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IMPROVEMENT OF THE ALGORITHM FOR SETTING THE CHARACTERISTICS OF INTERPOLATION MONOTONE CURVE

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Abstract. *Interpolation of a point series is a necessary step in solving such problems as building graphs de-scribing phenomena or processes, as well as modelling based on a set of reference points of the line frames defining the surface. To obtain an adequate model, the following conditions are imposed upon the interpolating curve: a minimum number of singular points (kinking points, inflection points or points of extreme curvature) and a regular curvature change along the curve. The aim of the work is to develop the algorithm for assigning characteristics (position of normals and curvature value) to the interpolating curve at reference points, at which the curve complies with the specified conditions. The characteristics of the curve are assigned within the area of their possible location. The possibilities of the proposed algorithm are investigated by interpolating the point series assigned to the branches of the parabola. In solving the test example, deviations of the normals and curvature radii from the corresponding characteristics of the original curve have been determined. The values obtained confirm the correctness of the solutions proposed in the paper.*

Keywords: interpolation, monotone curve, singular points, normal, centre of curvature, evolute, curvature radius

ULEPSZENIE ALGORYTMU WYZNACZANIE CHARAKTERYSTYKI INTERPOLACYJNEJ KRZYWEJ MONOTONICZNEJ

Streszczenie. *Interpolacja szeregu punktowego jest niezbędnym krokiem w rozwiązywaniu takich problemów, jak budowanie grafów opisujących zjawiska lub procesy, a także modelowanie w oparciu o zbiór punktów odniesienia układów liniowych definiujących powierzchnię. Aby uzyskać odpowiedni model, na interpolowaną krzywą stawia się następujące warunki: minimalną liczbę punktów osobliwych (punktów załamania, punktów przegięcia lub punktów skrajnej krzywizny) oraz regularną zmianę krzywizny wzdłuż krzywej. Celem pracy jest opracowanie algorytmu przypisania charakterystyk (położenia normalnych i wartości krzywizny) krzywej interpolacyjnej w punktach odniesienia, w których krzywa spełnia określone warunki. Charakterystyki krzywych nadawane są w obszarze ich możliwego położenia. Możliwości proponowanego algorytmu są badane poprzez interpolację szeregów punktów przypisanych do gałęzi paraboli. W rozwiązaniu przykładu testowego wyznaczono odchylenia normalnych i promieni krzywizny od odpowiednich charakterystyk pierwotnej krzywej. Otrzymane wartości potwierdzają poprawność zaproponowanych w pracy rozwiązań.*

Słowa kluczowe: interpolacja, krzywa monotoniczna, punkty osobliwe, normalna, środek krzywizny, ewolucja, promień krzywizny

Introduction

Interpolation of a series of fixed points is a necessary step in solving many geometric modelling problems. Such problems include creating graphs describing phenomena or processes, as well as modelling based on a set of reference points of the linear frameworks defining the surface.

The input data for process research are always discrete and can be presented on the graph by a sequence of points. Interpolation of reference points makes it possible to estimate the characteristics of the process at any point through the coordinates of the points of the interpolating curve. For this estimate to be correct, the interpolating curve configuration must comply with the layout of the sequence of reference points.

Such compliance is impossible without controlling the emergence of singular points along the interpolating curve. These can be:

- kinking points, where the curve has two tangent lines;
- inflection points, at which the convex and concave parts of the curve meet;
- extreme curvature points, where the direction of the increase in curvature along the curve changes;
- points at which the regularity of curvature changes along the curve is disturbed.

For a phenomenon or process to be adequately represented, the interpolation method must provide a number of singular points along the curve, minimum possible by task conditions. This means that the curve contains singular points in areas where, based on the configuration of the sequence of reference points, their presence is imperative. At the same time, there should be no singular points on sections that can be interpolated by a monotone curve along which curvature values change monotonously and regularly.

In case of uncontrolled emergence of singular points, the interpolating curve may deviate from the reference points to an uncontrolled distance. Therefore, the corresponding graph sections will not reflect the characteristics of the original phenomenon or process accurately.

