

## Identification of unmeasured variables in the set of model constraints of the data reconciliation in a power unit

MARCIN SZEGA  
GRZEGORZ TADEUSZ NOWAK\*

Silesian University of Technology, Institute of Thermal Technology,  
Konarskiego 22, 44-100 Gliwice, Poland

**Abstract** In generalized method of data reconciliation as equations of conditions beside substance and energy balances can be used equations which don't have precisely the status of conservation laws. Empirical coefficients in these equations are traded as unknowns' values. To this kind of equations, in application of the generalized method of data reconciliation in supercritical power unit, can be classified: steam flow capacity of a turbine for a group of stages, adiabatic internal efficiency of group of stages, equations for pressure drop in pipelines and equations for heat transfer in regeneration heat exchangers. Mathematical model of a power unit was developed in the code Thermoflex. Using this model the off-design calculation has been made in several points of loads for the power unit. Using these calculations identification of unknown values and empirical coefficients for generalized method of data reconciliation used in power unit has been made. Additional equations of conditions will be used in the generalized method of data reconciliation which will be used in optimization of measurement placement in redundant measurement system in power unit for new control systems.

**Keywords:** Mathematical modeling; Empirical characteristics; Data reconciliation

### Nomenclature

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\*Corresponding Author. E-mail: grzegorz.t.nowak@polsl.pl

$a_i$	–	empirical coefficients
$A$	–	area of a heat transfer, $m^2$
$c_i$	–	empirical coefficients
$\bar{D}^2$	–	weighted average variance of values after data reconciliation
$h$	–	specific enthalpy, $kJ/kg$
$m$	–	number of measurement variables
$\dot{m}$	–	mass flow, $kg/s$
$n$	–	number of unmeasured (estimated) variables
$p$	–	pressure, Pa
$P$	–	power, MW
$q$	–	number of additional conditional equations
$r$	–	number of a conditional equations
$R^2$	–	coefficient of determination
$s$	–	specific entropy, $kJ/(kg\ K)$
$T$	–	absolute temperature, K
$u$	–	uncertainty of an estimated value
$\bar{u}$	–	uncertainty of an estimated value after data reconciliation
$U$	–	average heat transfer coefficient, $kW/(m^2K)$
$z$	–	number of additional measurements or unmeasured variables

#### Greek symbols

$\eta_i$	–	adiabatic internal efficiency, %
$\sigma_i$	–	standard uncertainty of $i$ th measurement data to the data reconciliation calculations
$\bar{\sigma}_i$	–	standard uncertainty of $i$ th measurement data after data reconciliation

#### Subscripts

$el$	–	electric
$fw$	–	feed water
$in$	–	inlet
$out$	–	outlet
$pd$	–	pressure drop
$sf$	–	steam flow
$st$	–	steam

## 1 Introduction

Increasing nominal electric power of power plants and implementing of supercritical parameters of steam leads to more complicated structure of power systems. Concerning this path of power plants development, from the safety of operation point of view, there is a need of adequate number of measurements, which should give a information about technical condition of equipment and information about all processes which take place in the system. Beside the increase of information needed for the process control,

more information is available from the point of technical operation. This information allows to use control methods based on mathematical models of physical processes occurring in the system. These models are included in the decision support systems for technical operation control and the use of advanced engineering analysis. An essential element of such systems is to conduct validating measurement data from distributed control systems using measurement data reconciliation method [1–3]. This method, requires that the number of measurements in the distributed control system of the block was in excess relative to the minimum number of measurements needed for determination of unmeasured variables. New possibilities occur by using generalized method of data reconciliation which initially estimated the value of unmeasured variables and uncertainty of their estimation. This method can be applied in the design of the measurement system for newly built power plants. The information presented above shows that the number of measurements in the system must be redundant. Number of redundant measurements and their location in the thermal system can be optimized in terms of assumed criteria. Such criteria could be to minimize the uncertainty of calculated coefficient's of energy conversion in the process. Reducing measurement uncertainty after reconciliation, thereby reducing the uncertainty of calculated coefficient's of energy conversion can be obtained by the expansion of the system of condition equations. In reconciliation of measurement in thermal power unit to these equations can include equations of steam flow capacity in turbine and its internal efficiency, the equation of pressure drop in pipelines and heat flow in regenerative heat exchangers. In these equations to the values of unmeasured, initially estimated in the generalized method of validation, will also belong the empirical coefficients. These coefficients are determined mostly on the basis of special or operation measurements. For the optimization of the measurement system for designed unit this approach is not possible. Therefore those coefficients were determined by the numerical experiment. For this purpose, a mathematical simulation model of supercritical steam power unit with a capacity of 800 MW in the computer code Thermoflex was developed [6–8]. Using this simulation model calculations were carried out for different load capacity of the unit. The identification of the value of unmeasured factors and empirical coefficient in additional equations of conditions for the generalized method of measurement validation in thermal power unit.

