COMMENTARY ON HISTORY OF ADAM KOCHAŃSKI'S CIRCLE RECTIFICATION POPULARIZATION

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Abstract. The subject of study in this article is history of popularization of graphic notation of rectification of semicircle by Adam Kochański and changes introduced in this notation in following scientific publications. The author's aim is to show in which moment the original notation was replaced with the one that was commonly used in publications in XX c.

Keywords: squaring the circle,circle rectification, T. V. Clauso, L. Cremona, M. Fourneau, A. Kochański, J. É.Montucla, M. N. Navrotsky, Chr. Nehls, T. Żebrawski

1   Introduction

The meaning of A. Kochański's rectification or graphic way of finding a segment, which length approximately equals half of circle's length, for solving some engineering tasks, does not arouse suspicions.Kochański Publisher his rectification in 1685 in *Acta Eruditorum* [1, p.394-398] (Fig.1).Many scientific authorities consider it the most „*simple and beautiful*"[2, p.102].

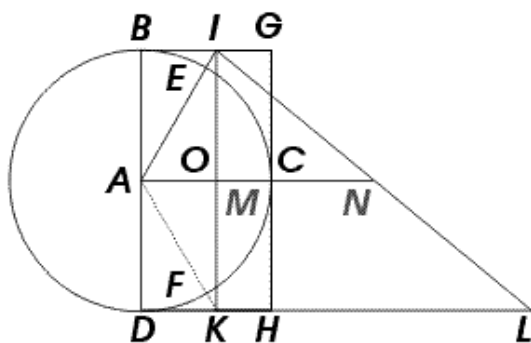

Figure 1: Adam Kochański's rectification, end of XVII century [1,p.397], based on[2, p. 103], study by A. Żaba
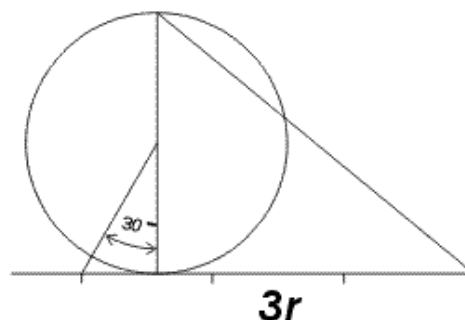
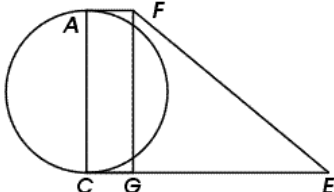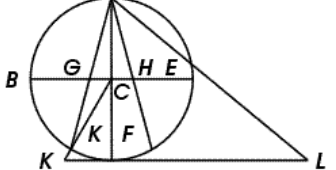Figure 2: Rectification of semicircle popularized in geometrical guidebooks of XX century, study by A. Żaba
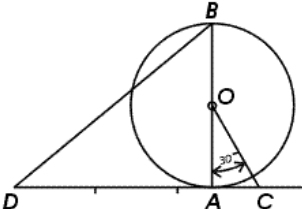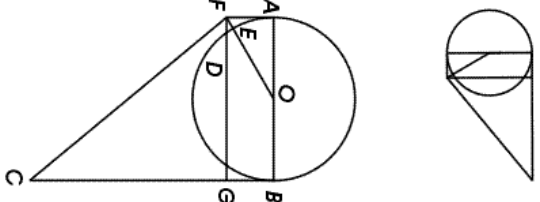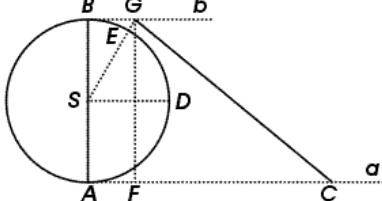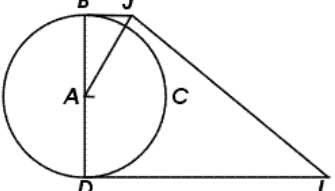
In XX century's textbooks on geometry and engineering graphics the original notation of Polish geometrician was replaced with construction, which is different from the original one. It is a transformed construction simplifying original Kochański's notation (Fig. 2). The author found the change interesting and conducted study aiming to fix: who and when, for the first time, introduced changes to the original notation.The problem of history popularization

was already described in Polish literature [3] and [4], but author's study made it possible to fix additional facts.

2   History of changes in original notation of Kochański's rectification

In mathematical literature, including geometry, rectification was connected with squaring the circle, which is not precise, but justified.Common element of both activities is to find the value ofπ. Many constructions called squaring or rectification referred to Kochański's rectification. Table 1 compiles these constructions, which where specified in literature as referring to Kochański's idea. Under the table there are short commentaries on each solution.

