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ORIGINAL PAPER

OPTIMAL DECISION MAKING FOR EMPTY CONTAINER MANAGEMENT AT SEAPORT YARD

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ABSTRACT. Background: In global trade, shipping companies are forced to manage empty containers due to imbalances in international trade activities. For decision-makers, the problems require considering restrictions and an uncertain environment and repositioning or leasing the containers to satisfy the rapidly changing global demands regardless of the epidemic outbreak's impact on the seaport. The proposed approach can help decision-makers manage the empty container in port yards more effectively under market uncertainty by employing the Bellman optimality principle for the stochastic dynamic system.

Methods: A stochastic production planning model is employed to cope with uncertainty and unexpected events to ensure a robust management strategy. Ito's formula describes the dynamic model for solving a stochastic differential equation. This paper uses stochastic optimal control theory to deal with efficient empty container management at the port yard. The findings have revealed the effectiveness of the proposed framework, which will provide a decision-making support scheme for efficient port operations.

Results: The presented algorithm is realized by a novel approach, employing the Hamilton-Jacobi-Bellman (HJB) equation for optimal stochastic control problems. When comparing the model with and without uncertainty events, the gap is just about 0.04 %, proving the robustness of the proposed model. The results provide a decision support system for port managers when managing the empty container in the seaport yard.

Conclusions: The proposed model not only figures out the optimal ordering of empty containers for each cycle but also points out the optimal safety stock level. Using a stochastic optimization approach, decision-makers can implement a strategic management policy to optimize seaport operational costs under market disruptions.

Keywords: Empty container management; Decision making; Inventory model; Stochastic optimal control; Hamilton-Jacobi-Bellman equation

INTRODUCTION

Marine transportation is the backbone of international trade and the most cost-effective way to move large quantities of goods and materials worldwide. With an estimated 90% of globalized trade by volume being carried out by sea transport, the maritime containerization market was projected to reach approximately 160 million TEUs (Twenty-foot Equivalent Units) in 2021 [UNCTAD, 2021]. Due to the dramatic increase in consumer demand for goods, the seaport terminals are the center of busy operations to ensure greater global supply chain resilience. More and more cargo will be filled in empty containers, and then the containers are transported between a terminal and the customer’s location by trucks, trains, etc. Shipping companies always assume the responsibility for supplying freight containers to their customers. The inshore depot and the seaport yard are the two central locations where the empty container can be stored. The container management cost reached approximately 17 billion dollars [Boilé, 2006]. Besides, the global shipping industry estimated around 110 billion dollars per year on managing the containers (purchasing, leasing, maintenance, etc.). [Rodrigue, 2006]. Furthermore, the trade imbalance between each shipping route is the primary reason that causes the management of empty containers to become more intense. Table 1 illustrates the container volume for the major routes between 2020 and 2021 [UNCTAD,

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2021]. From Table 1, the number of containers on the route from Asia to North America and from Asia to Europe is outstanding and over 17 million TEUs, twice or three times larger than on other routes.

Table 1. Container volume on shipping routes over the period 2020-2021 (million TEUs)

Year	Routes			
	Asia – North America	**North America – Asia**	**Asia – Europe**	**Europe - Asia**
2020	20.6	6.9	16.9	7.2
2021	24.1	7.1	18.5	7.8

Recently, empty containers have been piled up in seaports worldwide due to the COVID-19 supply chain disruptions. The trade imbalance between the hub ports leads to the unpredictable stochastic demand for empty containers in each terminal. In particular, if the number is small, the port will lack empty containers, leading to time delays and higher shipping costs. The port will experience increased storage costs and congestion problems with many empty containers. The revenue or profit depends strongly on the container management policy. In addition, the stochastic environment is another crucial factor that makes container management more complex, in which unexpected events will impact maritime transportation and trade. The uncertainty includes frequent congestion or extreme weather that occurs at any stage of the supply chain. The recent pandemic is a potent risk to maritime transportation and trade. Discretions might negatively impact port operations, and cargo cannot be delivered to the consignee on time, resulting in a reliability problem for shipping enterprises. The management of empty containers has recently become surprisingly complicated and is one of the most challenging problems in maritime logistics. As a result, this issue has drawn significant attention from researchers who focus on mitigating supply chain disruptions efficiently and cost-effectively, discussed by [Abdelshafie et al., 2022]. The first group finds the shipping route that transports containers in the hinterland port or the seaport network. The second focuses on the number of empty containers from suppliers or other ports. The third group deals with a sub-problem or constraint under different decision-making strategies.

