallel system, the value can be 68.61%.

Another significant feature can be also found in Table 5, is that the failure probability for the parallel system is much lower than that of the series system when the same approximate method is applied. It demonstrates the parallel system can outperform the series system. For example, the interval length of failure probability obtained by the wide bound method for the series and parallel system are 0.201×10^{-2} and 0.806×10^{-3} respectively, the reliability can be ameliorated by nearly 60%, and the result is about 10.92% when the narrow bound method is applied.

The whole process for the reliability analysis of multi-manipulators can be summarized in the flow chart drawn in Fig. 9 , in which the series system is converted into the parallel system based on the calibration technique and failure probability is calculated by use of the wide bound method and narrow bound method respectively.

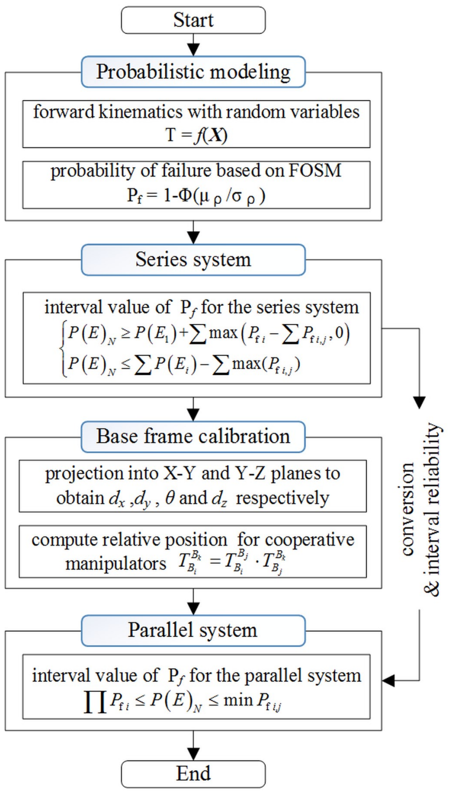


Fig. 9. Procedure for reliability analysis of multi-manipulators

5. Discussion

A deep insight is provided to better understand the problem of interval reliability estimation of the multi-manipulator system. Three aspects are focused and the corresponding specific details are given as follows.

1) Failure probability in kinematics

Unlike the dynamic analysis for multi-manipulators, of which the major objective is to develop a control algorithm to ensure the tracking error of the joint torque and position in working space can be converged close to zero. The reliability analysis in kinematics mainly focuses on the computation of probability that the multiple manipulators' positional error can satisfy the requirement together. Since there is the possibility for a single manipulator to fail both in kinematics and dynamics, it is rational to infer the multi-manipulator system can also have the failure possibility from the perspective of probability theory. No matter how robust and intelligent the control algorithm can be designed, there are always some uncertainties originated from parameters that are either ignored by us or modeled inappropriately, let alone the impact caused by the suddenly random disturbance. Therefore, both the model and control strategy are far away perfect to guarantee that the multi-manipulator system can never be failed over the service life. Similar to the study of the single manipulator, the failure probability analysis is essential and can be another important aspect to evaluate the performance of multi-manipulator system.

2) Approximate accuracy enhancement

As we can see from the Fig. 8, the approximate result can be quite different when the wide and narrow bound method are used respectively. The wide bound method lays the foundation that it is more practical to introduce the upper and lower bounds to construct an interval of the failure probability, rather than to compute a precise value which is almost impossible to obtain. However, in terms of the multi-manipulator system, the internal forces formed by the closed-chain configuration make each component dependent, the impact caused by the others on the manipulator itself is significant and therefore cannot be ignored. With this consideration, the narrow bound method further computed the joint failure probability of paired manipulators having the correlation relationship. That is the reason why the length of the interval reliability can be much shorter and the computation accuracy can be significantly increased.

3) Influence of the system type

When no connections are available, each manipulator just moves along its own trajectory independently. Actually, the errors originated from the failed manipulator can be propagated and even expanded through the closed –chain of internal force and finally bring about the negative influence on the others. The unintelligent cooperative way in the series system limits the ability of component to react to the disturbance. The base frame calibration is used to figure out the relative position, which can construct the relationship among the manipulators for the information transmission. The leader-follower scheme is applied to assign the role that each manipulator needs to play, the leading manipulator's movement is treated as the benchmark for the rest of manipulators to follow, which means the leader's actual behavior is always acceptable and safe during the transportation process. From this point of view, the series system composed by *m* manipulators can be converted into a parallel system consists of *m*-1 components, the failure may be only from the followers. In summary, the parallel system can perform more reliably than the series system mainly because of two folds: a) reduction of the number of components decreased the complexity of the system and b) strengthening in dynamic adjustment capability of each manipulator makes the parallel system more robust and reliable.

6. Conclusion

The purpose of our work is to analyze the reliability of a multi-manipulator system from the novel perspective of kinematics which is quite different from the traditional dynamic analysis. Some conclusions are made as follows.

(1) **Interval reliability estimation is suitable for the multi-manipulator system.** Since the manipulators are effected mutually by each other, which can bring about highly nonlinear, time-variant and even random parameter uncertainties, acquiring a precise value of the reliability becomes difficult. Therefore, working out the appropriate upper and lower bounds to construct an interval of failure probability is practical and economic. With consideration of the joint failure probability, the narrow bound method can provide the much more satisfied result for both the series and parallel system, of which the accuracy can be enhanced by 85.86% and 68.61% when compared to that of the wide bound method.

