Qualitative methods for research of transversal vibrations of semi-infinite cable under the action of nonlinear resistance forces

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Abstract. The aim of paper is to study the solution of the problem of nonlinear transverse vibrations of elastic elongated body under the force of resistance in unbounded domain. Such problems have applications in various technical systems - vibration of pipelines, railways, long bridges, electric lines, optical fibers. Unboundedness of the area creates more fundamental difficulties in the study of the problem. For the considered models of nonlinear oscillations have no general analytical techniques for determining the dynamic characteristics of the oscillatory process. Therefore it is suggested to use qualitative methods of the theory of nonlinear boundary value problems to obtain correct problem solution conditions (existence and uniqueness of the solution). In the paper conditions of the correctness of the solution of mathematical model for these nonlinear systems (sufficient conditions of the existence and uniqueness in the class of locally integrable functions) are obtained.

Methods of qualitative study of semi-infinite cable vibrations under the forces of resistance based on general principles of the theory of nonlinear boundary value problems - method of monotony and Galerkin method. Scientific novelty of the work lies in particular in the generalization of methods of studying nonlinear problems on a new class of oscillatory systems in unbounded domains, justifying the correctness of the solution with specified mathematical model, which has practical applications in real engineering oscillatory systems.

The technique allows not only for proving the correctness of the model solution, but also has an opportunity in its study to apply various approximate methods.

Key words: mathematical model, nonlinear vibrations, nonlinear boundary value problem, Galerkin method, method of monotony, unbounded domain.

INTRODUCTION. OVERVIEW OF THE MAIN RESULTS

Problems of studying dynamic processes in nonlinear oscillatory systems describing the transverse (longitudinal) vibrations with the movements of cargo by conveyor belt (cable) type are important problems of mechanics. Investigation of nonlinear oscillatory and wave phenomena in elastic rod structures under the action of various perturbations (power, inertial and kinematic) is one of the classic problems of structural mechanics. Revitalization of theoretical research in this direction is due to not only logic of the foundations of deformed systems dynamics of, but also the interests of a wide variety of practical applications in the construction and engineering.

It should be noted, that the problem of studying of the influence of system parameters (such as speed of belt movement) on vibrations, sufficiently investigated in the case of constant velocity and linear law of elastic material. Specified is due to the fact that such situations are modeled by linear partial differential equations [4, 11, 18]. Asymptotic methods of nonlinear mechanics allowed to explore a wide class of mechanical oscillation systems for the case of quasi-linear dependence of amplitude of oscillations from the resistance force [17, 23]. In the case of non-linear law of elastic material, essentially nonlinear dependence of amplitude of vibrations from the resistance forces and variable speed of belt (cable) movement, problem is associated with the principled mathematical difficulties because there are no general analytical methods for solving this class of problems. Therefore, there is no general techniques of evaluation of amplitude - frequency characteristics of the oscillatory process. On the other hand, qualitative methods of general theory of nonlinear boundary value problems allow for a wide class of oscillatory systems to obtain correct solution results of the problem (existence, uniqueness and continuous dependence on the initial data). The above technique allows to substantiate the correctness of the model solution and allows in further investigation to use various approximate methods. Thus, the problem of qualitative research methods for nonlinear systems is relevant.

This article focuses on the qualitative study of mathematical models of nonlinear oscillations of semi-infinite cable under the action of nonlinear resistance forces. Similar problems arise in various technical applications such as vibrations of the pipelines on (nonlinear) elastic soil, railways, long bridges, tightly stretched electric lines, optical fibers embedded in the nonlinear elastic body, etc. [5, 6, 10, 15, 16, 21, 22].

In case of essentially nonlinear dependence of the amplitude from resistance forces problem associated with the principled mathematical difficulties even for the case of oscilatory model studies in a bounded domain. This problem is generally solved only for a very narrow class of problems.

Unboundedness of domain creates additional fundamental problems.

Particular problem for nonlinear wave equations in a form:

$$\frac{\partial^2 u}{\partial t^2} - \alpha \frac{\partial^2 u}{\partial x^2} + \beta |u|^{\rho-2} u = f(x,t), \rho > 2,$$

α, β - some functions (constants) in unbounded domains was considered in [3, 1, 9, 7, 2, 27, 18, 15, 13, 20]. At the same time limitation of elliptic operator coefficients are assumed. The results of existence and uniqueness of the solution of problems in unbounded domains in these works are obtained under the assumption of certain behavior of solution, initial data and of the right side of the equation at the infinity or without such assumptions. Currently qualitative results about the correctness of the solutions mentioned above mathematical models could be obtained only for a rather narrow class of problems in unbounded domains, because in unbounded domains we need to modify the methods of the general theory of nonlinear boundary value problems.