In the first group, some studies focus on reducing the transferability of empty containers between each terminal in ports by shipping enterprises. Depot-direct and street-turn policies introduced by [Jula et al., 2006] are two primary strategies for reallocating empty containers in the port network. As optimal tools for shipping lines to save management costs, the Inland-Depots-for-Empty-Containers (IDEC) system is employed to find storage nodes in the empty container supply chains attempting to decrease distance-related movement, fuel consumption and congestion in the port area proposed by [Mittal et al., 2013]. Otherwise, unloaded containers will be imported directly to ports to continue loading cargo, and this policy is known as the “street turn” introduced by [Furió et al., 2013]. To achieve the proposed policy, a substantial level of cooperation between each link in the supply chain network. On the one hand, the leasing industry exploded in the 1970s due to the economic benefit and flexibility tools to carriers, mainly because of the tremendous demand for empty containers investigated by [Theofanis et al., 2008]. The leasing container industry can be categorized into three main types: master lease, long-term lease, and short-term lease. Related to the final research group, long-term and short-term leasing decisions are compared to implement a realistic shipping network conducted by [Hu et al., 2020]. Each duty requires a similar and coincident time, type of cargo, shipping enterprise, and destination. In some cases, empty containers must be inspected (cleaning, repairing, etc.) before reusing them for the next cycle [Hoffmann et al., 2020]. Fig. 1 illustrates the generic container flows in the shipping network.

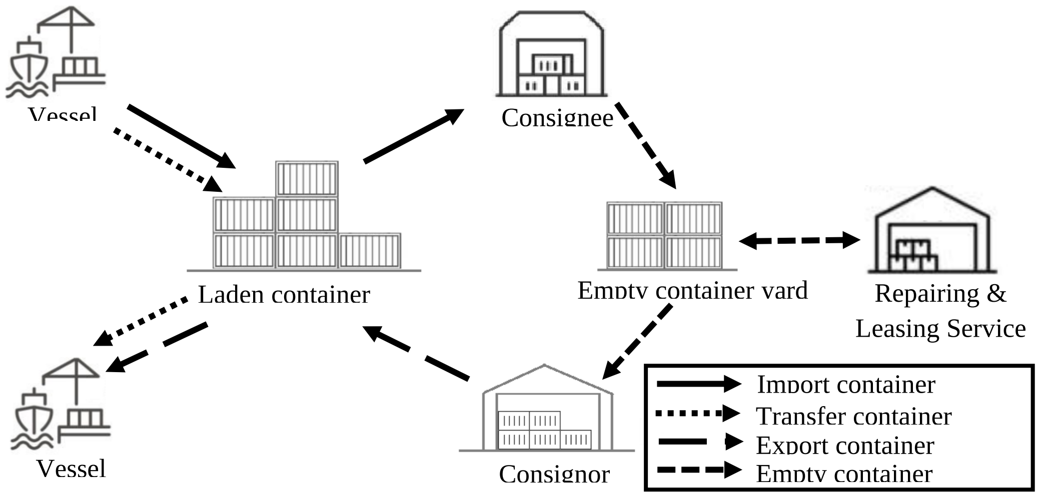

Fig. 1. Basic concepts of the generic container transport chain.

However, the studies focused on only one target for decision-making on the lease or repair and maintenance (R&M) of damaged containers without uncertainty. In addition, the latest articles deal with the deterministic scheme outnumbering compared to the stochastic approach. The deterministic models might be enough to realize optimal solutions for ideal cases without disruptions. [Di Francesco et al., 2009] proposed the deterministic mathematical model that examines multiple scenario policies' effect on empty container re-allocation. In contrast, uncertainty programming was offered to meet the time-dependent demand, supply, and capacity and minimize operation costs simultaneously presented by [Hosseini et al., 2018]. However, those studies may lack consideration if repairing and leasing costs are not routinely included in dynamic models. The liner shipping network problem has been introduced by [Shintani et al., 2007] for empty container repositioning. The study proposed a container shipping routing network using the Knapsack model approach with a Genetic Algorithm (GA) to optimize profit for the liner shipping enterprises. To extend the problem with the business flow approach, the two-layer collaborative optimization model was discussed by [Zhang et al., 2019]. This study addresses the decision-making problem by combining the tactical and operation layers. Within the framework of metaheuristic algorithms, [Belayachi et al., 2017] applied the heuristic method by neighborhood – Tabu Search (TS) to solve the transfer problem of empty containers from port to serve client demand. Particle Swarm Optimization (PSO) and GA are some of the most well-known techniques. There are also some combinations of inventory and mathematical programming models to solve complicated problems. In an inland transportation system, an inventory control problem was employed considering probabilistic demand and supply for the ordering and leasing policy proposed by [Yun et al., 2011].