(2) **Base frame calibration can convert the series system to the parallel system efficiently.** Since only two different points are required to figure out the relative position and the computation process is quite simple, connections among the manipulators can be established efficiently through the base frame calibration, each manipulator is out of isolation and can acquire the dynamic adjustment capability. Therefore, the conversion from the series system to a parallel system can be realized.

(3) **The parallel multi-manipulator system can behave more stably and reliably compared with the serial mode.** Because of the outstanding dynamic adjustment capability and decrease of the complexity of the parallel system, the reliability can be much ameliorated when compared to the series system. The percentage of optimization can be nearly 59.86% and 10.92% for the wide bound and narrow bound method respectively.

Appendix A

Because of the importance of system type conversion, to better illustrate the base frame calibration procedure, the computation of the transformation from $O_1 - X_1Y_1Z_1$ to $O_2 - X_2Y_2Z_2$ is detailed with much more specific. The geometric relationship projected into the X-Y plane can be drawn in Fig. 10.

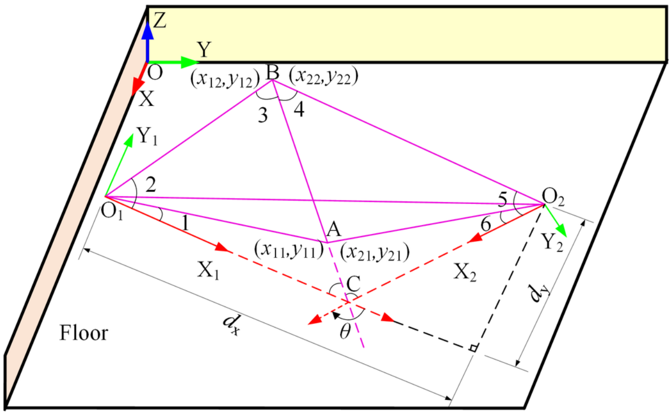


Fig. 10. Projection into the X-Y plane

The formulas for all the parameter calculation are listed in Table 6. For simplicity, the arctangent and arccosine functions are denoted as *atan* and *arcos* respectively.

Table 6. Geometric parameters in the calculating process

parameter	formula	result
$\lvert AO_1 \rvert$	$\sqrt{x_{11}^2 + y_{11}^2}$	404.9438
$\lvert BO_1 \rvert$	$\sqrt{x_{12}^2 + y_{12}^2}$	410.9163
$\angle 1$	$atan2(y_{11}, x_{11})$	13.6905°
$\angle 2$	$atan2(y_{12}, x_{12})$	76.049°
$\angle BO_1A$	$\angle 2 - \angle 1$	62.3593°

$\lvert AB\rvert$	$\sqrt{\lvert AO_1\rvert^2+\lvert BO_1\rvert^2-2\times\lvert AO_1\rvert\times\lvert BO_1\rvert\times cos\left(\angle AO_1B\right)}$	422.4207
$\angle 3$	$arcos\left(\frac{\lvert BO_1\rvert^2+\lvert AB\rvert^2-\lvert AO_1\rvert^2}{2\times\lvert BO_1\rvert\times\lvert AB\rvert}\right)$	58.1273°
$\lvert AO_2\rvert$	$\sqrt{x_{21}^2+y_{21}^2}$	216.9311
$\lvert BO_2\rvert$	$\sqrt{x_{22}^2+y_{22}^2}$	592.1767
$\angle 4$	$arcos\left(\frac{\lvert BO_2\rvert^2+\lvert AB\rvert^2-\lvert AO_2\rvert^2}{2\times\lvert BO_2\rvert\times\lvert AB\rvert}\right)$	15.5199°
$\angle O_1BO_2$	$\angle ABO_1+\angle ABO_2$	73.6472°
$\lvert O_1O_2\rvert$	$\sqrt{\lvert BO_1\rvert^2+\lvert BO_2\rvert^2-2\times\lvert BO_1\rvert\times\lvert BO_2\rvert\times cos\left(\angle O_1BO_2\right)}$	618.4680
$\angle 5$	$\lvert atan2\left(y_{22},x_{22}\right)\rvert$	82.3030°
$\angle BCO_2$	$180°-\left(\angle 4+\angle 5\right)$	82.1771°
$\angle BCO_1$	$180-\left(\angle 2+\angle 3\right)$	45.8229°
$\boldsymbol{\theta}$	$\angle BCO_1+\angle BCO_2$	**128.00°**
$\angle BO_1O_2$	$arcos\left(\frac{\lvert BO_1\rvert^2+\lvert O_1O_2\rvert^2-\lvert BO_2\rvert^2}{2\times\lvert BO_1\rvert\times\lvert O_1O_2\rvert}\right)$	66.7448°
$\angle CO_1O_2$	$\angle 2-\angle AO_1O_2$	9.3050°
$\boldsymbol{d_x}$	$\lvert O_1O_2\rvert\times cos\angle CO_1O_2$	**610.33**
$\boldsymbol{d_y}$	$\lvert O_1O_2\rvert\times sin\angle CO_1O_2$	100.00
$\boldsymbol{d_z}$	$\left[\left(z_{11}-z_{21}\right)+\left(z_{12}-z_{22}\right)\right]/2$	2.92

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