PROBLEM STATEMENT

The article presents qualitative research methodology of mathematical model for nonlinear oscillations of elastic semi-infinite cable under condition of linear (variable by spatial variable) elastic law and nonlinear resistance force. In its simplest formulation model is described by the mixed problem for the equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left(a(x) \frac{\partial u(x,t)}{\partial x} \right) + g\left(x, \frac{\partial u(x,t)}{\partial t} \right) = f(x,t), \qquad (1)$$

with initial conditions:

$$u(x,0) = u_0(x), \qquad (2)$$

$$\frac{\partial u(x,0)}{\partial t} = u_1(x), \qquad (3)$$

and boundary conditions:

$$u(x,0) = u_0(x), \qquad (4)$$

in unbounded domain $Q = (0,+\infty) \times (0, T)$, $0 < T < +\infty$. Further in this paper we denote $Q_{R,\tau} = (0, R) \times (0, \tau)$, $Q_\tau = (0, +\infty) \times (0, \tau)$ for arbitrary $R > 0$, $\tau \in (0, T]$. We will use the following Sobolev space of functions:

$$H_0^1(0,R) = \left\{ u \in H^1(0,R) : u\big|_{x=0} = u\big|_{x=R} = 0 \right\}, \quad \|u\|^2_{H_0^1(0,R)} = \int_0^R \left(\frac{\partial u}{\partial x} \right)^2 dx,$$

$$H^1_{0,loc}(0,+\infty) = \left\{ u \in H^1(0,R) \text{ for arbitrary } R > 0, \quad u(0,t) = 0 \right\},$$

$$L_{loc}(\overline{Q}) = \left\{ u \in L^r(Q_{R,T}) \text{ for arbitrary } R > 0 \right\}, \quad r \in (1,+\infty).$$

The generalized solution of the problem we will call a function that satisfies conditions (1), (2), (4) and the integral identity:

$$\int_{Q_\tau} \left[-\frac{\partial u}{\partial t}\frac{\partial v}{\partial t} + a(x) \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} \right] dxdt + \int_{Q_\tau} \left[\left| \frac{\partial u}{\partial t} \right|^{p-2} \frac{\partial u}{\partial t} v - fv \right] + \int_0^{+\infty} \frac{\partial u}{\partial t}(x,\tau) v(x,\tau) dx - \int_0^{+\infty} u_1(x) v(x,0) dx = 0. \qquad (5)$$

For arbitrary $\tau \in (0, T]$ and for an arbitrary function with limited carrier such that:

$$v \in L^2((0,T); H^1_{0,loc}(0,+\infty) \cap L^p_{loc}(\overline{Q}), \quad \frac{\partial v}{\partial t} \in L^2_{loc}(\overline{Q}).$$

Concerning the coefficients of the right side of the equation (1) and the initial data let's assume the fulfillment of following conditions.

(I) Function $a(x)$ belongs to the space $C([0,+\infty)$, $a(x) \geq a_0$, $a_0 = const > 0$ for all $x \in ([0,+\infty)$; $|a(x)| \leq M(1 + x^\alpha)$ for $x \to +\infty$, where $M > 0$, $0 \leq \alpha < 1 - \frac{p-2}{2p}$.

Remark. In the above relation is taken into account, that the modulus of elasticity can grow at sufficiently large x very slow (slower than the linear law) or stays constant.

(II) Function $g(x, \xi)$ - measurable by x and continuous by ξ moreover for arbitrary ξ, $s \in R$ and almost all $x \in (0, +\infty)$ we will obtain:

$$|g(x,\xi)| \le g_1 |\xi|^{p-1}, \quad p > 2,$$
$$g_1 = const > 0,$$
$$(g(x,\xi) - g(x,s))(\xi - s) \ge g_0 |\xi - s|^p,$$
$$g_0 = const > 0,$$

(III) $f \in L^q_{loc}(\overline{Q}), \quad \frac{1}{p} + \frac{1}{q} = 1.$

(IV) $u_0 \in H^1_{0,loc}(0,+\infty), \quad u_1 \in L^2_{loc}(0,+\infty).$

The aim of proposed work is to study the problem (1) - (4) for second order nonlinear wave equation, which in particular includes the equation of forced oscillations of the rod in the medium of resistance [14, p. 234] and obtain conditions for the correctness of solution of the mathematical model - sufficient conditions for existence and uniqueness of solution in the class of locally integrable functions.