Most existing studies are based on combining different models or optimization techniques to deal with empty container problems. Some studies introduced hybrid optimizations or enhanced traditional methods, which are inappropriate for a large-scale uncertain system. A four-echelon supply chain network was introduced under uncertainty with a hybrid mathematical model-based approach by [Douaioui et al., 2021]. To address the limitations and challenges outlined above, this article will explore different cost factors associated with the management of empty shipping containers, such as the fixed and variable costs involved in moving them and expenses related to ordering, repairing, leasing, and storing each container. To accomplish this, using a combination of inventory modeling and a methodology, the Hamilton-Jacobi-Bellman equation determines the optimal number of containers to order and the appropriate safety stock level for the seaport yard. Additionally, the study will incorporate stochastic optimal control and street turn policy to optimize the overall objective function more effectively.

The rest of this paper is organized as follows. Section 2 presents the dynamic models related to empty containers on the seaport yard by employing the inventory approach and the cost of the repositioning operation. Section 3 analyzes the problem using the HJB equation and introduces the proposed policy regarding optimality. Section 4 presents the numerical simulation of the optimization models with experimental data. Section 5 concludes this research and discusses the future direction of the research.

METHODOLOGY

The inventory model is typically used to optimize profit or cost problems by adapting the safety level introduced by [Li et al., 2015]. Some factors directly affect management costs, such as ordering costs, leasing costs, holding costs, repairing costs, etc. To calculate the optimal empty container level, this paper considers container deterioration over the holding period, leasing, the export demand, the uncertain imports of the container, and the stochastic factor at the seaport terminals. The dynamic model should accommodate those factors to explore a more realistic scenario when describing the superior stock level and optimal cost. Table 2 describes the critical variables with parameters to describe the multi-period empty container management problem and analyze inventory policies concerning distinct objectives.

Table 2. Notation and definition

$u(t)$	Number of empty containers being ordered each period
$x(t)$	Number of empty containers in the yard
$\hat{x}$	Goal or safety number of empty containers in the yard
$\omega_e(t)$	Number of empty containers exported from the yard
$\omega_i(t)$	Number of empty containers imported after unloading cargo
α	Coefficient of the damaged rate of a container after unloading cargo
$\beta_{x(t)}$	Coefficient of the deterioration of empty containers in the seaport
$I_{u(t)}$	Coefficient of the different prices when hiring the empty containers from the suppliers
γ	Coefficient of the number of containers being followed by the street-turn policy
$\varepsilon(t)$	Stochastic factors occurred in the operation of the seaport. (Delay time when importing empty containers)
c_o	Parameter of the ordering cost per container
c_l	Parameter of the leasing cost per container and day
c_h	Parameter of the holding cost per container and day
c_r	Parameter of the repairing cost per container

It is especially noteworthy that most previous studies on the inventory model do not consider the perturbation factor. Adding uncertainty to the yard model might demonstrate fluctuation and disturbance of all components, making the mathematical model more realistic at terminal ports. By incorporating stochastic disturbance for controlling empty containers, the dynamic model is described by Ito's approach to stochastic differential equation:

$$dx = \left[u(t) - \beta_{x(t)} x(t) - \omega_e(t) + \gamma \omega_i(t) \right] dt + \varepsilon(t) dz(t) \quad (1)$$

where the boundary condition is given by, $x(0) = x_0$, describing the starting number of containers in the seaport yard. The number also expresses the container capacity at the terminal port. The $\varepsilon(t)$ illustrates the stochastic component related to unexpected events in container management. The delay in loading and unloading goods might lead to the wrong number of empty containers and incompatibility between departments during operations. The street-turn policy is one of the crucial policies when managing the transportation of empty containers. This policy allows a carrier to use a container after unloading the goods rather than bringing them to the empty container yard. This container can be used directly to load the goods to prepare for transporting new goods. This strategy obviously saves cleaning costs, repair costs, and other costs. Using the street-turn policy will make yard management more effective through the warehouse model. To formalize the policy, there is always the proportion of containers unloaded without damaged use for the next period, where the parameter γ will denote the proportion. The empty container ordering cost is the expense that creates and processes orders to leasing companies.