Modelling complex surfaces is based on forming linear frameworks [5, 14, 25, 26]. In many cases, framework lines can only be obtained by interpolating a sequence of points. The number of reference points may be significant, and the sequence of points may have a complex setup. This happens when the input data for the surface model is an array of points whose coordinates are obtained by measuring an existing prototype (reverse engineering) [16, 24, 27, 28, 32]. Another example is grid-based modelling, where two families of lines intersect to define the array of points that must be interpolated.

Complex surfaces tend to bound products whose function is to interact with the environment. These are surfaces with elevated aero- or hydrodynamic properties. The laminar flow of such surfaces is ensured by the characteristics of the curves forming the surface framework [2, 6, 9, 10, 15, 18, 23]. The minimum by task conditions number of singular points of the curves forming the linear framework is the basic condition providing elevated dynamic properties of the surface.

Imposing additional conditions on the interpolating curve requires increasing the degree of its equation. Such conditions may include the number of points through which the curve passes, location of tangents, and curvature values of the curve at specific points. The more conditions are imposed on the curve, the higher the possibility that it will have singular points, and the harder it is to control their presence and location.

If the interpolating curve is formed as a polynomial [1], the degree of its equation is less by one than the number of reference points. If the polynomial degree is higher than 3, it is impossible to control the emergence of inflection points. The emergence of inflection points can be prevented by interpolating second-order curves [3, 8, 19, 22, 30] or a B-spline [4, 17, 20, 21]. However, these methods are insufficient in terms of controlling the emergence of extreme curvature points.

The contradiction between imposing additional conditions on the interpolating curve and the control of the emergence of singular points on its sections is objective. The analytical representation of the curve lies at the basis of this contradiction. The equation defining the curve specifies its configuration, smoothness, and the pattern of changes in curvature values along the curve. If these patterns are out of compliance with the configuration of the reference points sequence and the characteristics assigned to them, then the emergence of singular points is inevitable. At present, this contradiction does not have a system solution.

Papers [11–13] propose a solution to the problem of controlling the emergence of singular points on the interpolating curve based on avoiding the analytical representation of its sections.

The author of [13] proposes a solution to the problem of forming a curve in the form of the area of possible location of its monotone parts. The area of possible location of the curve is formed as a sequence of closed contours, joined together at reference points. The dimensions of each of the contours are determined by the distance between the respective reference points, the absence of singular points on the section of the curve and the characteristics of the curve assigned at the reference points.

The area of possible location of section $i \ldots i+1$ of the curve with no inflection points is defined by triangles i; N; $i+1$ (Fig. 1). The triangles are bounded by lines (i-1;i), (i;i+1) and (i+1;i+2) passing through three pairs of consecutive reference points.

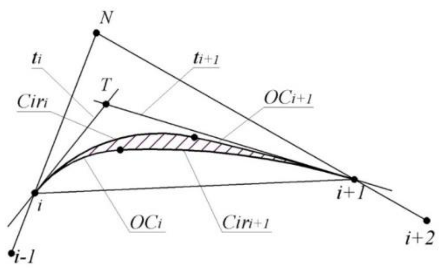

Fig. 1. Area of possible location of the interpolating curve

If the positions of the tangents are assigned to the reference points, the possible location of the interpolation curve section is triangle i; T; i+1, bounded by segment [i, i+1] and tangents t_i and t_i+1.

The area of possible location of the section of the curve with no extreme curvature points is bounded by arcs of circles. The boundaries of the area consist of arcs of circles osculating to the curve at reference points (OC_i and OC_{i+1}) and arcs of circles tangent to the osculating circle and a tangent at a different reference point (Cir_{i+1} and Cir_i) (Fig. 1).

Research has established that the value of the area of possible location of the curve, determined based on the assumption that there are no extreme curvature points, is 2-3 times less than the area determined by the convexity condition of the curve.

The analytical description of the boundaries of the area of possible location of the monotone curve was obtained in [11]. The article also offers a solution for the task of providing a given interpolation accuracy. The assignment of intermediate points within the reference sections corresponding to the tangent line and osculating circles leads to an increase in the number of sections and a decrease in their dimensions. The location area of the curve is considered to be formed when the dimensions of the maximum of its sections do not exceed the given value. In this case, the graph describing the phenomenon or process can be represented as an area of possible location of the curve.

