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Evaluation of Qualimetrical Measurements Quality Based on the Uncertainty Concept

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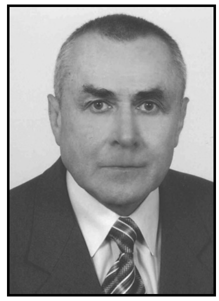

Abstract

The main problems of qualimetrical measurements for quality assessment as a new metrology trend are considered in this paper. The conceptual notion of the qualimetrical measurement is proposed and developed. The main problems of realization procedure are considered and its solution variants are analyzed. The methodology of results uncertainty assessment of qualimetrical measurements is developed. The correlations between individual single studied product quality values are taken into account. It helps to assess objectively the quality of qualimetrical measurements.

Keywords: qualimetrical measurements, uncertainty, product quality level, virtual quality measure, multidimensional scaling.

Ocena dokładności pomiarów kwalimetrycznych oparta na teorii niepewności

Streszczenie

W pracy przedstawiono główne problemy dotyczące metodologii pomiarów kwalimetrycznych, które można określić jako nowy trend w dyscyplinie metrologia. Zaproponowano i opracowano pojęcie mierzenia kwalimetrycznego jako pośredniego pomiaru danej wielkości, której wartość określa się poprzez opracowanie wyników pomiarów według metodologii skalowania wielowymiarowego. Przeprowadzono analizę metodologii oceny pomiarów kwalimetrycznych, przy wykorzystaniu teorii niepewności. Przedstawiono podstawowe problemy realizacji procedur pomiarów kwalimetrycznych i przeanalizowano sposoby ich rozwiązania. Zgodnie z teorią pomiaru, rozwiązano zagadnienie syntezy miary jakości produktu, jako jednego z kluczowych elementów realizacji pomiarów kwalimetrycznych. Wprowadzono pojęcie wirtualnej miary jakości produktu, która jest odpowiednikiem teoretycznym realnej, fizycznej miary jakości. Przedstawiono problem wykorzystania metodologii skalowania wielowymiarowego w pomiarach kwalimetrycznych, w tym analizę i uzasadnienie zastosowania modelu trójmodalnego skalowania wielowymiarowego. Otworzyło to możliwość oceny poziomu jakości badanego produktu. Zaproponowano metody szacowania niepewności wyników pomiarów kwalimetrycznych, biorąc pod uwagę różne stopnie skorelowania pomiędzy poszczególnymi wskaźnikami jakości produktów badanych. Stworzyło to możliwość oceny jakości tych pomiarów. Ponadto, dzięki nowej metodologii pomiarów kwalimetrycznych jest możliwość osiągnięcia jednoznaczności pomiarów.

Słowa kluczowe: pomiary kwalimetryczne, niepewność wyniku pomiaru, jakość produktu, wirtualna miara jakości, skalowanie wielowymiarowe.

1. Introduction

The measurement is some kind of activity in consequence of which the specific product (measurement result) is obtained. Therefore measurements and measurement results are assessed by specific qualitative characteristics. The measurement accuracy is the most used qualitative characteristic.

Main numerical assessments of the measurement accuracy are the error and the measurement result uncertainty. Today the uncertainty concept is dominant in measurements quality assessment in the metrology. Accordingly numerical assessment of measurement accuracy is the uncertainty of measurement result. It is defined as parameter linked with the measurement result, and which characterizes value spread that can be reasonably ascribed to the measurement value [1]. It should be noted that the uncertainty concept usage in measurements quality assessment has been sufficiently studied in classical metrology.

Appropriate effective normative documents affirm it, for instance ISO/IEC Guide 98-3:2008 [2] and numerous publications in scientific and technological journals. However qualimetrical measurement quality assessment nowadays as the new trend in the metrology has not been practically studied, and as a result it became topical in this work.

2. The meaning of qualimetrical measurement notion and the main research goals formulation

Qualimetrical measurements refer to the sphere of qualimetry – the branch of science where research subject is quantitative assessment of product quality [3]. The authors proposed and developed the qualimetrical measurement conceptual notion in the work. This is indirect measurement of products quality level. Its meaning can be obtained by the way of measurement results processing of its characteristics according to the multidimensional scaling methodology. Product quality level is a relative characteristic of product quality, which is based on the comparison of values of products quality assessed indexes and basic values of the corresponding indexes.

Generally qualimetrical measurement as any other measurement consists of two main stages:

- carrying out the measuring experiment during which different characteristics (mechanical, dimensional, electric, magnetic, thermic, and chemical composition etc.) of studied products are measured;
- measuring experiment results processing during which the studied products quality level Q is defined. This is the qualimetrical measurement results.

