A method of feature selection in the aspect of specific identification of radar signals

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Abstract. This article presents an important task of classification, i.e. mapping surfaces which separate patterns in feature space in the scope of radar emitter recognition (RER) and classification. Assigning a tested radar to a particular class is based on defining its location from the discriminating areas. In order to carry out the classification process, it is necessary to define metrics in the feature space as it is essential to estimate the distance of a classified radar from the centre of the class. The method presented in this article is based on extraction and selection of distinctive features, which can be received in the process of specific emitter identification (SEI) of radar signals, and on the minimum distance classification. The author suggests a RER system which consists of a few independent channels. The task of each channel is to calculate the distance of the tested radar from a given class and finally, set the correct identification coefficient for each recognized radar. Thus, a multi-channel system with independent distance measurement is obtained, which makes it possible to recognize particular radar copies. This system is implemented in electronic intelligence (ELINT) system and tested in real battlefield conditions.

Key words: radar emitter recognition (RER), specific emitter identification (SEI), minimum distance classification, ELINT system.

1. Introduction

In the theory of radar signals recognition and their emission sources, the main task is to set distinctive patterns of such signals and work out methods for their distinguishing [1, 2]. In the literature, terms such as source emission patterns separating surfaces in the measurable features space are widely used to describe pattern recognition and pattern classification [3, 4, 5]. Taking into account the fact above, radar emitter recognition and classification is based on defining the location of the emission source from the above separating surfaces. Here, it is essential to indicate a very significant fact, namely, in order to set a distinctive radar signal pattern, radar metrics needs to be defined in the measurable feature space of this signal; only after that the pattern vector of such a signal in the database can be designed [1, 6, 7]. However, what needs to be taken into account is the specificity of such a database for ELINT systems and electronic warfare (EW), [8, 9, 10]. The specificity of such a database is not the subject of the content of this article. Radar metrics is based on a distinctive radar signal pattern used to calculate the distance of this pattern from the centre of the class. Classification in this case is based on calculation of the distance of the recognized radar pattern from the centre of the class and classifying it to the class with the smallest distance. In order for this problem of radar signal classification to be solvable, it is necessary to define the class of a radar. Defining classes is not only setting the expected average values of basic measurable parameters of a radar signal, i.e., radio frequency (RF), pulse width (PW) and pulse repetition interval (PRI). This process also requires defining variance and correlation of these features [11]. The second significant element of the classification pro-

cedure is the problem to define how to estimate the distance of a tested radar emitter signal from the centre of the class taking into consideration variance and correlation of vector's features. The RER method also provides a solution when the features of radar patterns are not linearly separable. RER method bases on the analysis of basic measurable parameters of the radar signal (such as RF, PW, PRI) as result of which it is possible to extract additional distinctive features. The RER process is called specific emitter identification (SEI).

Additionally extracted distinctive features, which are received in the process of RER, may be a product of out-of-band radiation of radar devices [12]. These features may be of fractal type, which is presented in the works [13, 14, 15]. The received features may also be a product of inter-pulse modulation [16] and intrapulse analysis of a radar signal [17]. Of course, there are more complicated approaches, which offer effective methods for solving the classification task (i.e. mapping separating surfaces). These are based on solving the linear approximation task recurrently, using gradient methods and nonlinear approximation [18], nonlinear approximation of random function [19] and other methods for adaptive regression splines, classification and approximation [20, 21]. This is a typical solution for identification systems such as perceptrons or artificial neural network (ANN), e.g., support vector machine networks (SVM) [22] using Widrow-Hoff learning algorithms, Adaline ANN or the method based on back-propagating errors and neural network classifier with low discrepancy optimization [23, 24]. Also, the Fourier transform has been widely used in radar signal and image processing. Another work [25] presents joint time-frequency (JTF) domain analysis as a useful tool for improving radar signal and image processing for time-and frequency-varying cases. In [26], quadratic time-frequency analysis and sequential recognition for SEI is presented. Another approach based on three-dimensional distribution feature is

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presented in [27]. Radar emitter signal identification based on hierarchical agglomerative clustering is analyzed in [28]. Vector neural network (VNN) with a supervised learning algorithm suitable for signal classification is very useful for emitter identification and radar classification process, as shown in [29, 30].