While modelling a surface, lines contours are formed within the areas of the location of its framework, representing the curves with assigned characteristics with specified accuracy. The solution of the problem within the area of possible location of the monotone curve is proposed in [11]. The contour is formed by smoothly joined arcs of circles. Location of the contour within the specified area is provided by:

– the given direction of monotone increase of the radii of circles along the contour;
– the contour's contingency with the interpolating curve at reference points.

The disadvantage of solutions proposed in [11] is a complex algorithm for deter-mining the boundaries of the curve's location and a large number of necessary calculations.

The size of the areas of possible location of the interpolating curve is determined by the positions of its tangents and the curvature values assigned by the designer at reference points. The option of assigning these characteristics proposed in [13] ensures that there is such an area at each of the sections. The task of controlling the width ratio of the areas of different sections was not considered in the work. As a consequence, an area is likely to be formed where narrow sections alternate with areas of significant width. The curve formed within such an area will consist of areas, along which curvature changes rapidly, as well as areas where it changes insignificantly. Such a solution would reduce both the interpolation accuracy and the dynamic properties of the modelled surfaces.

The possibility to level the width of the adjacent sections of the area of the curve by correcting the characteristics of the interpolating curve at the point separating the sections was investigated in [11, 12]. The possibility to improve the obtained solution as a result of successive iterations was established.

Implementing the iterative approach in computer software implies the existence of consecutive cycles providing a step-by-step approximation of the curve's characteristics to the required values. The sequence of interpolated points can be in the thousands. Consequently, the iteration process can take a considerable amount of time, making it difficult to form the area of the curve's location in interactive mode.

The need for software implementation for the method under development re-quires increasing its effectiveness through:

1. developing a method for assigning the correct characteristics of the interpolating curve at reference points;
2. developing a simpler algorithm for determining the area of possible location of the monotone curve.

The article is aimed at the development and approbation of the algorithm for as-signing the positions of curvature centres corresponding to reference points, at which it is possible to provide regular and uniform change in curvature values along the interpolating curve. To achieve this aim, it is necessary to:

– develop a method of assigning the location of the normals based on the assumption that the curve has no singular points;
– develop a method for assigning the positions of the curvature centres on already assigned normals, which would provide a uniform change in curvature values along the curve;
– test the proposed algorithm when assigning curvature centres to interpolating curves at reference points assigned to a monotone curve.

The development of these methods requires solving the following tasks:

– determining the area of possible location of the normal of the interpolating monotone curve at reference points;
– determining the optimal location of each normal within the corresponding area, based on the conditions of the task;
– determining the areas of possible location of curvature centres and assigning their final positions.

1. Materials and methods

Let us consider how a sequence of curvature centres (C_i) of the monotone curve interpolating a given point series is formed, by the example of the curve with increasing radii of curvature.

The osculating circle corresponding to point i (OC_i) divides the monotone curve into two parts (Fig. 2).

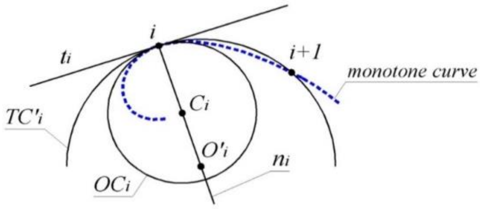

Fig. 2. Position of osculating and tangent circles with respect to the monotone curve

The part of the curve at which the radius of curvature is smaller than the radius of OC_i (R_i) is located inside it. The rest of the curve is outside OC_i. Point i, tangent to the curve at this point (t_i) and point $i+1$, located on the curve outside OC_i define the tangent circle TC'_i. The radius of TC'_i is greater than R_i. Point $i\text{-}1$ belonging to the curve and located inside OC_i defines $'TC_i$, whose radius is smaller than R_i. Points $i\text{-}1$, i, $i+1$ define the adjacent circle (AC_i) (Fig. 3). The radius of AC_i can be both larger and smaller than R_i.