Therefore, the mentioned above approach of qualimetrical measurements essence and purpose enables to treat them as one of the indirect measurements types and to use the main principles of the measuring theory for their analysis.

The basis of any measurement is the comparison of measured value with measure which presents and (or) keeps some physics value of given meaning [4]. Qualimetrical measurements specificity is the absence of specific physical quality measures of some products. Available basic (standard) samples of the studied product don't always meet the metrological requirements elaborated for measures. It is not always methodologically possible to make comparison of the studied product with such basic sample, that is the main problem of qualimetrical measurements realization.

Product quality virtual measure is proposed to use for the methodological implementation of qualimetrical measurements procedure realization and the assessment of their accuracy and quality [5]. Therefore the main research tasks are the following:

- the analysis of the qualimetrical measurement procedure realization methodology with the use of virtual quality measure;
- development and analysis of the assessment methodology of measurement results accuracy of product Q quality level on the basis of the uncertainty concept.

3. The methodology of qualimetrical measurement with the use of virtual measure of product quality

3.1. The definition of the product quality virtual measure notion

We use the main principles of virtual measuring instruments technology as one of the most modern high information technology [6], and set theory as corresponding part of mathematics to define the virtual product quality measure notion [7].

The essence of virtual measuring instruments technology is in computer program imitation of real physical measuring instruments, measuring systems and control systems. In this case the virtuality is expressed in the sense of virtual imitation of some instrument functions by mathematical and program means.

Consequently, the virtual product quality measure is the reflection of real physical measure of the current product quality expressed by mathematical and program means. From the other hand, the virtual product quality measure can be expressed as some set (totality, combination) of several random objects (elements) combined by their general properties (indications).

In qualimetry such objects (elements) are single absolute P_i and relative K_i product quality values characterizing its quality under elaborating, producing, exploitation and usage [5]. Absolute product quality index P_i (i =1,2,…,n, where n - properties number) characterizes individual product properties and is equal to i-property of product p_i and expressed in its units.

Relative product quality index K_i (i =1,2,…,n, where n - properties number) characterizes individual product properties in form of correlation between some property values expressed by its absolute indexes. So K_i is intangible parameter. Sets are the study subject of the set theory – the part of mathematics studding sets abstracting from specific elements sets nature [7].

For instance, one of the set theory chapter studies the problem of point sets forming in n - Euclidean measurable space (here n - coordinate number). The product quality assessment is made in the space as product P_i quality single absolute indexes (i = 1,2,…,n, where n - single indexes number which is equal to coordinate number of Euclidean multidimensional space) have different physical nature and dimension. So they are the points on corresponding axis of multidimensional space. It should be noticed that scales of individual i coordinates axis are different and are defined by the ponderability coefficients m_i of absolute single quality indexes P_i.

In qualimetry as part of metrology the study subject of which is the issue of product quality numerical assessment, some random objects (elements) set that is examined above has the title of the quality profile. The quality profile is the totality numerical single product quality index [5]. Thus, the profile product quality is an individual complex characteristic of its quality and can be used for virtual quality measure formation.

3.2. The synthesis methodology of the product quality virtual measure

Quality profiles Π can be formed from the absolute single product P_i quality indexes and from the relative single product K_i quality indexes. But absolute single product P_i quality indexes, i = 1,2,…, n equal to corresponding product properties values gained under measuring experiment. So they have different physical nature and dimensions. Relative single product K_i quality indexes should be used for forming virtual product quality measure, i =1,2,…, n. Let us consider the indexes systematization of quality according to some properties.

All quality indexes depending on function they carry out are divided on absolute estimated $P_{o,i}$ and relative $K_{o,i}$, basic absolute $P_{b,i}$ and relative $K_{b,i}$.There are also regulated limited values of absolute quality product indexes $P_{o,i}$ - minimum $P_{i,min}$ and maximum $P_{i,max}$. The single relative estimated quality indexes value $K_{o,i}$ always lies within limits $0 \le K_{o,i} \le 1$. But depending on how indexes $K_{o,i}$ value impacts on product Q quality level, they change differently and two groups of product quality indexes can be selected.