## 2 Conditions of the additional conditional equations application

The generalized method of data reconciliation, as described above, can use as a conditional equations (equality constraints) additional equations which describe the physics of occurring processes. Expansion of the system of equations in the process of data reconciliation brings additional benefits presented in [9]. Advantages of using additional equations are for satisfying not only the substance and energy laws of conservation, but also the principle of entropy increasing. Development of the system of equations with additional equations also brings additional benefit for reducing the measurement uncertainty after measurements data reconciliation. Evaluation of the effect of reducing uncertainty through the use of additional equations in generalized data reconciliation method are presented in [9]. For evaluation of reducing uncertainty a coefficient was, implemented presented in Eq. (2), which is calculated by dividing Eq. (1) (sum of the weighted variance measurements and estimated values) by the number of variables which as a sum of measurements, described by formula

$$\sum_{i=1}^m \frac{\bar{\sigma}_i^2}{\sigma_i^2} + \sum_{j=1}^n \frac{\bar{u}_{yj}^2}{u_{yj}^2} = m + n - r \quad (1)$$

by the actual number of variables which is taken into consideration is a sum of measurements including actual and estimated values. This coefficient is defined as the weighted average of the variance of the variables in a generalized method of data reconciliation [9]

$$\bar{D}^2 = 1 - \frac{r}{m + n}, \quad (2)$$

On the base of Eq. (2) the value of the variance is uniquely determined by the number of equations, number of measurements and the estimated values in the validation task. Value of the coefficient presented in this equation in generalized method of reconciliation is always contained in the interval (0,1), because  $r < m + n$ . Introduction to validation analyses additional equations, measurements or estimated values would be beneficial if the variance resulting from Eq. (2) is less than the variance of the analyses without these expansions. Condition of advantage considering the reduction of uncertainty in measurement after data reconciliation in this case is [9]

$$\frac{z - q}{z} < \bar{D}^2, \quad \text{for } z, q \geq 0 \quad (3)$$

It is clear from inequality (3), that suitable effect of introduction of additional conditional equations depends on the number of equations and the number of estimated values, which will be the empirical coefficients, which are present in these equations. For example, implementing additional conditional equation with one estimated an empirical coefficient ( $z = 1, q = 1$ ) will always result in reducing the uncertainty of the reconciled variables. For these values of  $z$  and  $q$  left-hand side of inequality (3) is in fact in this case zero. Thus, a positive value for the variance, the condition (3) is always satisfied.

### 3 Additional conditional equations in data reconciliation method

For the purpose of reconciliation as additional equations for reconciliation method the equation for steam flow in turbine and its internal efficiency, the equation of pressure drop in pipelines and heat flow in regenerative heat exchangers were chosen. Mathematical form of those are as follows:

- equation of adiabatic internal efficiency

$$\eta_i = \frac{h_{in} - h_{out}}{h_{in} - h_{out s}}, \quad (4)$$

- equation of steam flow capacity of a turbine

$$\dot{m}_{st} \frac{\sqrt{T_{in}}}{p_{in}} = c_{sf} \sqrt{1 - \left(\frac{p_{out}}{p_{in}}\right)^2}, \quad (5)$$

- equation of pressure drop in a flow pipeline

$$\Delta p = p_{in} - p_{out} = c_{pd} \dot{m}^2, \quad (6)$$

- heat flow in regenerative heat exchangers

$$\dot{m}_{fw} (h_{fw out} - h_{fw in}) = UA \Delta T_m. \quad (7)$$

The average temperature difference  $\Delta T_m$  has been calculated as a thermodynamic temperature difference of fluid streams flow through the heat exchanger

$$\Delta T_m = \left(\frac{\Delta h}{\Delta s}\right)_{st} - \left(\frac{\Delta h}{\Delta s}\right)_{fw}. \quad (8)$$

Each of the Eqs. (4)–(7) contains a value which requires initial estimation. They are: adiabatic internal efficiency  $\eta_i$ , empirical coefficients  $c_{sf}$  and  $c_{pd}$ , and average heat transfer coefficient  $U$ , under the condition (3). Introduction of these additional equations for the data reconciliation process, which includes initially only mass and energy balances, will reduce uncertainty of measured and estimated variables after data reconciliation.