2. Multi-channel recognition system with an independent distance measurement

During the measurement procedure, radar signals in the form of basic vector parameters $\mathbf{V}_B$ are defined according to (1), where n_w is the number of pulses in samples which are classified to the analysis and k is the number of pulses in a measurement sample. As a general rule, in one sample there are 5000 pulses while a superheterodyne ELINT receiver is used to record a package of radar signals [12].

$$\mathbf{V}_B = \left[\frac{1}{n_w} \sum_{k=1}^{n_w} \mathrm{RF}(k), \ \frac{1}{n_w} \sum_{k=1}^{n_w} \mathrm{PW}(k), \ \frac{1}{n_w} \sum_{k=1}^{n_w} \mathrm{PRI}(k) \right] \quad (1)$$

The suggested RER system consists of independent channels. Each of the channels is assigned to one class of the recognized radars. The main task of every single channel is to estimate the distance of a tested radar signal from its class. The distance of the tested radar signal (defined by features' values of its basic measurable parameters) $\mathbf{V}_B \in R^n$ from a particular class with i index marked as $d_i(\mathbf{V}_B)$. It should be mentioned that it is the value estimated as a distance from "$\mathbf{V}_B$ point" to the centre of i-class. As a result of the procedure above, the measurement of the distance received from channels is the base for recognition. The radar is recognized as belonging to a particular class with w index, if $d_w(\mathbf{u}) = \min\{d_1(\mathbf{V}_{B1}), d_2(\mathbf{V}_{B2})\}$. A significant problem which comes out in the method above concerns the rule of choosing a decision, which is based on defining values of different $d_i(\mathbf{V}_B)$ functionals. This is a problem which concerns mathematical reasoning [31]. In order to solve this problem it is assumed that $\mathbf{V}_{Bk}$ is a random variable which defines features' values of elements of a radar class with k index and $\mathrm{E}d_j(\mathbf{V}_{Bk})$ is an expected value of the distance d_j for the radar class with k index.

3. Defining the distance on impartiality condition

To start with, a particular class G of radars is chosen for which the centre of the class $\overline{\mathbf{V}}_B = \mathrm{E}(\mathbf{V}_B)$ is defined and covariance matrix of the vector features $\mathbf{V}_B$ from G class, according to (2), where T is a sign of matrix transposition.

$$\mathbf{R} = \mathrm{E}(\mathbf{V}_B - \overline{\mathbf{V}}_B)(\mathbf{V}_B - \overline{\mathbf{V}}_B)^T \quad (2)$$

The eigenvalues denoted as λ_k and orthogonal eigenvectors denoted as $\mathbf{t}_k$ for each covariance matrix R are calculated according to (3) and (4).

$$\mathbf{R}\,\mathbf{t}_k = \lambda_k\,\mathbf{t}_k \quad where \quad k = 1,2,...,n \quad (3)$$

$$\mathbf{t}_i^T \mathbf{t}_j = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases} \quad (4)$$

In order to define the distance of $\mathbf{V}_B$ vector from G class, Karhunen-Loeve (K–L) transform with transformation matrix T are used. The lines of the T matrix are the R matrix's eigenvectors, according to (5), where for $\mathbf{V}_B$ argument there is a result in a form of a transformation vector $\mathbf{TV}_B$ according to (6).

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T \\ \mathbf{t}_2^T \\ ... \\ \mathbf{t}_n^T \end{bmatrix} \quad (5)$$

$$\mathbf{TV}_B = \begin{bmatrix} \mathbf{t}_1^T \mathbf{V}_B \\ \mathbf{t}_2^T \mathbf{V}_B \\ ... \\ \mathbf{t}_n^T \mathbf{V}_B \end{bmatrix} \quad (6)$$

Taking into consideration the fact that the base for estimation of the distance of $\mathbf{V}_B$ vector feature from G class is the difference $\mathbf{r} = \mathbf{V}_B - \overline{\mathbf{V}}_B$, the next step is to define K–L transform of the estimated difference, according to (7).

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_n \end{bmatrix} = \mathbf{Tr} = \begin{bmatrix} \mathbf{t}_1^T \mathbf{r} \\ \mathbf{t}_2^T \mathbf{r} \\ ... \\ \mathbf{t}_n^T \mathbf{r} \end{bmatrix} \quad (7)$$

The decorrelation process of the vector's features' co-ordinates according to (8) and the normalization process of dispersion of co-ordinates' values according to (9) results in a correlation matrix equal to an identity matrix according to (10).