Table 1: Compilation of figures of construction specified as Kochański's rectification

Item	Figure	Date of publ.	Author [Source] *Spelling of the name*
1.		1754	J. É. Montucla [5, p. 64i Chart 1, Fig. 6] *P. Kolhanski*
2.		1802	T. V. Caluso [8, p. 582 ip. 584 - Chart XI] *P. Kolhanski*
3.		1890	L. Cremona [11, p. 117 - Fig. 131] *Kochansky*
4.		1850	L. C. Schulz vonStrassnitzki [13, p. 304 - Fig. 317] *Kochanski*
5.		1862	T. Żebrawski [3, p. not numbered - diagram I] *Kochański*
6.		1913	Hobson E. W. [14, p. 33 - Fig. 16] *Kochansky*

2.1   The authors of chosen articles on Kochański's rectification and commentaries on its references

Kochański'srectification in its simplified version was included in 1754 in his book *Histoire des recherchessur la quadrature du cercle* [5] by **Jean Étienne Montucli** (1725-1799) (see Table 1, item 1). The author evaluates the solution of *'Polishgeometricia'* very high, but he changes the name A. Kochański to P. Kolhanski [5, p.64]. This spelling mistake is often found in many other articles, for example in French[6, p.415][1], Dutch[8, p.739] and Italian [9, p.582] (see Table 1).

Tommaso Valperga di Caluso (1737-1815) in 1802 publishes *Della Impossibilitàdella Quadratura...* [9] a picture, in which he mixed constructions of Christian Huygens (1629-1695) and Kochański (see Table 1, item 2). Although the notation of Kochański's construction in Caluso's redaction is very similar to the one used in XXth century geometry textbooks, it is hard to assert, on current state of study, that this is the first notation of its kind.We can only state, that from the beginning of XIX century that simplification was present in literature.

In 1875 **Luigi Cremona** (1830-1903) published his work *Rektificationeines Kreisbogen* [10]. It contained a figure, which resembles a figure in Caluso's book without any doubts. However, its description says, that this is construction presented to the author by an engineerCeradini of Scuolad' Applicazione[2] in Rome. The author of German translation of the original Italian Cremona's[3] book was Maximilian Curtze[4] (1837-1903). This German philologist, historian of mathematics and researcher of Nicolaus Copernicus while working on his translation worked in Toruń, the place, in which Kochański graduated from secondary school. However he did not associate construction presented by Cremona with Kochański. In 1890, in English edition of his book [11] Cremona gives information, that construction which was first ascribed to Ceradini is construction of Kochański (see Table 1, item 3). Cremona does not appeal to original work of Kochański. He appeals to work of Dr Bottcher[5] published in 1880 [11, p. 117].

Simplified notation of Kochański's construction can be found in Italian literature as work of Kolhański and Ceradini[6], and later in English literature as Kochansky's.

In German literature we can find described rectification in work *Handbuch der Geometrie...* [13] of **Leopold Carla Schulz von Strassnitzki** (1803–1852). Rectification is presented as item 4 inTable 1. On the left side it is shown, in which order the picture can be found in publication. On the right side there is a picture turned by 90^0 with description remaining original letter combination. According to the author, Starssnitzki's notation is very similar to the original one.

In Polish literature to aforementioned German study appeals work of **Teofil Żebrawski** (1800-1880) from 1862, entitled *Wiadomość o Adamie Kochańskim...*[3]. It contained construction presented in Table 1, item 5. Polish author informed, that Kochański'srectification was mentioned in publications already known to us: Montucla's [5][7]

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[1]Describing work of NicolasFourneau (1722-1792) [7], that could not be found by author.
[2]It was engineer's school of applied studies.
[3]Probably *Corso di staticagrafica* from the year 1868.
[4]Also known as Ernst Ludwig Wilhelm Maximilian Curtze.
[5]The study was published in year 1883 by Johannes Eduard Böttcher (1847-1919) w *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, XVI c.
[6]As Ceradini's rectification it is functioning construction proposed by Cremona as far back as in XX c., for example in Ernest Herman Koch's works (1875-?) [12, p. 206].
[7]Żebrawski [3, s. 7] mantions that Montuclas work was published in the year 1756 (see [5]).
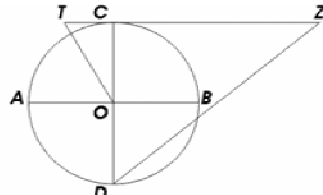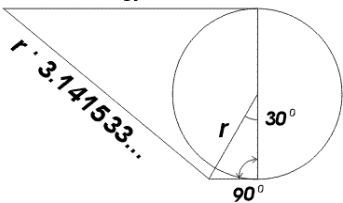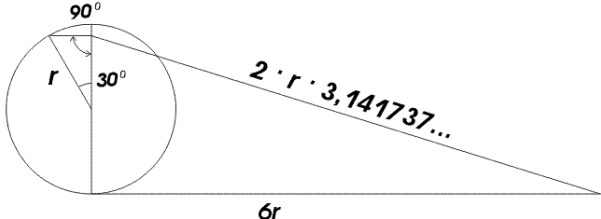
and „Leopold Szultz von Strasznicki" [13][8i9]. He also said, that according to his information, in Polish literature nobody had ever written about Kochański's rectification. Maybe he thought that it should be disseminated and that is why later it can be found in for ex ample engineering dictionaries by him [14] and [15]. In *Słownik wyrazów technicznych...* published in 1883 Żebrawski presents Kochański'sconstruction, however he uses only a semicircle [15, p. 196 – Fig. 67].