In the first quality indexes group the rise of single estimated absolute quality index value $P_{o,i}$specifies the increase of studied product quality level. And accordingly it leads to the increase of single relative quality index $K_{o,i}$. So in the first product quality indexes group the basic values of relative quality indexes $K_{b,i}$=1. That is why single relative quality $K_{o,i}$indexes values are reasonable to calculate in the next formula:

$$K_{o,i} = \frac{P_{o,i}}{P_{i,max}} = \frac{P_{o,i}}{P_{b,i}}, P_{o,i} \le P_{b,i} . \quad (1)$$

In the second quality indexes group the rise of studied product quality level leads to the decrease of single relative quality index $P_{o,i}$ and single relative quality index $K_{o,i}$ accordingly. So in the second product quality indexes group the basic values of relative quality indexes $K_{b,i}$=0. That is why single relative quality indexes $K_{o,i}$ values are reasonable to calculate in the next formula:

$$K_{o,i} = \frac{P_{o,i} - P_{i,min}}{P_{i,max} - P_{i,min}} = \frac{P_{o,i} - P_{b,i}}{P_{i,max} - P_{b,i}}, P_{o,i} \ge P_{b,i} . \quad (2)$$

Studied product quality profiles formed from single relative quality indexes are reasonable to divide into two groups:

- estimated quality profiles $\Pi_{K,o}$ formed of single weighed estimated relative product quality indexes $K_{oz,i}$, i=1,2,…n:

$$\Pi_{K,o} = \left\{K_{oz,1}; K_{oz,2}; \ldots; K_{oz,n}\right\}; \; K_{oz,i} = K_{o,i} \cdot m_i , \quad (3)$$

  where $K_{o,i}$ - i single estimated relative product quality indexes numerical values which are determined by formulas (1) or (2) accordingly to measured corresponding studied product properties $P_{o,i}$; m_i - normalized ponderability index $K_{o,i}$ coefficient, in other words $\sum_{i=1}^{n} m_i = 1$;

- basic quality profile $\Pi_{K,b}$ formed of single weighed estimated basic relative product quality indexes $K_{bz,i}$, i=1,2,…n:

$$\Pi_{K,b} = \left\{K_{bz,1}; K_{bz,2}; \ldots; K_{bz,n}\right\}; \; K_{bz,i} = K_{b,i} \cdot m_i , \quad (4)$$

  where $K_{b,i}$ - i single basic relative product quality indexes numerical values of which as have been mentioned above for first indexes group is equal to 1 and for the second indexes group – 0.

Basic quality profile $\Pi_{K,b}$ is formed of single weighed basic relative product quality indexes and is the virtual product quality measure.

3.3. The methodology of determination of product quality level with the use of quality virtual measure

It is necessary to carry out the comparison of estimated studied product $\Pi_{K,o}$ quality profile with basic quality profile $\Pi_{K,b}$ which is the virtual quality measure to estimate the studied product Q quality level (complete qualimetrical measurement procedure realization). Comparison of quality profiles $\Pi_{K,o}$ and $\Pi_{K,b}$ is realized by multidimensional scaling methodology [8].

We make the choice of multidimensional scaling methodology model on the basis of the analysis of statistic link (correlation) presence or absence between single studied product quality values.

In the case of statistic independent (non-correlated) single relative estimated quality indexes $K_{o,i}$ for comparison estimated studied product $\Pi_{K,o}$ quality profile with basic quality profile $\Pi_{K,b}$ with virtual quality measure it should be used weighed Euclidean individual differences model. To determine the differences between corresponding single weighed relative quality indexes values $K_{oz,i}$ and basic $K_{bz,i}$, and absolute difference or deviation function $\Delta\Pi$ can be found in the next formula:

$$\Delta\Pi = \sqrt{\sum_{i=1}^{n}\left(K_{oz,i} - K_{bz,i}\right)^2} = \sqrt{\sum_{i=1}^{n} m_i^2\left(K_{o,i} - K_{b,i}\right)^2}. \quad (5)$$

Single estimated absolute product $P_{o,i}$ quality indexes (corresponding studied product properties) values of which are measured during the research process in general are relative correlated values than relative correlated values are also single estimated relative product $K_{o,i}$ of quality indexes.

In this case to determine product quality level the multidimensional scaling model should be used [8]. It enables to take into account the correlation presence between single estimated product quality indexes $K_{o,i}$ and $K_{o,j}$, and as result absolute difference or deviation function $\Delta\Pi$ can be found by the next formula:

$$\Delta\Pi = \sqrt{\begin{array}{l}\sum_{i=1}^{n} m_i^2\left(K_{o,i} - K_{b,i}\right)^2 + \\ +2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} m_i\, m_j\left(K_{o,i} - K_{b,i}\right)\left(K_{o,j} - K_{b,ij}\right)\cdot r_{K_{o,i},K_{o,j}}\end{array}} \quad (6)$$

where $r_{K_{o,i},K_{o,j}}$ - correlation coefficient between single estimated relative product $K_{o,i}$ and $K_{o,j}$ quality indexes values.