$$\mathrm{E}(\mathbf{yy}^T) = \mathrm{E}[\mathbf{Tr}][\mathbf{Tr}]^T = \mathbf{T}\mathrm{E}(\mathbf{rr}^T)\mathbf{T}^T = \mathbf{TRT}^T =$$

$$= \begin{bmatrix} \mathbf{t}_1^T \\ \mathbf{t}_2^T \\ ... \\ \mathbf{t}_n^T \end{bmatrix} \mathbf{R}[\mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_n] = \begin{bmatrix} \mathbf{t}_1^T \\ \mathbf{t}_2^T \\ ... \\ \mathbf{t}_n^T \end{bmatrix} \cdot [\mathbf{Rt}_1, \mathbf{Rt}_2, ..., \mathbf{Rt}_n] =$$

$$= \begin{bmatrix} \lambda_1 \mathbf{t}_1^T \mathbf{t}_1 & \lambda_2 \mathbf{t}_1^T \mathbf{t}_2 & ... & \lambda_n \mathbf{t}_1^T \mathbf{t}_n \\ \lambda_1 \mathbf{t}_2^T \mathbf{t}_1 & \lambda_2 \mathbf{t}_2^T \mathbf{t}_2 & ... & \lambda_n \mathbf{t}_2^T \mathbf{t}_n \\ ... & ... & ... & ... \\ \lambda_1 \mathbf{t}_n^T \mathbf{t}_1 & \lambda_2 \mathbf{t}_n^T \mathbf{t}_2 & .. & \lambda_n \mathbf{t}_n^T \mathbf{t}_n \end{bmatrix} = \quad (8)$$

$$= diag(\lambda_1, \lambda_2, ..., \lambda_n) = \mathbf{\Lambda}$$

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_n \end{bmatrix} = \begin{bmatrix} \dfrac{y_1}{\sqrt{\lambda_1}} \\ \dfrac{y_2}{\sqrt{\lambda_2}} \\ \dots \\ \dfrac{y_n}{\sqrt{\lambda_n}} \end{bmatrix} \tag{9}$$

$$\mathrm{E}(\mathbf{z}\mathbf{z}') = E \begin{bmatrix} z_1 z_1 & z_1 z_2 & \dots & z_1 z_n \\ z_2 z_1 & z_2 z_2 & \dots & z_2 z_n \\ \dots & \dots & \dots & \dots \\ z_n z_1 & z_n z_2 & \dots & z_n z_n \end{bmatrix} =$$

$$= \mathrm{E} \begin{bmatrix} \dfrac{y_1 y_1}{\lambda_1} & \dfrac{y_1 y_2}{\sqrt{\lambda_1 \lambda_2}} & \dots & \dfrac{y_1 y_n}{\sqrt{\lambda_1 \lambda_n}} \\ \dfrac{y_2 y_2}{\sqrt{\lambda_2 \lambda_1}} & \dfrac{y_2 y_2}{\lambda_2} & \dots & \dfrac{y_2 y_n}{\sqrt{\lambda_2 \lambda_n}} \\ \dots & \dots & \dots & \dots \\ \dfrac{y_n y_2}{\sqrt{\lambda_n \lambda_1}} & \dfrac{y_n y_2}{\sqrt{\lambda_n \lambda_2}} & \dots & \dfrac{y_n y_n}{\lambda_n} \end{bmatrix} = diag(1,1,\dots,1) \tag{10}$$

The $d(\mathbf{V_B})$ distance of $\mathbf{V_B}$ vector from G class is defined according to (11), where $d(\mathbf{V_B})$ functional defines the distance from $\mathbf{V_B}$ features' vector of a tested radar to the $\overline{\mathbf{V}}_B$ centre as a Mahalanobis distance.

$$d(\mathbf{V_B}) = \mathbf{z}^T \mathbf{z} = \sum_{k=1}^{n} z_k^2 = \sum_{k=1}^{n} \frac{y_k^2}{\lambda_k} = \sum_{k=1}^{n} \frac{(\mathbf{t}_k^T \mathbf{r})^2}{\lambda_k} =$$

$$= \sum_{k=1}^{n} (\mathbf{r}^T \mathbf{t}_k) \frac{1}{\lambda_k} (\mathbf{t}_k^T \mathbf{r}) =$$

$$= \mathbf{r}^T [\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_n] \begin{bmatrix} \lambda_1^{-1} & 0 & \dots & 0 \\ 0 & \lambda_2^{-1} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_n^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{t}_1^T \\ \mathbf{t}_2^T \\ \dots \\ \mathbf{t}_n^T \end{bmatrix} \mathbf{r} = \tag{11}$$

$$= (\mathbf{V_B} - \overline{\mathbf{V}}_B)^T \mathbf{T}^T \mathbf{\Lambda}^{-1} \mathbf{T} (\mathbf{V_B} - \overline{\mathbf{V}}_B) = (\mathbf{V_B} - \overline{\mathbf{V}}_B)^T \mathbf{R}^{-1} (\mathbf{V_B} - \overline{\mathbf{V}}_B)$$