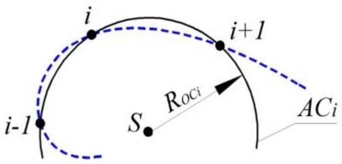

Fig. 3. Position of OC_i with respect to the monotone curve

The points assigned to the monotone curve define a sequence of adjacent, tangent, and osculating circles whose radii increase monotonously along the point series:

$$\ldots<R\,AC_{i-1}<R\,TC'_{i-1}<R\,'TC_i<R\,AC_i<R\,TC'_i<R\,'TC_{i+1}<R\,AC_{i+1}<\ldots \quad (1)$$

$$\ldots<R_{i-1}<R\,TC'_{i-1}<R\,'TC_i<R_i<R\,TC'_i<R\,'TC_{+1}<R_{i+1}<\ldots \quad (2)$$

The position of the curvature centres of the interpolating monotone curve corresponding to the reference points shall be determined and assigned based on (1), (2).

The centres of AC … S_{i-1}, S_i, S_{i+1}, … are located at the intersection of the lines drawn through bisecting points of the segments (point K_i) perpendicularly to the segments connecting the adjacent reference points (Fig. 4).

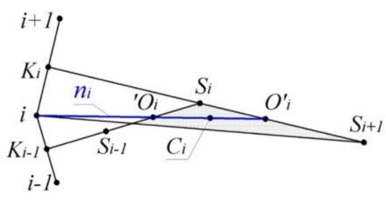

Fig. 4. Original area of the i-th centre of curvature

According to (1), the normal of the monotone curve at point i (n_i) must intersect sections [S_{i-1}, S_i] and [S_i, S_{i+1}] at points $'O_i$ and O'_i – centres of $'TC_i$ и TC'_i respectively. According to (2), points $'O_i$ and O'_i bound the range of the centre of curvature C_i on normal n_i. The original area of location of C_i is a triangle bounded by lines (K_{i-1}, S_i), (K_i, S_{i+1}) and that of lines (i, S_{i+1}) or (i, S_{i-1}), which is at a smaller angle to (K_{i-1}, S_i).

The reference criterion for the mutual arrangement of the normals assigned to adjacent points i and $i+1$ is the ratio of the lengths of the segments (Fig. 5):

$$|S_i, O'_i| \le |C_i, 'O_{i+1}|. \quad (3)$$

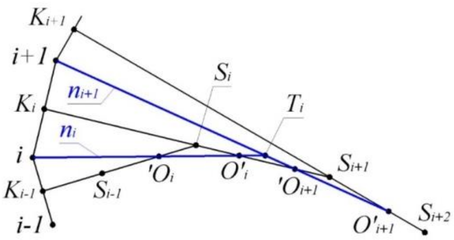

Fig. 5. Mutual arrangement of normals assigned to adjacent points

The equality of the specified segments means that point T_i belongs to line (K_i, S_i). In this case, section (i, $i+1$) of the monotone curve is the arc of the circle with centre T_i.

When assigning the position of normals at reference points, we will strive for a uniform arrangement of the centres of TC within the segment bounded by the centres of the corresponding AC.

Let us look at the variant of assigning such positions to the normals.

1. Assign a preliminary position of n_i, at which point $'O_i \equiv n_i \times (K_{i-1}, S_i)$ separates segment [S_{i-1}, S_i] at a ratio of $|'O_i, S_i| : |S_{i-1}, S_i| = 1:3$.
2. Determine point O'_i, at which n_i intersects line (K_i, S_i). If the value of the ratio of the lengths of segments $|S_i, O'_i|:|S'_i, S_{i+1}|$ does not exceed 1:2, then the position of n_i is considered to be finalized. Otherwise, we assign the final position of n_i, at which point O'_i divides segment [S_i, S_{i+1}] at a ratio of $|O'_i, S_i| : |S_i, S_{i+1}| = 1:3$.
3. The position of normal n_{i+1} is selected from two options:
   - point $'O_{i+1} \equiv n_{i+1} \times (K_i, S_{i+1})$ divides segment [O'_i, S_{i+1}] into equal parts;
   - point $O'_{i+1} \equiv n_{i+1} \times (K_{i+1}, S_{i+1})$ divides segment [S_{i+1}, S_{i+2}] at a ratio of $|O'_{i+1}, S_{i+1}| : |S_{i+1}, S_{i+2}| = 1:3$.