To Kochański'soriginal also appeal later studies, as for example *Squaring the Circle…*[16] by **Ernest William Hobson** (1856–1933) of 1913 (seeTable 1, item 6).

3   On two rectifications connected with A. Kochański's solution

According to the author in description of history of object rectification's popularization it is worth to pay attention to two rectifications presented in Table 2.

Table 2: Compilation of constructions similar to Kochański's rectification

Ite	Drawing	Year of publication	Author [Source]
1.		1844	N. M. Nawrotzki [19, Fig. 1]
2.		1882	C. Nehls [21, p.483]
3.			

The author of the first one is **Mykola Nikonorovicz Navrotsky** (1803-1859) (see: tab. 2, item. 1). Doctor Navrotsky was Russian military man, appreciated mathematician and leader of scientific societies of high rank[10]. He published his rectification in Russian (for example in [18]) and German language [19].

Teofil Żebrawski gives information on publication of Nawrocki's rectification in'*eleventh scientific journal of Moscow'*[3, p. 9] as far back as the year 1834. The author evaluates, that Nawrocki could newly discover rectification after 150 years[11] or that the

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[8]Name of born in Cracow Strassnitzki (in both parts) Żebrawski spells differently than we can find in literature.
[9]The author did not manage to fix, about which work of Strassnitzki had Żebrawski written. It is possible that it's about quoted in this article [13] or about published in 1835 *Elemente der reinen Geometrie*.
[10]More information on Navrotsky gives P. Sergeev [17].
[11]About rectification of Nawrocki also wrote J. Daniiin *Zagadnienie kwadratury koła...*[4] in year 1956.

changes applied to Kochanski's construction notation sufficiently important to announce to the world, that he is the author of the construction.

Another interesting rectification is the construction published in 1882 by **Christian Nehls**[12] (1841-1897) in *Uebergraphische Rectification...*[20]. In the same year Nehls publishes this construction in popular engineering journal [21, p. 483].

In the study, a construction similar to Kochański's[13] (see Table 2, item 3) was compared to Nehls' idea (see Table 2, item 3). It was emphasized, that the result, although it was less precise, it was better from the practical point of view, because it considers straightening the whole circle. However, Nehls' rectification would not get such a big credit in literature on the history of mathematics as Kochański's. In the article, that was published after Nehls' death [22, p.417] this rectification was not mentioned, although it mentioned his other mathematical achievements.

4   Summary

Attempts to solve the problem of squaring the circle and rectification of the circle were taken as far back as antiquity. In modern times extensive study on these matters were taken in XIX century. They were summed up in the year 1906 by **Maximilian Simon** (1844-1918) in *Über die Entwicklung der Elementar-Geometrieim 19. Jahrhundert;...*[23]. The author presents, as he declared, *'randomly compiled solutions'* [23, p. 64],however, according to the author of this article, it is a very interesting collection, very rich and, most important, representative. Simon also did evaluate solutions from basic criteria point of view, which was the value of deviation from the number $\pi=3,14153$.   However he does not present Kochański's construction itself, he wrote about it: *'After all* (after studies of XIX century solutions – authors annotation) ***Kochański's construction occurred to be unattainable'*** [23, p. 66].

It is also important to state, that from the beginning of publishing Kochański's construction the authors were aiming for simplifying original construction. The author fixed, that in the year 1802 in Italian-speaking study Caliso there was presented simplified notation of Kochański's construction, which was commonly used in publications of XX century. The solution was popularized in German- and English-speaking editions of Cremona's book. According to the author, it is too soon to state unambiguously, that discussed simplifying was connected only with 'Italian school of mathematics'.

5   References

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[12]Actually Johann Christian Nehls, publishing as Christian Nehls [22].
[13]It is Kochański's rectification in similar version to Caluso's editing (see tab. 1, item 2).

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UWAGI NA TEMAT HISTORII POPULARYZACJI REKTYFIKACJI OKRĘGU ADAMA KOCHAŃSKIEGO (1631-1700)

Przedmiotem opracowania w artykule są dzieje popularyzacji rektyfikacji Adama Kochańskiego oraz zmian wprowadzanych w jej zapisie graficznym w kolejnych publikacjach naukowych. Celem, jaki stawia sobie autorka jest próba ustalenia, w którym momencie nastąpiło przekształcenie zapisu oryginalnego na stosowany w publikacjach z XX wieku.