As a result of averaging out the received functional according to (11), $Ed(\mathbf{u})$ value is received according to (12).

$$\mathrm{E}d(\mathbf{V_B}) = \mathrm{E}[(\mathbf{V_B} - \overline{\mathbf{V}}_B)^T \mathbf{R}^{-1} (\mathbf{V_B} - \overline{\mathbf{V}}_B)] = \mathrm{E}[\mathbf{r}^T \mathbf{T}^T \mathbf{\Lambda}^{-1} \mathbf{T} \mathbf{r}] =$$

$$= \mathrm{E}[y^T \mathbf{\Lambda}^{-1} y] = \mathrm{E}\left\{ \mathbf{y}^T \begin{bmatrix} \lambda_1^{-1} & 0 & \dots & 0 \\ 0 & \lambda_2^{-1} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_n^{-1} \end{bmatrix} \mathbf{y} \right\} = \tag{12}$$

$$= \mathrm{E} \sum_{k=1}^{n} \frac{y_k^2}{\lambda_k} = \sum_{k=1}^{n} \frac{\mathrm{E} y_k^2}{\lambda_k} = n$$

The analysis above makes it possible to use the definition of distance $d_i(\mathbf{V_B})$ for the channel with index $i \in \{1, 2, \dots, L\}$ according to (13), where $\overline{\mathbf{V}}_{Bi}$ is the expected value of $\mathbf{G}_i$ features' class vector, while $\mathbf{R}_i = \mathrm{E}(\mathbf{V}_{Bi} - \overline{\mathbf{V}}_{Bi})(\mathbf{V}_{Bi} - \overline{\mathbf{V}}_{Bi})^T$ is the covariance matrix of $\mathbf{G}_i$ class. It is also necessary to mention that d_i functionals, which are defined by (14), where $\mathbf{V}_{Bi}$ is a random variable which defines the values of $\mathbf{G}_i$ class elements, fulfill the impartiality condition $\mathrm{E}d_1(\mathbf{V}_{B1}) = \mathrm{E}d_2(\mathbf{V}_{B2})$.

$$d_i(\mathbf{V_B}) = (\mathbf{V_B} - \overline{\mathbf{V}}_{Bi})^T \mathbf{R}_i^{-1} (\mathbf{V_B} - \overline{\mathbf{V}}_{Bi}) \tag{13}$$

$$\mathbf{E}d_i(\mathbf{V}_{Bi}) = n \qquad i \in \{1, 2, \dots, L\} \tag{14}$$

In spite of $\mathbf{R}_i$ parameter diversity, the values of the distance estimated according to (13) can be comparable and are the base to define the class of the nearest recognized radar emitter signal.

4. The process of learning

The presented radar recognition process is supposed to be preceded by the process of learning. This process aims at estimation of the expected value of class patterns according to (15), estimation of covariance matrix of classes' patterns according to (16) for each of the collections $\mathbf{G}_i$ class patterns and defining eigenvalues of covariance matrix $\mathbf{R}_i$ and belonging to them, orthogonal eigenvectors according to (17, 18).

$$\overline{\mathbf{V}}_{Bi} = \frac{1}{\|G_i\|} \sum_{\mathbf{x} \in G_i} \mathbf{x} \qquad i \in \{1, 2, \dots, L\} \tag{15}$$

$$\mathbf{R}_i = \frac{1}{\|G_i\|} \sum_{\mathbf{x} \in G_i} (\mathbf{x} - \overline{\mathbf{V}}_{Bi})(\mathbf{x} - \overline{\mathbf{V}}_{Bi})^T \qquad i \in \{1, 2, \dots, L\} \tag{16}$$

$$\mathbf{R}_i \mathbf{t}_k^{(i)} = \lambda_k^{(i)} \mathbf{t}_k^{(i)} \qquad k \in \{1, 2, \dots, n\} \tag{17}$$

$$\mathbf{T}_i = \begin{bmatrix} (\mathbf{t}_1^{(i)})^T \\ (\mathbf{t}_2^{(i)})^T \\ \dots \\ (\mathbf{t}_n^{(i)})^T \end{bmatrix} \qquad i \in \{1, 2, \dots, L\} \tag{18}$$

The analysis requires the assumption that covariance matrices might be nonsingular. Sometimes tests show that covariance matrix can be singular. As a result, it is not possible to normalize coordinates of K–L transform according to (8). Thus, there is a need to normalize coordinates of K–L transform according to (19).