#### 4 Identification of the unmeasured variables in the additional conditional equations

For identification of unmeasured variables occurring in the listed additional equations the simulation model was developed of the analyzed power unit in program Thermoflex. As a result of multivariate simulation the values of unmeasured variables were obtained in a wide range of changes of electric generator power. Calculations of internal efficiency, coefficients in steam flow capacity equation for different groups of turbine stages, coefficients in the equation of the pressure drop in pipelines connecting steam bleeding of turbine with regenerative heat exchangers and heat transfer coefficient for so-called Peclet equation (7) [10]. As a result of numerical experiment stated sensitivity of these coefficients to the change of electric power of generator. Analysis of those sensitivity showed the possibility of developing empirical functions which will allow to estimate the values of unmeasured variables depending on electric generator power. This function is described by polynomials of the second degree

$$y = a_0 + a_1 P_{el} + a_2 P_{el}^2 . \quad (9)$$

Using the least squares method, determined were the empirical coefficients ( $a_0, a_1, a_2$ ) in (9) for the internal efficiency  $\eta_i$  and for the steam flow equation  $c_{sf}$  for ten groups turbine stages. The coefficients of determination  $R^2$  internal efficiency characteristics obtained are in the range from 75% to over 99%, while the coefficient of the steam flow capacity equation is between 97.9–99.8%. Identification of the empirical characteristics coefficients  $c_{pd}$  of the nine equations pressure drops in pipelines connecting steam bleeding of turbine with regenerative heat exchangers. Obtained coefficients of determination  $R^2$  is in the range from 97% to almost 99.8%. For seven heat regenerative heat exchangers calculated was the range of variation of the product of heat transfer coefficient  $U$  and surface heat flow  $A$  in Eq. (7). As in previous cases, high values of determination coefficients

of  $R^2$  exceeding the value of 99% were obtained. Tables 1–4 presents the exemplary results of the identification of the characteristics of unmeasured variables in Eqs. (4)–(7) for the first group of stages of the high-pressure turbine, the pipeline connecting the first steam bleeding of turbine with the regenerative heat exchanger and regenerative heat exchanger before steam cooler (Fig. 1).

Table 1. The results of calculations of coefficients and statistical analysis of the empirical characteristics of the adiabatic internal efficiency of the first group stages of the high-pressure turbine.

Parametr	Value
Coefficients of empirical characteristics:	
$a_0$	91.052
$a_1$	0.0005
$a_2$	0.0000006
Assessing parameters for the energy performance:	
– coefficients of determination $R^2$	0.9528
– standard deviation of the residual component	0.1019
– maximum deviation	0.1218

Table 2. The results of calculations of coefficients and statistical analysis of the empirical characteristics of steam flow capacity equation of the first group stages of the high-pressure turbine.

Parametr	Value
Coefficients of empirical characteristics:	
$a_0$	54.064
$a_1$	0.0024
$a_2$	0.0000006
Assessing parameters for the energy performance:	
– coefficients of determination $R^2$	0.9981
– standard deviation of the residual component	1.4274
– maximum deviation	1.778

## 5 Mathematical simulation model

As was mentioned before as a mathematical model for the supercritical power unit with a capacity of 800 MW has been developed. Concept of this

Table 3. The results of calculations of coefficients and statistical analysis of the empirical characteristics of pressure drop in pipelines connecting the first steam bleeding of turbine with the regenerative heat exchanger.

Parametr	Value
Coefficients of empirical characteristics:	
$a_0$	0.0045
$a_1$	0.000008
$a_2$	0.000000004
Assessing parameters for the energy performance:	
– coefficients of determination $R^2$	0.998
– standard deviation of the residual component	0.0101
– maximum deviation	0.016

Table 4. The results of calculations of coefficients and statistical analysis of the empirical characteristics of UA parameter for regenerative heat exchanger.