$$O(u(t)) = c_o u(t),\ \ u(t) > 0 \tag{2}$$

Managing the number of empty containers is markedly tricky because many factors affect the exact number. To ensure the shipping time from the consignor to the consignee and increase the reliability of port operations, the container yard always provides that the number of empty containers in the yard is greater than the number of containers needed for the loading process at all times. Whenever the number of empty containers in the seaport yard is insufficient for the safety level, the manager will order the corresponding amount from the leasing company. In reality, the amount of leasing directly affects the management cost. For example, if the manager hires just one or two empty containers, the cost will eventually be more expensive than hiring many empty containers. Thus, the leasing cost is described as follows:

$$L(u(t)) = c_l I_{u(t)} u(t),\ \ u(t) > 0 \tag{3}$$

where $I_{u(t)}$ describes the percentage of the discount rate of container leasing price from the companies. A practical formula can be used to rent empty containers, considering percentage discounts when renting large containers; the more hiring numbers, the more discount value. The discount rate will be separated into two sides with 1000 TEUs as a middle point, and it is given as follows:

$$I_{u(t)} = \begin{cases} 1, & u(t) \geq 1000 \\ 0.7, & u(t) < 1000 \end{cases} \tag{4}$$

When too many empty containers are stored in the port yard, they will occupy many areas that reduce the storage capacity of the laden container, thus reducing the number of container transportation at the port. The holding cost covers all costs of each empty container in the yard, such as storage, depreciation, personnel, and rental space charge, and the more containers are stored, and the more the holding cost will account for. Hence, the holding cost is described as follows:

$$H(x(t), u(t), \omega_e(t)) = c_h \left(x(t) - \hat{x} - \omega_e(t) + u(t) \right)^2 \tag{5}$$

To make the model more realistic, the containers will be dirty and damaged for repair after each unloading of goods from the containers. Therefore, to use them for the subsequent loading of goods and long-term use, port authorities will spend the costs of repairing and cleaning containers. This leads to another extension of the primary problem.

$$R(\omega_i(t)) = \alpha c_r \omega_i(t) \tag{6}$$

Let $J(x(t), u(t))$ denote the expected value of the total cost per cycle, which comprises the leasing, holding, ordering, and repairing costs. After some algebraic manipulation, the inventory system's total cost per unit of time is obtained as follows.

$$J(x(t),u(t),\omega_e(t),\omega_i(t)) = E\left[\int_0^\infty \left(O(u(t)) + L(u(t)) + H(x(t),u(t),\omega_e(t)) + R(\omega_i(t))\right)dt\right]$$
$$= E\left[\int_0^\infty \left(c_o u(t) + c_l I_{u(t)} u(t) + c_h\left(x(t) - \hat{x} - \omega_e(t) + u(t)\right)^2 + \alpha c_r \omega_i(t)\right)dt\right] \quad (7)$$

STOCHASTIC OPTIMIZATION

The value function should satisfy the Hamilton-Jacobi-Bellman (or HJB) principle for a stochastic differential equation inspired by [Sethi, 2021]. The optimization problem is described as follows:

$$J = \min_{u(t)}\left[c_o u(t) + c_l I_{u(t)} u(t) + c_h\left(x(t) - \hat{x} - \omega_e(t) + u(t)\right)^2 + \alpha c_r \omega_i(t) + J_t \right.$$
$$\left. + \left(u(t) - \beta_{x(t)} x(t) - \omega_e(t) + \gamma\omega_i(t)\right) J_x + \frac{1}{2}\varepsilon^2(t) J_{xx}\right] \quad (8)$$

Recalling that the HJB equation depends on the value of $x(t)$, it is necessary to solve a set of equations for the case of $x(t) \geq 0$. There is still a certain number of empty containers in the seaport yard, which leads to $\beta_{x(t)} x(t) > 0$. Otherwise, $x(t) < 0$ the terminal port might have an empty container to serve the demand. Therefore, the optimal solution for decision-making can be realized by employing the stochastic optimal control theory.