Correlation coefficients values $r_{K_{o,i},K_{o,j}}$ can be determined as ratio of indexes $K_{o,i}$ and $K_{o,j}$ covariance (covariance moment) $R_{K_{o,i},K_{o,j}}$ to multiplication of their standard deviations $s_{K_{o,i}}$ and $s_{K_{o,j}}$ estimations:

$$r_{K_{o,i},K_{o,j}} = \frac{R_{K_{o,i},K_{o,j}}}{s_{K_{o,i}}\cdot s_{K_{o,j}}} = \frac{\sum_{\chi=1}^{\eta}\left(K_{o,i_\chi} - \bar{K}_{o,i}\right)\cdot\left(K_{o,j_\chi} - \bar{K}_{o,j}\right)}{\sqrt{\sum_{\chi=1}^{\eta}\left(K_{o,i_\chi} - \bar{K}_{o,i}\right)^2\cdot\sum_{\chi=1}^{\eta}\left(K_{o,j_\chi} - \bar{K}_{o,j}\right)^2}} \quad (7)$$

where $K_{o,i}$ and $K_{o,j}$ - indexes $K_{o,i}$ and $K_{o,j}$ measurements results which under normal distribution of measuring experiments results K_{o,i_χ} and K_{o,j_χ} (χ = 1,2,…, η) can be calculated as medium arithmetic values of corresponding samples:

$$\bar{K}_{o,i} = \frac{1}{\eta}\sum_{\chi=1}^{\eta} K_{o,i_\chi} \text{ and } \bar{K}_{o,j} = \frac{1}{\eta}\sum_{\chi=1}^{\eta} K_{o,j_\chi}. \quad (8)$$

On the basis of obtained deviations $\Delta\Pi$ the scale of product Q quality level determination can be build, and as result bigger numerical quality Q level value will correspond for higher product quality:

$$Q = 1 - \Delta\Pi \text{ or } Q = (1 - \Delta\Pi)\cdot 100\%. \quad (9)$$

So product Q quality level value that was determined by elaborated methodology can change from 0 to 1 or from 0 to 100% that is useful and methodologically grounded for usage in product quality assessment practice. In addition to product quality assessment the proposed method on the basis of obtained deviation function $\Delta\Pi$ values and product Q quality level enables to provide its sorting by the quality level and to fix different prices on it accordingly.

4. The assessment methodology of measurement accuracy of the product quality level on the basis of the uncertainty concept

4.1. The conceptual principles of the assessment methodology of measurement accuracy of product quality level with the use of virtual quality measure

Product quality level accuracy assessment with virtual quality measure usage can be carried out by means of the uncertainty assessment of obtained measurement result, in other words, by the studied product Q quality level. The premise is that result accuracy primary assessment x of any measurement is its standard uncertainty $u(x)$ [1].

As it is clear from (9) the standard uncertainty $u(Q)$ of obtained product Q quality level value is specified by the uncertainty $u(\Delta\Pi)$ of deviation function $\Delta\Pi$ value, in other words $u(Q)$=$u(\Delta\Pi)$. To assess product quality level accuracy determination let consider the method of deviation function $\Delta\Pi$ uncertainty $u(\Delta\Pi)$ values determination. For analysis usability the deviation function formula (5) should be written as follows:

$$\Pi = \left(\Delta\Pi\right)^2 = \sum_{i=1}^{n} \mathrm{M}_i\cdot\mathrm{K}_i, \quad (10)$$

$$\mathrm{M}_i = m_i^2;\ \mathrm{K}_i = \left(K_{o,i} - K_{b,i}\right)^2 = \left(\Delta K_i\right)^2;\ \Delta K_i = K_{o,i} - K_{b,i}. \quad (11)$$

The analysis of uncertainty determination method of parameter Π value should be provided basing on the next opinions:

- normalized ponderability coefficients m_i are constant and non-correlated values; accordingly parameters $M_i$are constant and non-correlated;
- single basic relative product quality $K_{b,i}$ indexes are constant and non-correlated values;
- single estimated relative product quality $K_{b,i}$ indexes are random correlated values and their values depend on single estimated absolute product $P_{o,i}$ quality values measured during the experiment;

- the values of estimated and basic product quality indexes difference $\Delta K_i = K_{o,i} - K_{b,i}$ are random correlated values;
- the analysis can be provided in terms of one-time measurements of all single estimated absolute product $P_{o,i}$ quality values, in other words the measurement results uncertainty assessment should be determined only by type **B**.