The main result of this paper: if the mathematical model of oscillating process is described by problem (2) - (4) for equation (1), then under the conditions (I), (II), (III),

(IV) exist unique generalized solution of problem (1) - (4) for which:

$$u \in C([0,T]; H^1_{0,loc}(0,+\infty)),$$
$$\frac{\partial u}{\partial t} \in C([0,T]; L^2_{loc}(0,+\infty) \cap L^p_{loc}(\overline{Q}). \tag{6}$$

METHODS OF OBTAINING RESULTS

Let u^1, u^2 - generalized solutions of problem (1) - (4) and problem that differs from (1) - (4) by the fact that in the right side of (1) force f is replaced by $\overline{f} \in L^q_{loc}(\overline{Q})$ respectively. Then for arbitrary τ, R, R_0 such, that $0 < R_0 < R$, $\tau \in (0,T]$, we can obtain the following evaluation:

$$\int_0^{R_0} \left(\frac{\partial u^1(x,\tau)}{\partial t} - \frac{\partial u^2(x,\tau)}{\partial t} \right)^2 dx +$$
$$+C_1 \int_0^{R_0} \left(\frac{\partial u^1(x,\tau)}{\partial x} - \frac{\partial u^2(x,\tau)}{\partial x} \right)^2 dx +$$
$$+C_2 \int_{Q_{R_0,\tau}} \left| \frac{\partial u^1}{\partial t} - \frac{\partial u^2}{\partial t} \right|^p dxdt \le \left(\frac{R}{R-R_0} \right)^\beta \times$$
$$\times \left\{ C_3 R^{1+(\alpha-1)\frac{2p}{p-2}} + C_4 \int_{Q_{R,\tau}} \left| f - \overline{f} \right|^q dxdt \right\}, \tag{7}$$

where: $\beta > \frac{2p}{p-2}$ - arbitrary number; C_1, C_2, C_3, C_4 - positive constants that depend only from p, β.

Let's explain the inequality (7). Let $R > R0 > 0, \tau \in (0,T]$ - arbitrary numbers. We define the function φ as follows:

$$\begin{cases} \frac{R^2 - x^2}{R}, & x \le R, \\ 0, & x > R. \end{cases}$$

Directly convince ourselves that the evaluation of function φ holds:

Let u^1, u^2 - generalized solutions of problem (1) - (4) and problem (1) - (4), where in the right side of equation (1) function f is replaced by $\overline{f} \in L^q_{loc}(\overline{Q})$. We set further $w = u^1 - u^2$ and follow the procedure of regularization described in [14 pp. 238 - 239]. Based on some calculations and transformations similar to those made in [14], after limit passage when $l,k \to \infty$ one can get:

$$\frac{1}{2}\int_0^R \left[\left(\frac{\partial w(x,\tau)}{\partial t} \right)^2 + a(x) \left(\frac{\partial w(x,\tau)}{\partial x} \right)^2 \right] \varphi^\beta dx +$$
$$+ \int_{Q_{R,\tau}} a(x) \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} \frac{\partial \phi^\beta}{\partial x} dxdt +$$
$$+ \int_{Q_{R,\tau}} \left(g\left(x, \frac{\partial u^1(x,t)}{\partial t} \right) - g\left(x, \frac{\partial u^2(x,t)}{\partial t} \right) \right) \times$$
$$\times \left(\frac{\partial u^1(x,t)}{\partial t} - \frac{\partial u^2(x,t)}{\partial t} \right) \varphi^\beta dxdt =$$
$$= \int_{Q_{R,\tau}} (f - \overline{f}) \frac{\partial w}{\partial t} \varphi^\beta dxdt. \tag{8}$$