The final position of n_{i+1} is that, at which the normal is closer to point S_{i+1}.

Having assigned the positions of normals at all reference points according to the given scheme, we obtain a sequence of centres of TC whose radii comply with condition (1). The criterion for the final selection of the positions of normal is the ratio of the lengths of the parts into which the normals divide the segments connecting the centres of the corresponding AC. The position of normals in which the specified segments are divided into three equal parts shall be considered optimal.

The position of the centres of curvature is determined on corresponding normals based on the properties of the evolute of the monotone curve [7, 29]:

- evolute is a convex curve with no inflection points or spinodes;
- the normals of the curve are tangent to its evolute in the respective centres of curvature;
- the length of any section of the evolute is equal to the difference in values of the radii of curvature at points bounding the corresponding section of the original curve.

The position of the centres of curvature C_i is assigned within the respective segments [$'O_i$, O'_i]. Consider the option of assigning these positions.

1. For each section of the interpolating curve, the minimum length of the evolute is determined. For section ($i…i+1$) such an evolute is segment [C'_i, $'C_{i+1}$], the length and location of which are determined by the conditions:

$$\begin{cases} |C'_i, 'C_{i+1}| = \Delta r_i; \\ |C'_i, T_i| = |'C_{i+1}, T_i|, \end{cases} \quad (4)$$

where $\Delta r_i = |i+1, 'C_{i+1}| - |i, +1, C'_i|$ – the difference in the radii values of the circles whose arcs make up the section of contour (i, $i+1$).

Segment [C'_i, $'C_{i+1}$] can be defined by finding its position in relation to triangle $A, 'O_{i+1}, T_i$ (Fig. 6).

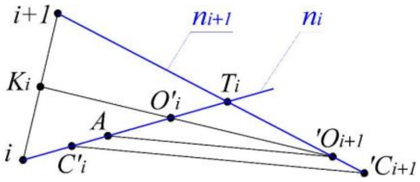

Fig. 6. Finding the minimum evolute of a section of the interpolating curve

Point A is assigned on line n_i based on the equality of segments $|A,T_i|=|{}'O_{i+1},T_i|$. The position of segment $[C'_i, {}'C_{i+1}]$ in relation to triangle $A, {}'O_{i+1}, T_i$ determines the coefficient:

$$f = \frac{|C'_i, T_i|}{|A, T_i|} = \frac{|{}'C_{i+1}, T_i|}{|{}'O_{i+1}, T_i|} = \frac{|C'_i, {}'C_{i+1}|}{|A, {}'O_{i+1}|}, \tag{5}$$

whose value is calculated by the formula:

$$f = \frac{\Delta}{2|T_i, {}'O_{i+1}| - |A, {}'O_{i+1}|} + 1, \tag{6}$$

where $\Delta = |i, A| + |A, {}'O_{i+1}| - |i+1, {}'O_{i+1}|$ is equal to the difference in lengths of evolute $[C'_i, {}'C_{i+1}]$ and segment $[A, {}'O_{i+1}]$.

2. After determining the minimum evolutes for each section of the curve, a sequence of segments …, $[C'_{i-1}, {}'C_i]$, $[C'_i, {}'C_{i+1}]$, … is obtained (Fig. 7) determining the involute as smoothly joined arcs of circles. This involute interpolates the reference point series by locating within the area of possible location of the monotone curve and is similar to the contour formed by method proposed in [11].