4.2. The uncertainty finding of the obtained measurement result of product Q quality level

Taking into account the mentioned above combined standard uncertainty by type **B** $u_{cB}(\Pi)$ parameter Π value can be determined in the next formula:

$$u_{cB}(\Pi) = \sqrt{\sum_{i=1}^{n} C^2_{\mathrm{M}_i} \cdot u^2_B(\mathrm{M}_i) + \sum_{i=1}^{n} C^2_{\mathrm{K}_i} \cdot u^2_B(\mathrm{K}_i)} \quad , \qquad (12)$$

Where: $C_{\mathrm{M}_i} = \frac{\partial \Pi}{\partial \mathrm{M}_i} = \mathrm{K}_i = (\hat{E}_{o,i} - \hat{E}_{b,i})^2$ - the uncertainty $u_B(\mathrm{M}_i)$ impact coefficient of the parameter M_i value on the uncertainty $u_{cB}(\Pi)$ of the parameter Π value; $u_B(\mathrm{M}_i) = \frac{\partial \mathrm{M}_i}{\partial m_i} \cdot u_B(m_i) = 2m_i \cdot u_B(m_i)$ - the standard uncertainty by type **B** of the parameter M_i value; $u_B(m_i)$ - the standard uncertainty by type **B** of the normalized ponderability coefficient m_i value; $C_{\mathrm{K}_i} = \frac{\partial \Pi}{\partial \mathrm{K}_i} = \mathrm{M}_i = m_i^2$ - the uncertainty $u_B(\mathrm{K}_i)$ impact coefficient of the parameter K_i value on the uncertainty $u_{cB}(\Pi)$ of the parameter Π value; $u_{cB}(\mathrm{K}_i) = \frac{\partial \mathrm{K}_i}{\partial(\Delta K_i)} \cdot u_{cB}(\Delta K_i) = 2\Delta K_i \cdot u_{cB}(\Delta K_i)$ - the combined standard uncertainty by the type **B** of the parameter K_i value; $u_{cB}(\Delta K_i)$ - the combined standard uncertainty by the type **B** of the estimated and basic product quality indexes difference $\Delta K_i = K_{o,i} - K_{b,i}$.

It should be taken into account that single estimated relative product $K_{o,i}$ quality indexes are random correlated values. Accordingly random correlated values are the estimated and basic product quality indexes difference $\Delta K_i = K_{o,i} - K_{b,i}$. Then the standard uncertainty $u_{cB}(\Delta K_i)$ by the type **B** of the differences ΔK_i value can be found in the next formula:

$$u_{cB}(\Delta K_i) = \sqrt{\sum_{i=1}^{n} u^2_B(K_{o,i}) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} u_B(K_{o,i}) \cdot u_B(K_{o,j}) \cdot r_{ij}} \quad , (13)$$

Where: $u_B(K_{o,i})$ - the standard uncertainty by the type **B** of the single estimated relative product $K_{o,i}$ quality index; r_{ij} - correlation coefficient between values of the single estimated relative product $K_{o,i}$, $i = 1,2,3,\ldots,n$ quality indexes.

Deviation $\Delta\Pi$ function in the accordance with (10) is linked to the parameter Π of the ratio $\Delta\Pi = \sqrt{\Pi}$. Then the combined uncertainty $u_{cB}(\Delta\Pi)$ value by the type **B** and the combined uncertainty $u_{cB}(Q)$ value by the type **B** of studied product Q quality level value can be determined by the next formula:

$$u_{cB}(Q) = u_{cB}(\Delta\Pi) = \frac{\partial(\Delta\Pi)}{\partial \Pi} \cdot u_{cB}(\Pi) = \frac{1}{2\sqrt{\Pi}} \cdot u_{cB}(\Pi) \, , \quad (14)$$

where $u_{\tilde{n}B}(\Pi)$ - the combined standard uncertainty by the type **B** of the parameter Π value that was determined by the formula (12).

Then to determine expended uncertainty $U_p(Q)$ of the studied product Q quality level value:

$$U_p(Q) = k_p \cdot u_{cB}(Q), \qquad (15)$$

where k_p – expansion (coverage) coefficient, the value of which depends on confidence level p and density of the product Q quality level possible values distribution.

5. Conclusions

1. It is advisable to organize the qualimetrical measurement quality assessment as a new metrology trend on the basis of the uncertainty concept that meets the modern world metrology development tendencies.

2. One of the main qualimetrical measurement methodology problems is the problem of the product quality measure theory. The virtual product quality measure that is formed on the basis of the basic product quality profile and it should be used to solve the mentioned above problem.

3. The qualimetrical measurements results accuracy assessment by means of their uncertainty determination, taking into account the correlation between single products quality indicators, provides the fulfillment of the qualimetrical measurements unity conditions.

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