Theorem 1: If there is no constraint on the control input $u(t)$, then the optimal order number of empty containers at the seaport yard is described as the following equation:

$$u^*(t) = -\frac{\left(3 + 2\beta_{x(t)} + S\right)\left(e^{(T-t+C_1)S} - 1\right)}{2\left[1 - \left(e^{(T-t+C_1)S}\right)\frac{\left(3 + 2\beta_{x(t)}\right) + S}{\left(3 + 2\beta_{x(t)}\right) - S}\right]} x(t)$$
$$- \frac{1}{2c_h P}\left(\frac{PB - 2Q(t)N - 2c_h\left(A - \omega_e(t) - \hat{x}\right)}{e^{P(T-t+C_2)}} + 2Q(t)N + 2c_h\left(A - \omega_e(t) - \hat{x}\right)\right)$$
$$+ \frac{-c_o - c_l I_{u(t)} + 2c_h\left(\omega_e(t) + \hat{x} - x(t)\right)}{2c_h} \quad (9)$$

where

$$S = \sqrt{(3 + 2\beta_{x(t)})^2 + 4},\ P = \left(2 + \frac{Q(t)}{c_h} + \beta_{x(t)}\right),\ N = \left(\gamma\omega_i(t) - \frac{c_o + c_l I_{u(t)}}{2c_h} + \hat{x}\right),\ Q(t) > 0$$

Proof See Appendix 1 for more details.

According to Theorem 1, the number of empty containers is optimal. In practice, the ordering number of empty containers will be non-negative; thus, a boundary being set in minimum is zero. Moreover, the safety number of containers stored in the seaport yard is positive for serving the demand for empty containers. The optimal strategy should focus on minimizing the management cost and guaranteeing the number of containers within the safety range. Hence, an optimal policy is described as follows:

$$u(t) = \max\left[0, -\frac{\left(3+2\beta_{x(t)}+S\right)\left(e^{(T-t+C_1)S}-1\right)}{2\left[1-\left(e^{(T-t+C_1)S}\right)\frac{\left(3+2\beta_{x(t)}\right)+S}{\left(3+2\beta_{x(t)}\right)-S}\right]}x(t) - \frac{1}{2c_h P}\left(\frac{PB-2Q(t)N-2c_h\left(A-\omega_e(t)-\hat{x}\right)}{e^{P(T-t+C_2)}}+2Q(t)N+2c_h\left(A-\omega_e(t)-\hat{x}\right)\right) + \frac{-c_o-c_l I_{u(t)}+2c_h\left(\omega_e(t)+\hat{x}-x(t)\right)}{2c_h}\right] \quad (10)$$

Using Karush-Kuhn-Tucker (KKT) conditions and the inequality constraint $u(t) \geq 0$ [Sethi, 2021], the optimal ordering number of empty containers is given in equation (10). It is worth noting that the KKK condition describes the optimality requirement in dynamic programming. Therefore, the optimal ordering policy can be determined by Theorem 1.

SIMULATION RESULT AND DISCUSSION

Numerical analysis is conducted on a personal computer with an AMD Ryzen 5 5600G processor, a base clock speed of 3.9 GHz, and 16 GB RAM. A code used for simulation results is built into the MATLAB program. All data used in the simulation have been collected from the port authority of Busan Port, which ranked sixth in the world's container throughput and is the primary port in South Korea. A numerical simulation is carried out for the evaluation of the proposed model. Input data are given in Tables 3 and 4 based on the fundamental market analysis and are typical in port operations. The experiment is sampled with 3650 days, starting from an initial value of 3000 TEUs.