Let's estimate the integrals of equation (8) due to:

$$\frac{1}{2}\int_0^R a(x) \left(\frac{\partial w(x,\tau)}{\partial x} \right)^2 dx \ge \frac{a_0}{2} \int_0^R \left(\frac{\partial w(x,\tau)}{\partial x} \right)^2 dx,$$

$$\int_{Q_{R,\tau}} a(x) \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} \frac{\partial \varphi^\beta}{\partial x} dxdt \le M \int_{Q_{R,\tau}} \sum_{i,j=1}^n \frac{\partial w}{\partial x} \frac{\partial w}{\partial t} \frac{\varphi^{\frac{\beta}{2}} \varphi^{\frac{\beta}{p}}}{\varphi^{\frac{\beta}{2}+\frac{\beta}{p}}} \times$$
$$\times \frac{\partial \varphi^\beta}{\partial x} R^\alpha dxdt \le M \left(\int_{Q_{R,\tau}} \left(\frac{\partial w}{\partial x} \right)^2 \varphi^\beta dxdt \right)^{\frac{1}{2}} \times$$
$$\times \left(\int_{Q_{R,\tau}} \left| \frac{\partial w}{\partial t} \right|^p \varphi^\beta dxdt \right)^{\frac{1}{p}} \times \left(\int_{Q_{R,\tau}} \left| \frac{\frac{\partial \varphi^\beta}{\partial x} R^\alpha}{\varphi^{\frac{\beta}{2}+\frac{\beta}{p}}} \right|^{p_1} dxdt \right)^{\frac{1}{p_1}} \le$$
$$\le C_1 \delta_1 \int_{Q_{R,\tau}} \left(\frac{\partial w}{\partial x} \right)^2 \varphi^\beta dxdt + \delta_2 \int_{Q_{R,\tau}} \left| \frac{\partial w}{\partial t} \right|^p \varphi^\beta dxdt +$$
$$+C_2 \int_{Q_{R,\tau}} \frac{\beta \varphi^{(\beta-1)p_1} R^{\alpha p_1}}{\varphi^{\beta(p_1-1)}} dxdt \le C_1 \delta_1 T \int_0^R \left(\frac{\partial w}{\partial x} \right)^2 \varphi^\beta dx +$$
$$+\delta_2 \int_{Q_{R,\tau}} \left| \frac{\partial w}{\partial t} \right|^p \varphi^\beta dxdt + C_3 \int_{Q_{R,\tau}} \varphi^{\beta - p_1} R^{\alpha p_1} dxdt \le$$

$$\leq C_1\delta_1 T\int_0^R\left(\frac{\partial w}{\partial x}\right)^2\varphi^\beta dx+\delta_2\times\int_{Q_{R,\tau}}\left|\frac{\partial w}{\partial t}\right|^p\varphi^\beta dxdt+C_4R^{\beta-\frac{2p}{p-2}+\frac{2p\alpha}{p-2}+1},$$

where: δ_1, δ_2 - arbitrary sufficiently small positive constants, C_1, C_2, C_3, C_4 - some positive constants that depend on the p, β. Note, that in the last evaluation we used Young inequality [8] and the properties of the φ function. Lets estimate the following integrals from equation (8):

$$\int_{Q_{R,\tau}}\left(g\left(x,\frac{\partial u^1(x,t)}{\partial t}\right)-g\left(x,\frac{\partial u^2(x,t)}{\partial t}\right)\right)\times$$
$$\times\left(\frac{\partial u^1(x,t)}{\partial t}-\frac{\partial u^2(x,t)}{\partial t}\right)\varphi^\beta dxdt\geq$$
$$\geq g_0\int_{Q_{R,\tau}}\left|\frac{\partial w}{\partial t}\right|^p\varphi^\beta dxdt,$$

$$\int_{Q_{R,\tau}}(f-\overline{f})\frac{\partial w}{\partial t}\varphi^\beta dxdt\leq C_\delta\times$$
$$\times\int_{Q_{R,\tau}}\left|f-\overline{f}\right|^q\varphi^\beta dxdt+\delta\int_{Q_{R,\tau}}\left|\frac{\partial w}{\partial t}\right|^p\varphi^\beta dxdt,$$

where: constant $C_\delta > 0$, and constant $\delta > 0$ can be made an arbitrarily small.