$$\mathbf{z}_k = \begin{cases} \dfrac{y_k}{\sqrt{\lambda_k}} & \text{if} \quad \lambda_k \neq 0 \\ 0 & \text{if} \quad \lambda_k = 0 \end{cases} \tag{19}$$

This causes that $d(\mathbf{V_B})$ distance of the $\mathbf{V_B}$ vector from G class, defined according to (11), is modified to the form of (20), where p is the number of non-zero eigenvalues of co-

variance matrix, $\mathbf{R}^+$ is Moore-Penrose inverse of covariance matrix, according to the relation $\mathbf{R}^+ = \mathbf{T}^T\mathbf{MT}$, where $\mathbf{M}$ is the matrix with (21).

$$
\begin{aligned}
d(\mathbf{V}_B) = \mathbf{z}^T\mathbf{z} &= \sum_{k=1}^{p} z_k^2 = \sum_{k=1}^{p} \frac{y_k^2}{\lambda_k} = \sum_{k=1}^{p} \frac{(\mathbf{t}_k^T\mathbf{r})^2}{\lambda_k} = \\
&= \sum_{k=1}^{p} (\mathbf{r}^T\mathbf{t}_k)\frac{1}{\lambda_k}(\mathbf{t}_k^T\mathbf{r}) = \\
&= \mathbf{r}^T[\mathbf{t}_1...\mathbf{t}_p...\mathbf{t}_n]
\begin{bmatrix}
\lambda_1^{-1} & 0 & ... & 0 \\
0 & \lambda_p^{-1} & ... & 0 \\
... & ... & ... & ... \\
0 & 0 & ... & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{t}_1^T \\
\mathbf{t}_p^T \\
... \\
\mathbf{t}_n^T
\end{bmatrix}
\mathbf{r} = \\
&= (\mathbf{V}_B - \overline{\mathbf{V}}_B)^T \mathbf{T}^T\mathbf{MT}(\mathbf{V}_B - \overline{\mathbf{V}}_B) = \\
&= (\mathbf{V}_B - \overline{\mathbf{V}}_B)^T \mathbf{R}^+(\mathbf{V}_B - \overline{\mathbf{V}}_B)
\end{aligned}
\tag{20}
$$

$$
\mathbf{M} =
\begin{bmatrix}
\lambda_1^{-1} & 0 & ... & 0 \\
0 & \lambda_p^{-1} & ... & 0 \\
... & ... & ... & ... \\
0 & 0 & ... & 0
\end{bmatrix}
\tag{21}
$$

The analysis above makes it possible to create a scheme of recognition system with the use of K–L transform, which is presented in Fig. 1.
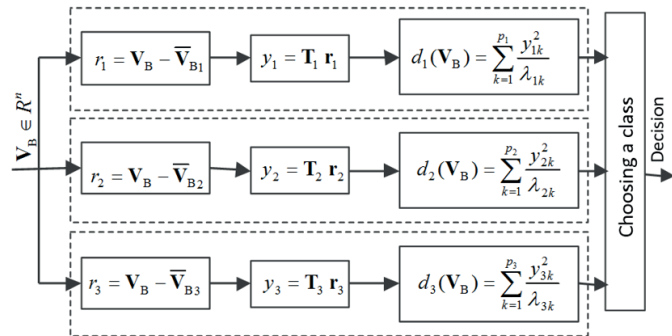

Fig. 1. A scheme of a triple-channel RER system based on K–L transform

5. Calculation results and received RER results

The defined distance $d_i(\mathbf{V}_B)$ for particular recognition channels with a coefficient $i \in \{1, 2, ..., L\}$ according to (13) and modification of this distance according to (20), is a base for classification of the tested radar to an appropriate class. In order to compare the received results of RER method with other methods described in works [7, 12, 13, 15–17, 32], correct identification coefficient (CIC) is set according to (22), where n_{B-P} is the

number of correct comparisons of basic features' vectors $\mathbf{V}_B$ to pattern vectors $\mathbf{V}_P$ in a particular class, and N is the number of all comparisons divided by the number of test collections.

$$
\text{CIC} = \frac{n_{B-P}}{N}
\tag{22}
$$

The number of n_{B-P} correct comparisons is set according to (23), where γ_i^j function assigns to a pair of vectors $(\mathbf{V}_B^i, \mathbf{V}_P^j)$ the value which equals '1' if i = j, or the value which equals '0' if i ≠ j. The received CIC values are presented in Table 4.