Table 3. Summary statistics for numerical analysis

Types of data	Source	Numerical evaluation
ordering costs	market analysis	[$1-5]/TEU
holding costs	market analysis	[$1-5]/TEU/day
leasing costs	market analysis	[$5-20]/TEU/day
repair distribution	industry interviews	Poisson Distribution
repair costs	market analysis	[$50-300]/TEU
street-turns	industry interviews	[0-30%] of a container after unloading cargo

The demand is based on data from leasing companies near Busan Port. The exponent distribution can be described as the number of empty containers in the seaport yard. The trending exporter demand is increased by following the simulation time:

$$\omega_e(t) = 1500 + 200\sin\left(\frac{t}{70}+4\right) + e^{0.002t} + rand(100, 500) \quad (11)$$

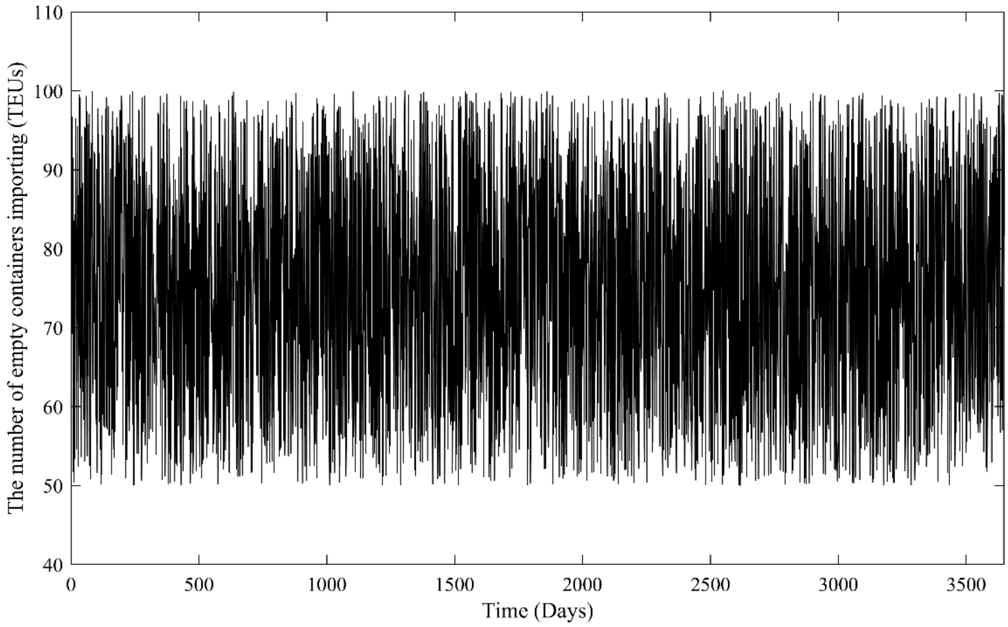

Fig. 2. The number of empty containers after unloading cargo

In this scenario, the volume of empty containers importing after unloading cargo fluctuates between 50 and 100 TEUs daily. There are a lot of unexpected events occurred during the unloading stage. Obviously, the importing or exporting numbers often fluctuate unpredictably as these numbers impact transportation costs. In this case, the parameter will be described as random numbers from the uniform distribution on the interval shown in Fig. 2. The stochastic variable will be expressed as the Gaussian distribution demonstrating unexpected container shipping events. The parameter values are given in Table 4, prepared for the numerical simulation that illustrates stochastic events. All the values are selected by the many experiment tests, and some are typical in port operations.

Table 4. Parameter values for numerical analysis.

Parameter	α	β	γ	x_0	c_o	c_l	c_h	c_r
Value	0.2	0.02	0.7	3000	2	10	2	200

For the setting of the given parameter, the numerical test is performed based on changing the safety stock levels in the yard from 1000 TEUs to 4000 TEUs. The computed costs are summarized in Table 5, where the holding costs account for most of the costs of container management. Changing the safety stock levels gives the total management cost according to the parabolic shape. Therefore, the safety level is determined to optimize the management cost for the seaport operations. When the safety level is lower, the cost of renting empty containers will increase to meet the demand. On the contrary, more empty containers will be stored in the seaport yard when the safety level is higher. Then it will increase the holding cost.