Parametr	Value
Coefficients of empirical characteristics:	
$a_0$	0.0089
$a_1$	-4.9041
$a_2$	1548.3
Assessing parameters for the energy performance:	
– coefficients of determination $R^2$	0.9942
– standard deviation of the residual component	49.707
– maximum deviation	132.6

power unit was assumed as a newly built power unit. That is due to the trends observed in the European Union where units equipped with chemical absorption of carbon dioxide are regarded as new designs. The waste heat from the process is used for district heat production. The detailed description of this model is presented in [7]. Simulation model was developed in the computer code Thermoflex which is equipped with many predefined equipment for power unit. Simulation of this unit was divided into two parts. In the first one a whole power unit was developed with the main assumptions, which are specified in Tab. 5. The results obtained from the simulation model were verified white examples presented in the literature and industrial application. The next step was to transfer this model into off-design mode which allowed to simulate different electric power of gen-



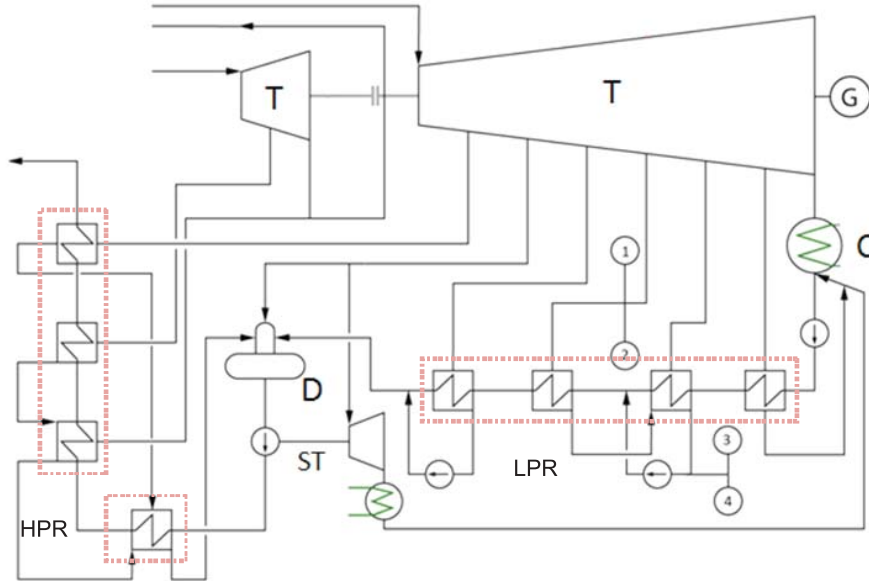


Figure 1. Scheme of the conception of steam cycle in the power unit: G – electric generator, T – steam turbine, D – deaerator, C – condenser, HPR – high-pressure regeneration, LPR – low-pressure regeneration, 1 – steam to district heat exchanger, 2 – steam to chemical absorption of CO<sub>2</sub>, 3 – condensate from district heat exchanger, 4 – condensate from chemical absorption of CO<sub>2</sub>.

Table 5. Selected input data for the simulation calculations

Parameters	Unit	Value
Live steam temperature	K	923
Live steam pressure	MPa	30
Reheated steam temperature	K	943
Reheated steam pressure	MPa	6
Condenser pressure	MPa	0.00686
Ambient parameters:		
Temperature	K	288
Pressure	MPa	0.1013
Humidity	%	60
Cooling water temperature	K	293

erator and observe the plant behavior depending on the change of electric power. These results were used for calculation of the empirical coefficients in additional conditional equations used in data reconciliation.

## 6 Conclusions

The introduction of additional equations to the process of validating the measurement data can bring the benefits of reducing the overall measurement uncertainty in the thermal power unit. Condition to obtain this benefit is to satisfy inequality (3). For additional equations used in the data reconciliation of measurement in thermal power unit, improvement will occur due to the presence in them of only one unmeasured variable. The improved accuracy of reconciliation of measurements data also determines the value of its uncertainty. For very large interval of uncertainty estimation for this variable, adding an additional conditional equation with this variable in relation to analyses without this equation, improving the accuracy of measurements is relatively small. Therefore, for the estimation of unmeasured variables and reduce their uncertainty ranges were determined the empirical functions based on numerical experiment based on the mathematical model. The resulting empirical characteristics unmeasured variables as a function of the electric generator power is characterized by a high correlation of the dependent variable with the independent variable. Proved by the high values of the calculated coefficients of determination  $R^2$ . Good quality empirical relationships also provide a small average values of obtained approximation errors (standard deviation of the residual component).

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