$$
n_{B-P} = \sum_{i=1}^{I}\sum_{j=1}^{J} \gamma^{ij}
\tag{23}
$$

During the test, the total comes to a few hundreds of radar signals recorded in collections with the length of 5000 pulses. A superheterodyne ELINT receiver was used in the measuring procedure. This receiver makes it possible to define value of RF with measurement accuracy 0.5 MHz and value of PRI in the scope from 2 µs to 20 ms, with measurement accuracy 0.05 µs. The recordings came from a dozen or so radars of the same type. Three copies were chosen to further analysis and their measurable parameters were as follows: RF = 1415 MHz, PW = 11.2 µs and the following PRI values i.e. PRI_{min} = 1400.96 µs, PRI_{ave} = 1745.93 µs and PRI_{max} = 2090.91 µs. The pulse repetition interval value (measured between the arrival times of two consecutive pulses on the level of the detection threshold) changes sequentially every pair of pulses, taking seven different PRI values. The complete cycle of value changes of PRI was as follows: PRI_1 = 2091 µs, PRI_2 = 1738 µs, PRI_3 = 1575 µs, PRI_4 = 1491 µs, PRI_5 = 1447 µs, PRI_6 = 1420 µs and PRI_7 = 1401 µs. This is a complete cycle of value changes of the PRI in a measurement sample. The method by which it was possible to receive a complete cycle of changes of PRI values is described in the works [12, 28]. As a result of initial measurement data processing, $\mathbf{V}_B$ vectors are received according to (1). On the basis of the received $\mathbf{V}_B$ vectors, pattern vectors $\mathbf{V}_P$ are also received. The method which is also used in this process is called the Holdout method. This method divides the available collection into two disjoint sets i.e. a set which is used to learn the classificator and a set to test the classificator. The usual division of the available data collection is as follows: 2/3 of available data is in the learning set and 1/3 is in the test set [12]. The identification process is carried out on the basis of the distance measurement with the use of the minimum-distance classification, according to the presented method. In order to assess the quality of the classification process the CIC index is defined. In the final process of identification, fast-decision identification algorithm (FDIA) of radar emission source is used in relational ELINT database (DB). The FDIA algorithm mentioned above is a three-stage parametrized by implementation of three different ways to define the similarity degree of the signal vector to the pattern in the database and by the possibility to define three-stage value of decision function. As a result this algorithm working in the classification process

of radar types, the following CIC values are received i.e.: 88% and 92% for three different values of decision function. In case of identification of copies, the following CIC values are received: 26%, 47% and 63%. The mechanism of FDIA algorithm and the received identification results are described in [7] in detail.

The received estimation results are presented in Tables 1–3 and illustrated in Figs. 2–4. Appropriately crossed columns and lines of each V_B vector and radar patterns in the form of V_P vectors, present the degree of their similarity defined by the distance value. The less value of this distance means the bigger similarity of V_B vector to the pattern. Also, in Figs. 2–4 there are minimum values of the distance $d_i(V_B)$ marked with a red dotted ellipse in each G_i class. The received minimum

Table 1

The results of distance calculations for a radar copy No. 1 in L1 channel for 16 comparisons in G1 class

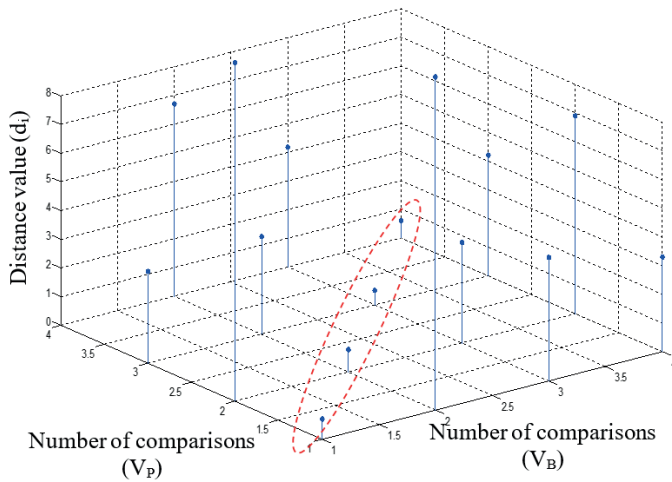| $d_1(V_B-V_P)$	V_B (for G_1 class)			
V_P (for G_1 class)	0,714	11,647	4,302	3,374
	11,821	0,798	3,567	6,995
	3,267	3,414	0,516	4,273
	5,278	6,728	4,178	0,612

Fig. 2. 3D graphic illustration of the received values of distances in L1 channel for 16 comparisons of the radar copy No. 1 in G1 class