Table 5. Operational cost analysis considering a different number of safety levels of empty containers (Unit: Thousand USD)

Safety level $\hat{x}$	1000	1500	2000	2500	3000	3500	4000
Holding cost	2500.5	1031.5	538.9	259.7	191.1	336.1	694.7
Total cost	5069.4	4992.4	4974.8	4975.7	4998.2	5040.4	5102.2

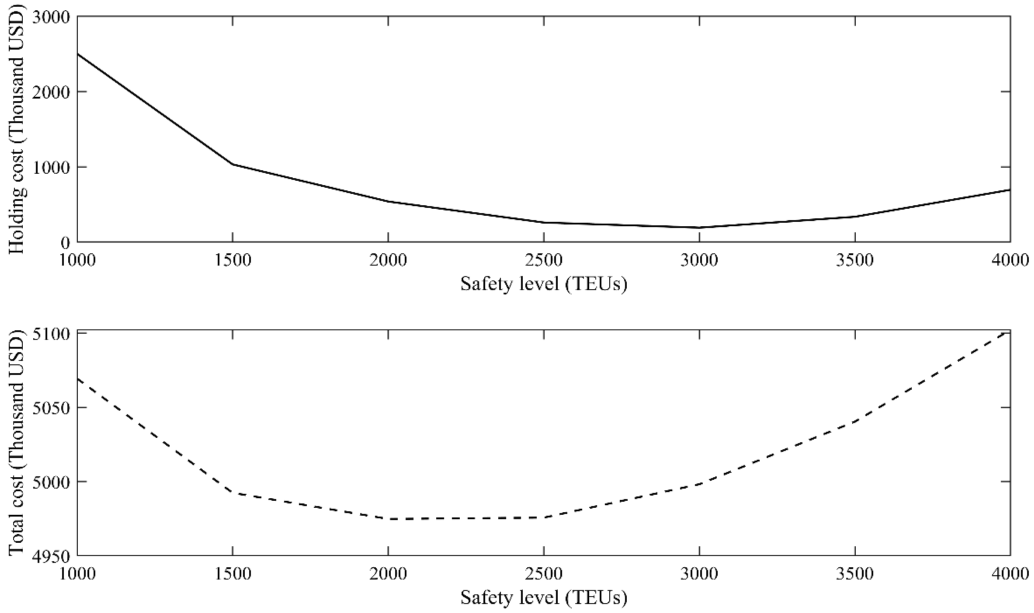

Fig. 3. Cost change for different safety levels ($\hat{x}$).

With gradually increasing safety levels, the holding cost and total cost for managing empty containers in the yard are calculated as shown in Table 5 and Fig. 3. When the number of safety empty containers slightly increases from 1000 TEUs to 4000 TEUs, and there is a significant decrease in the operational costs. It then reaches the lowest point, approximately 191.1 thousand dollars at 3000 TEUs and 4975.7 thousand dollars at 2500 TEUs, respectively. The holding cost witnesses a minimal growth to about 694.7 thousand dollars at the safety level of 4000 TEUs. The total cost dramatically rises to 5102.2 thousand dollars at the same safety level. These findings illustrate the sensitivity of the costs when the safety level changes. Furthermore, the test results can provide information on the safety level needed for optimizing yard operational costs.