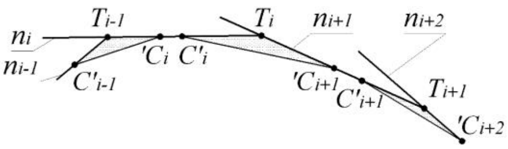

Fig. 7. Sequence of minimum evolutes

3. The curvature centres of the interpolating curve C_i, corresponding to the reference points, are assigned within the boundaries of segments $[{}'C_i, C'_i]$. Assigning the i-th centre of curvature within the range of segment $[i, C'_i]$, and the $i+1$-th centre of curvature outside line $[i+1, {}'C_{i+1}]$ enables forming the section of the evolute as a smooth convex line (Fig. 8), which defines the involute as a monotone curve with a regular curvature change.

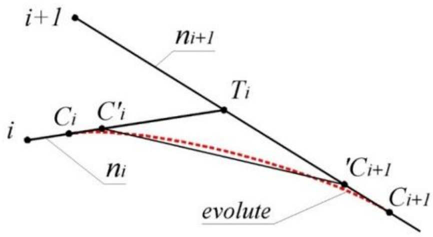

Fig. 8. Evolute of the section of the monotone curve

The criterion for the final assignment of curvature centres within the reference ranges $[{}'O_i, O'_i]$ is the ratio of the lengths of the parts into which these centres divide the corresponding segments $[{}'C_i, C'_i]$. The position of the centres of curvature in the middle of these segments shall be considered optimal.

The normals of the interpolating curve assigned at reference points and the chords connecting the curvature centres assigned on these normals bound the sequence of triangles (Fig. 9).

The sides of each triangle correspond to:

$$|C_i, C_{i+1}| \le R_{i+1} - R_i \le |C_i, T_i| + |C_{i+1}, T_i|, \tag{7}$$

and their sequence is the area of possible location of the evolute of the monotone curve that interpolates the set reference points.

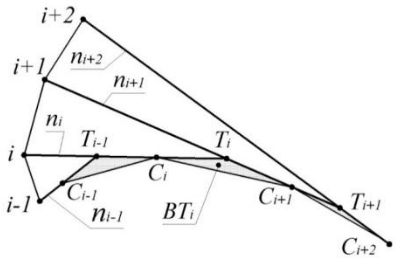

Fig. 9. Area of location of the evolute of the discretely presented curve

The solutions presented above allow assigning the characteristics of the interpolating curve at reference points according to the following algorithm:

1. The original point series is divided into parts that can be interpolated by a mono-tone curve.
2. For the reference points defining the monotone sections of the interpolating curve, the areas of location of normal are determined, at which it is possible to ensure the absence of singular points for these sections.
3. The position of each of the normals is determined based on the greatest possible approximation of the criterion determining its location within the area to the value accepted as optimal.
4. The ranges of possible location of the centres of curvature are determined on the assigned normals, based on the task of ensuring a uniform change of curvature values along the curve.
5. The centres of curvature are assigned within the respective ranges, based on the given ratio of lengths of the segments into which the range is divided by the centre of curvature.

2. Results and discussion

Consider solving the problem of assigning curvature centres to an interpolating curve based on forming the area of possible location of the evolute of the monotone curve, on the example of a sequence of reference points assigned to the branch of the parabola defined by the equation $y = \frac{x^2}{300}$. The characteristics of the reference point series are given in table 1.

Table 1. Characteristics of the reference point series

i	Point coordinates, mm		Chord length $|i, i+1|$, mm	Radius of AC RAC_i, mm	Range of location of the normal Δn_i, °
	x	y			
1	30	3	31.32	-	1.046
2	60	12	33.54	188.58	1.074
3	90	27	36.62	238.73	1.074
4	120	48	84.85	348.37	1.386
5	180	108	103.23	570.09	2.727
6	240	192	123.55	1002.64	2.244
7	300	300	-	-	-

The increasing values of the radii of AC along the sequence of reference points determine the direction in which the radii of curvature increase along the monotone curve, which can interpolate these points.

The values given in the sixth column of the table of the ranges of the normals of the interpolation curve are determined by the smaller of the angles between line (i, S_i) and one of the lines (i, S_{i-1}) or (i, S_{i+1}) (Fig. 4). For example, for point 4, range Δn_4 is equal to angle $S_4, 4, S_5$. For point 1, the range is defined by angle $B, 1, S_2$, where B is the intersection point of $(2, S_3)$ and (K_1, S_2). For the last point, the normal range is not determined because the AC_7 does not exist, and point S_7 does not bound the turn of normal n_7 towards increasing angle $6, 7, {}'O_7$. The equality of ranges Δn_2 and Δn_3 means that their value is determined by the condition of intersection by normals n_2 and n_3 of segment $[S_2, S_3]$.