Table 2

The results of distance calculations for a radar copy No. 2 in L2 channel for 16 comparisons in G2 class

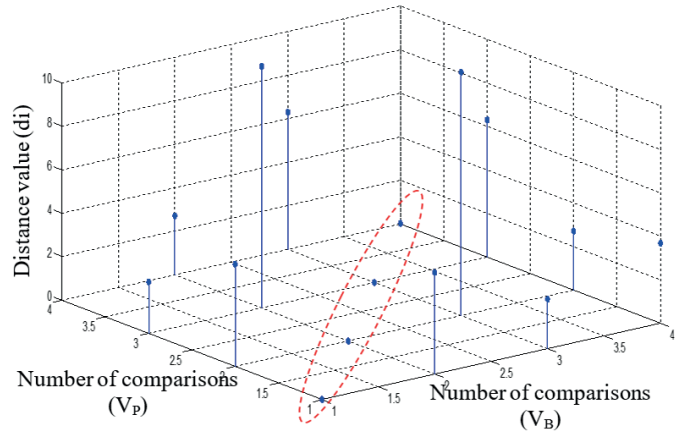| $d_2(V_B-V_P)$	V_B (for G_2 class)			
V_P (for G_2 class)	0,001	4,717	2,391	3,793
	4,732	0,022	11,207	2,721
	2,389	11,197	0,01	6,322
	4,796	2,717	6,329	0,008

Fig. 3. 3D graphic illustration of the received distance values in L2 channel for 16 comparisons of the radar copy No. 2 in G2 class

Table 3

The results of distance calculations for a radar copy No. 3 in L3 channel for 16 comparisons in G3 class

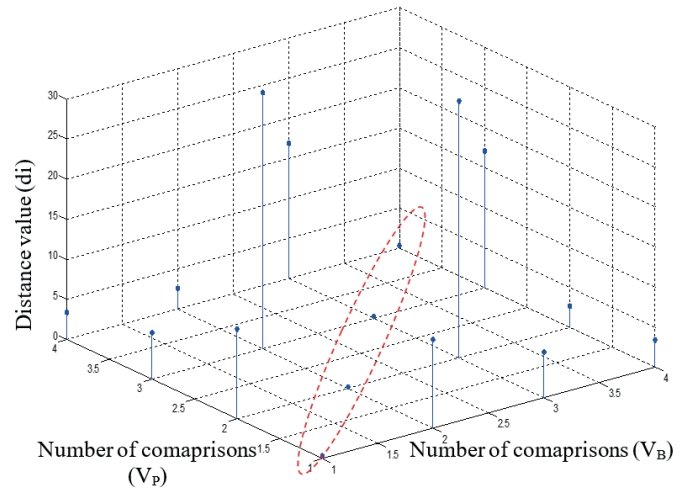| $d_3(V_B-V_P)$	V_B (for G_3 class)			
V_P (for G_3 class)	0,349	11,105	5,705	3,401
	11,258	0,217	32,061	2,715
	5,804	32,012	0,249	17,058
	3,432	2,617	16,994	0,315

Fig. 4. 3D Graphic illustration of the received distance values in L3 channel for 16 comparisons of the radar copy No. 3 in G3 class

Table 4

CIC values of RER method for three recognized radar copies in G1÷G3 classes

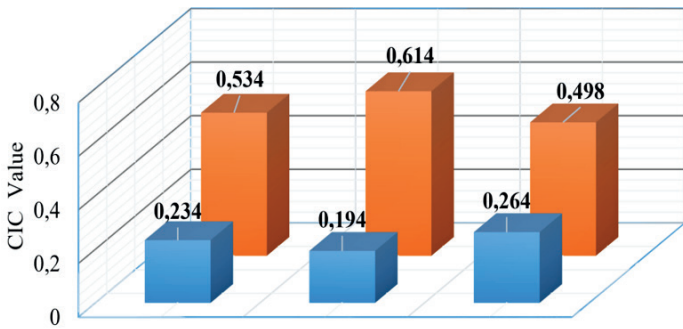| CIC_{RER}	G_1 class	G_2 class	G_3 class
V_B	0,234	0,194	0,264
V_P	0,534	0,614	0,498