Taking into account the above estimates and using them, we obtain:

$$(R-R_0)^\beta\left\{\int_0^{R_0}\left(\frac{\partial w(x,\tau)}{\partial t}\right)^2\varphi^\beta dx+\right.$$
$$\left.C_5\int_0^{R_0}\left(\frac{\partial w(x,\tau)}{\partial t}\right)^2\varphi^\beta dx+C_6\int_{Q_{R_0,\tau}}\left|\frac{\partial w}{\partial t}\right|^p dxdt\right\}\leq$$
$$\leq C_7R^{\beta+1+(\alpha-1)\frac{2p}{p-2}}+C_8R^\beta\int_{Q_{R,\tau}}\left|f-\overline{f}\right|^q dxdt,$$

$C_5 - C_8$ - positive constants. From the last inequality easy to receive inequality (7).

Consider the next sequence of domains $Q^k = (0,k) \times \times (0,T)$, $k = 1,2,\ldots$ and respectively in each domain Q^k the problem:

$$\frac{\partial^2u^k(x,t)}{\partial t^2}-\frac{\partial}{\partial x}\left(a(x)\frac{\partial u^k(x,t)}{\partial x}\right)+$$
$$+g\left(x,\frac{\partial u^k(x,t)}{\partial t}\right)=f^k(x,t), \tag{9}$$

$$u^k(x,0)=u_0^k(x), \tag{10}$$

$$\frac{\partial u^k(x,0)}{\partial t}=u_1^k(x), \tag{11}$$

$$u^k(0,t)=u^k(k,t)=0. \tag{12}$$

Note that in equation (9) functions $f^k(x,t)= =\begin{cases}f(x,t), x\leq k,\\ 0, x>k.\end{cases}$. Also, instead of functions u_0 functions u_0^k are discussed, where $u_0^k(x)=u_0(x)\cdot\xi_k(x)$, $\xi^k\in C^1(\mathrm{R})$,

$$\xi^k(x)==\begin{cases}1, x\leq k-1,\\ 0, x>k,\end{cases}\quad 0\leq\xi^k(x)\leq 1.$$

It is clear that functions $u_0^k\in H_0^1(0,k)$ and $\lim\limits_{k\to+\infty}\left\|u_0^k-u_0\right\|_{H_0^1(0,k)}=0$. Instead of the initial function u_1 consider the function u_1^k - narrowing of function u_1 on $(0,k)$, $u_1^k\in L^2(0,k)$, $\lim\limits_{k\to+\infty}\left\|u_1^k-u_1\right\|_{L^2(0,k)}=0$.

Under the generalized solution of problem (9) - (12) we mean function u^k, which satisfies (9) (10), (12) and the integral identity similar to the identity (5), which is treated in Q^k, where function v is chosen so that:

$$v\in L^2((0,T);H_0^1(0,k))\cap L^p(Q^k),$$
$$\frac{\partial v}{\partial t}\in L^2(Q^k).$$

Note that under the conditions of the theorem there exists a unique generalized solution of the problem (9) - (12) in Q^k [14, p. 234].

Consider now the sequence of problems of the form (9) - (12) for $k = 1$, $k = 2,\ldots$, $u^k = 0$, when $(x,t) \in Q\backslash Q$. We will obtain a sequence of solutions of problem (1) - (4) in Q, which for convenience we denote again $\{u^k\}$. Lets show that sequence $\{u^k\}$ is fundamental in space:

$$C([0,T];H_{0,loc}^1(0,+\infty),$$
and $\left\{\frac{\partial u^k}{\partial t}\right\}$ - fundamental in space:
$$C([0,T];L_{loc}^2(0,+\infty))\cap L_{loc}^p(\overline{Q}).$$

Consider the difference $u^l - u^m$, $l,m \in N$, $m > l$ in domain $Q_{R,\tau}(1 > R > R_0)$ and we will use the inequality (7), taking into account that $f^l - f^m \equiv 0$ in $Q_{R,\tau}$. Similarly to (8) we obtain:

$$\int_0^{R_0}\frac{\partial}{\partial t}\left(u^l(x,\tau)-u^m(x,\tau)\right)^2dx+$$
$$+C_1\int_0^{R_0}\frac{\partial}{\partial x}\left(u^l(x,\tau)-u^m(x,\tau)\right)^2dx+$$
$$+C_2\int_{Q_{R_0,\tau}}\left|\frac{\partial}{\partial t}\left(u^l-u^m\right)\right|^p dxdt\leq$$
$$\leq\frac{1}{\left(R-R_0\right)^\beta}C_3R^{\beta+1-\frac{2p}{p-2}}+$$
$$+C_4\left\|u_0^l-u_0^m\right\|_{H_0^1(0,R_0)}+$$
$$+C_5\left\|u_1^l-u_1^m\right\|_{L^2(0,R_0)}. \tag{13}$$