The position assigned to each of the normals determines the ratio of the lengths of the segments:

$$K_i = \frac{|S_i;O'_i|}{|S_i;S_{i+1}|}, \tag{8}$$

where O'_i – the intersection point of normal n_i with line (K_i,S_i) (Fig. 5).

The coefficient values of K_i for the normals of the interpolating curve are given in Tab. 2.

Table 2. Characteristics of the reference point series

i	1	2	3	4	5	6	7
K_i	-	0.333	0.292	0.283	0.301	0.276	-

The position of normal n_2, which corresponds to the smallest initial range of possible location, is assigned first. The position of normals $n_3 \dots n_6$ is defined according to the above method. The position of the normal n_1 at the first reference point is determined by the ratio of the lengths of segments $|O'_1, {}'O_2| = |{}'O_2, S_2|$. The position of normal n_7 at the last, seventh point is determined by the ratio of the lengths of segments $|S_6, O'_6| = |O'_6, {}'O_7|$.

Similar coefficients values of K_i reflect the correct assignment of the position of normals, which enables a monotonous and uniform change in the curvature values along the formed curve.

Once the normals have been assigned at reference points, the dimensions and position of the minimum evolutes are determined for each section of the interpolating curve, and then the curvature centres corresponding to the reference points are assigned. The results are shown in Tab. 3.

The proposed method makes it possible to determine the boundaries of the ranges of location of the curvature centre for all the reference points except the first and last ones. For the first point, the upper boundary of $({}'C_1)$, the extreme position of which is reference point 1 itself, is not determined. For the last point, the lower boundary of (C'_7), which can be at an arbitrary large distance from reference point 7, is not determined. We shall define the radius of the curvature centre based on the ratio of the lengths of similar ranges to the adjacent sections, as follows:

$$|{}'C_1,C'_1| = |{}'C_2,C'_2| \frac{|{}'C_2,C'_2|}{|{}'C_3,C'_3|}, \tag{9}$$

$$|{}'C_7,C'_7| = |{}'C_6,C'_6| \frac{|{}'C_6,C'_6|}{|{}'C_5,C'_5|}. \tag{10}$$

The parabola, on which a sequence of reference points was assigned, can be considered as a variant of the monotone curve that can interpolate these points. In the seventh column of the table, for comparison, the curvature radii of the parabola at reference points are given.

Deviation of the positions of normals and the relative deviation of the curvature radii for the interpolating curve are shown in Tab. 4.

The greatest deviation from the characteristics of the original curve occurred at the first and the last reference points. The reason for this error may be the fact that one of the boundaries of the characteristic ranges corresponding to these points is calculated based on the coordinates of the rest of reference points, and the other boundary is assigned intuitively, based on logical reasoning. This error can be reduced by reducing the distances between the reference points at the beginning and at the end of their sequence by increasing their number [11, 31].

The similarity of the values of the characteristics assigned by the proposed meth-od to the corresponding characteristics of the original curve confirms the correctness of the results presented in the article.

Table 3. Characteristics of the reference point series

i	Radius of TC, mm		Minimum evolute length, mm	Centre of curvature range, mm	Curvature radius of the interpolating curve	Curvature radius of the parabola R_i^{par}, mm
	$R'TC_i$	$R\,TC'_i$	$\lvert C'_i,{}'C_{i+1}\rvert$	$\lvert {}'C_i,C'_i\rvert$	R_i, mm	
1	-	166.74	29.46	5.30	155.10	159.09
2	176.10	201.94	55.23	10.02	184.56	187.40
3	218.66	260.65	110.82	18.94	239.79	237.90
4	319.22	384.88	218.92	40.58	350.61	355.03
5	468.62	693.55	417.33	75.50	569.53	571.71
6	837.83	1200.0	742.18	134.33	986.86	1007.55
7	1422.14	-	-	239.00	1729.04	1677.05