Fig. 5. Graphic illustration of CIC calculations for three radar copies in given classes G1÷G3 of triple-channel RER system

distances show at the same time the number of correct comparisons of basic features' vectors to pattern vectors in the class. In each class ($G_1 \div G_3$) 16 comparisons of these vectors are carried out and CIC is calculated. The results are presented in Table 4 and illustrated graphically in Fig. 5. According to the SEI methods listed in this section, the received RER results are as follows: the use of out-of-band radiation described in the work [12] the CIC value for RER is about 90%. The method based on fractal features described in the work [15] the CIC value is 91.6% for Mahalanobis metric and 96.7% for Euclidean and Hamming metrics. Very similar RER results are received in the work [13], where RER also bases on the analysis of fractal features. The method based on inter-pulse analysis described in the work [16] increases the CIC coefficient up to 70%, and the method based on intrapulse analysis described in the work [17] makes it possible to receive RER results reaching 90% level. Data modeling applied to RER and identification is presented in the work [32]. In this work the value of CIC equals 98%. In the work [7], the fast identification algorithm for RER is presented. This algorithm is parametrized in three stages by implementation of three different ways to define the similarity degree of the signal vector to the pattern in the database. Based on this algorithm, the CIC value is 63%.

In order to depict it clearly, CIC values received in other RER methods which appeared in this section are presented in Fig. 6.

It needs to be emphasized with full conviction that referring to the works above, during the RER procedure, the same recordings of a few hundred radar signals coming from the same type of radars are used. Only by this approach it is possible to compare the received results. It needs to be emphasized that the methods listed above, differ from each other as concerns the test procedure, the compilation level, calculation time and algorithm complexity. However, the main difference is that in the process of generating distinctive features, it is possible to achieve different distinctive features from a radar signal. In that way, a quasi-optimum radar signal pattern appeared.

6. Conclusion

The received measurement data have a significant influence on the specific identification process of radar, in which it is aimed to receive very high level of radar emitter signal identification. Ultimately, signal source identification which is 100%, should be characterized by the maximization of explicitness of RER procedure. It is not a trivial matter to achieve such a result. It is also known that stochastic context-free grammars (SCFG) appear promising for the recognition and threat assessment of complex radar emitters in radar systems, but the computational requirements for learning their production rule probabilities can be very onerous [33]. As shown in [34], a self-organizing map and the maximum likelihood gamma mixture model classifier and adopted Bayesian formalism are too complicated for direct analytical use in automatic radar recognition. The presented RER method is realized on the basis of MatLab software package and received vectors are recorded in a dedicated database for ELINT system. The received CIC value indicates that there has been a noticeable rise in the radar signal correct identification. Comparing the received results of the identification process with other methods, it may be admitted that
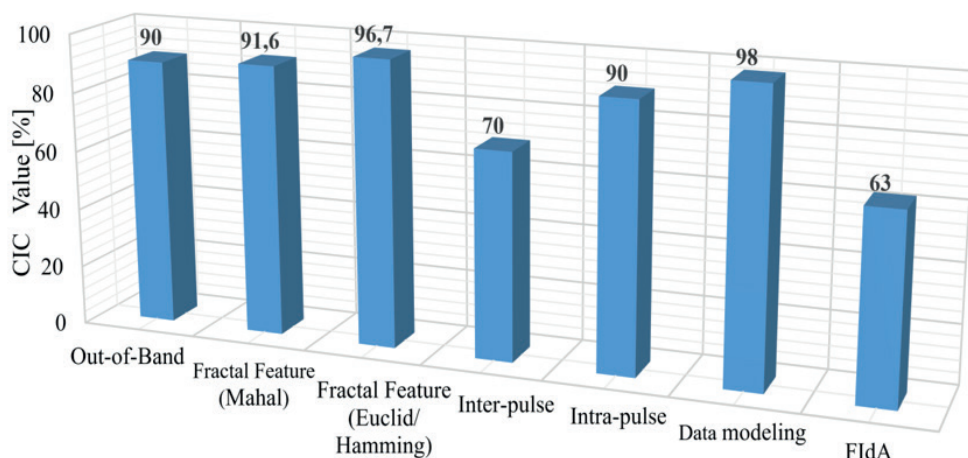

Fig. 6. Graphic illustration of CIC values for other RER methods

the presented RER method based on multi-channel recognition system makes it possible to receive the CIC coefficient which equals 61.4%. In order to increase the CIC coefficient value, in further works on RER use in SEI process, a common similarity matrix will be defined. This matrix will include the complexity of algorithms which are used in the RER method, estimation time, the requirements of the equipment platform and other requirements, which are significant in the process of quality estimation of a particular method. Thus, it will be possible to count automatically the similarities between vectors of basic measurable parameters for different radar copies of the same type. For ELINT systems working in real conditions, on a contemporary battlefield the automation of RER process and explicit identification of every single emitter in real time (with minimum time burden) are currently primary challenges for ELINT specialists.

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