From inequality (13) by proper a choice of sufficiently large $R > 0$:

$$\int_0^{R_0} \frac{\partial}{\partial t}\left(u^l(x,\tau)-u^m(x,\tau)\right)^2 dx +$$
$$+C_1\int_0^{R_0} \frac{\partial}{\partial x}\left(u^l(x,\tau)-u^m(x,\tau)\right)^2 dx +$$
$$+C_2 \int_{Q_{R_0,\tau}} \left|\frac{\partial}{\partial t}\left(u^l-u^m\right)\right|^p dxdt \le \varepsilon,$$

for any arbitrarily small $\varepsilon > 0$. Thus, $\{u^k\}$ is fundamental sequence in space $C([0,T];H^1_{0,loc}(\Omega))$, namely:

$u^k \to u$ strongly in $C([0,T];H^1_{0,loc}(0,+\infty))$,

and sequence $\left\{\frac{\partial u^k}{\partial t}\right\}$ is fundamental in space $C([0,T];L^2_{loc}(0,+\infty))\cap L^p_{loc}(\overline{Q})$, namely:

$\frac{\partial u^k}{\partial t} \to \frac{\partial u}{\partial t}$ strongly in space: $C([0,T];L^2_{loc}(0,+\infty))\cap$ $\cap L^p_{loc}(\overline{Q})$.

It is obvious that for function u, conditions (2) - (4) are satisfied. Thus, u is a generalized solution of problem (1) - (4) in the sense of the integral identity (5), for which inclusion performed (6) performed.

Uniqueness of the obtained solution follows from inequality (13) with $R \to +\infty$, if we consider two arbitrary solutions u^1 and u^2 of problem (1) - (4) and considering that:

$$u^1(x,0) = u^2(x,0),$$
$$\frac{\partial u^1(x,0)}{\partial t} = \frac{\partial u^2(x,0)}{\partial t}.$$

Note that for problem (1) - (4) it is easy to obtain sufficient conditions for the existence and uniqueness of periodic for the spatial variable generalized solution to problem (1) - (4).

Let conditions **(I), (II), (III), (IV)** are satisfied, $\alpha = 0$ and exist such number $\zeta > 0$, that:

a) $a(x + \zeta) = a(x)$ for all $x \in (0,l)$;
b) $f(x + \zeta,t) = f(x,t)$ for nearly all $(x,t) \in Q$;
c) $u_0(x + \zeta) = u_0(x)$, $u_1(x + \zeta) = u_1(x)$ for nearly all $x \in (0,+\infty)$.

Then problem (1) - (4) has a unique generalized solution u, which is periodic function by variable x with period ζ.

Really, since there is unique generalized solution of the problem (1) - (4) and function $u(x + \zeta,t)$, $(x,t) \in Q$ also is a generalized solution of problem (1) - (4) (it is easily verified), then from the uniqueness of the generalized solution immediately follows that $u(x + \zeta,t) = u(x,t)$ for nearly all $(x,t) \in Q$.

CONCLUSIONS

1. Firstly obtained correctness conditions for the solution in mathematical model for fluctuations of semiunbounded rope under the influence of nonlinear resistance force - sufficient conditions for the existence and uniqueness of solution in the class of locally integrable functions.
2. The above technique allows substantiate the correctness of the model also in the more complex case - the vibrations under the action of combined effects of nonlinear elastic foundation and resistance forces. This mathematical model is reduced to a qualitative analysis of the mixed problem in a bounded (in the case of a finite cable) or unbounded (as discussed in this article) domain for the equation:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(a(x)\frac{\partial u}{\partial x}\right) + g\left(x,u,\frac{\partial u}{\partial t}\right) = f(x,t).$$

3. These qualitative results justify, particularly the possibility of applying for such problem Galerkin numerical method and provide an opportunity to apply various approximate methods in further investigations of the dynamic characteristics of the considered oscillation mathematical models solutions.

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