Table 4. Deviation of characteristics of the interpolating and the reference curves

Point number, *i*	1	2	3	4	5	6	7
Deviation angle of the normal, °	0.32	0.14	0.11	0.23	0.18	0.25	1.07
Deviation of the curvature radius, %	2.51	1.50	0.79	1.24	0.38	2.05	3.10

3. Conclusions

The article offers an algorithm for assigning the positions of curvature centres of the interpolating curve at reference points. The curvature centres are assigned on the basis of a regular and uniform increase in curvature values along the sections of the curve where the initial conditions prevent the occurrence of singular points.

The algorithm is based on the following methods:

- assigning the positions of normals of the interpolating curve at reference points;
- assigning the positions of curvature centres on the assigned normals.

The position of each of the normals is assigned within an area, the boundaries of which are defined by the specified properties of the interpolating curve. The reference area of the possible location of the normal is uniquely defined by the coordinates of five consecutive reference points. The boundaries of the reference area shall be refined in accordance with the condition that the normals are assigned simultaneously at the previous and subsequent reference points. The proposed method for simultaneous assignment of normals to all the reference points provides:

- a proportional decrease of the reference area of the normal for all the points;
- location of each normal in the centre of the refined area.

These proportions are the compliance criterion of the assigned normals with the configuration of the reference point series.

The position of each centre of curvature is assigned within a predetermined segment that belongs to the corresponding normal. These segments are the ranges of curvature centres, and they take into account the whole area of a possible solution with the assigned positions of the normals. The proportions in which the curvature centres divide the corresponding ranges may serve as a criterion for the correctness of assigning their positions.

Assigning the centres of curvature at the points bounding the mentioned ranges provides a unique solution – an interpolating curve consisting of smoothly joined arcs of circles whose radii increase monotonously along the curve. Assigning the curvature centres within the ranges determines the possible location of the evolutes of the mono-tone regular curve that interpolates the reference points. The position of the centres of curvature and the points bounding the ranges of their possible assignment uniquely determine the possible location of the involute – the interpolating curve.

In this case, the solution is not unique. Based on the same location of the involute, it is possible to form a set of interpolating curves, whose characteristics comply with the conditions of the problem.

The possibilities of the proposed algorithm and its constituent methods have been investigated by interpolating a sequence of points assigned to the branches of the parabola. The standard positions of normals obtained by solving the test example deviate from the corresponding normals of the reference curve (parabola) within 0.32 degrees. The relative deviation of the assigned curvature radii from the corresponding values of the reference curve was within the range of 0.79–3.10 per cent. The values of these deviations confirm the correctness of the proposed solutions.

The proposed methods are based on geometric constructions, which resolve themselves to determining the intersection points of straight lines and dividing the segments in the fixed ratio. The necessary calculations consist in solving systems of linear equations. The simplicity of the geometric and computational schemes ensures high accuracy of calculations and does not require the application of iteration pro-cesses.

These features make the proposed algorithm the most appropriate for the task of its further implementation in the form of a computer program.

The disadvantages of the proposed solutions include the fact that the calculations of the division ratio of the initial ranges are based on logical reasoning and are currently insufficiently investigated. These ratios provided a positive result in solving the test example, but they cannot be established as optimum. The elimination of this deficiency requires further investigation of various sequences of points assigned on various curves.

The results obtained in this work complement and develop the research carried out in previous works [11–13]. Assigning the characteristics of the interpolation curve based on the area of possible location of its evolute simplifies the interpolation problem, reduces computational error, making the solution more reliable. However, this paper does not address the issue of ensuring the given accuracy of interpolation. The problem of forming the evolute of the curve, which contains a minimum based on the initial data number of singular points and interpolates a sequence of reference point with given accuracy, is to be solved in further research.

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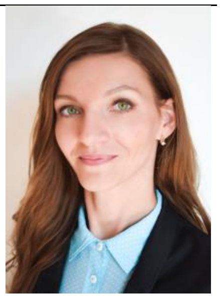

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