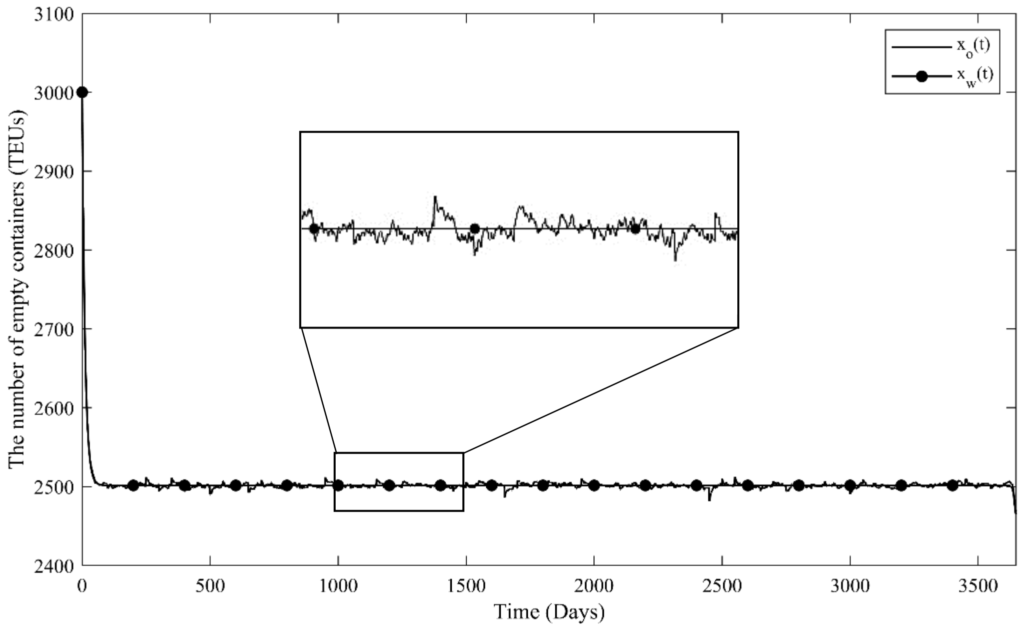

Fig. 4. Number of empty containers with and without stochastic components with a safety level of 2500 TEUs.

Fig. 4 presents the comparative analysis of the operational costs with (x_o) and without (x_w) stochastic events. Under the optimal ordering policy, the number of empty containers $x_o(t)$ decreased from the initial value of 3000 TEUs to the expected number of 2500 TEUs in 30 days. Then it fluctuated around the expected number until the end of the period. Without stochastic factors $x_w(t)$, the terminal operation is stable from the beginning until the end of the simulation period. With optimal ordering strategy, the total cost is calculated as 5714.8 thousand dollars which is a little bit bigger compared to the total cost without random variables in the dynamic model, as 5712.9 thousand dollars. The efficacy of the proposed model is illustrated in Fig. 4, where the actual number of empty containers fluctuates around the ideal value unaffected by the stochastic disturbance. The percentage of the difference between the management cost with and without uncertainty components is approximately 0.04 %, which might prove the effectiveness of the proposed model. Especially the number of empty containers approaches the desired safety level rapidly and keeps the safety level with little variability until the end of the period. The findings have revealed the effectiveness of the proposed framework, which will provide a decision-making support scheme for efficient port operations.

Next, the street-turn analysis based on the safety level at the seaport terminal is given in Table 6 and Fig. 5. The change in the street-turn coefficient directly affects the total cost. At a hundred percent with a safety level of 2500 TEU, the optimal management cost is approximately 4960.3 (thousand USD). However, the street-turn coefficient never reaches the maximum ratio in practice because an unexpected event when unloading might cause damage to the container. Fig. 5 shows that the percentage increase in the street-turn coefficient may bring further costs down. The optimal safety level and the coefficient value for the percent street-turn policy are determined by the results from Figs. 3 and 5. Efforts to maximize the coefficient might result in the optimal cost in the short-term period. However, it is not the best option in the long term since containers have been in use for an extended period, leading to increased repair costs and total costs. Additionally, Fig. 4 illustrates the efficiency of the proposed method when simulating with and without uncertainty considerations in the built model.

CONCLUSION

This study employs the inventory model to consider the stochastic optimization method to manage empty containers in the seaport yard. The repairing option for damaged containers and street-turn policy are incorporated in formulating objective functions. Based on the stochastic optimization method, the optimal number of ordering empty containers is implemented by solving the stochastic HJB equation. Furthermore, numerical experiments are provided to evaluate the efficiency and capability of the proposed strategies. By comparing the dynamic model with and without uncertainty components, the gap is just about 0.04 %, which might prove the robustness of the proposed model. Using a stochastic optimization approach, decision-makers can realize strategic management policy to optimize operational costs. The main goal of this study is to find the decision support system to solve the empty container management in the seaport yard. This study has limitations due to constraints, methodology, materials, etc. For the model to be more applicable in real-world scenarios, emissions and fuel consumption should be considered in the dynamic model to deal with problems like greenhouse gas emissions and global warming, imposing additional costs such as emissions costs. Additionally, the study could benefit from employing a method to approximate the exporter demand function to bridge the gap between theory and practical application. These issues can be solved in future research using an upgraded methodology.

Table 6. Total costs considering the different safety levels of the empty container and the street-turn coefficient in the seaport yard. (Unit: thousand USD).

Safety level $\hat{x}$	Street-turn coefficient γ	Total cost	Safety level $\hat{x}$	Street-turn coefficient γ	Total cost
1000	0.5	5125.7	2500	0.5	4997.6
	0.6	5118.1		0.6	4990.2
	0.7	5110.8		0.7	4982.6
	0.8	5103.2		0.8	4975.2
	0.9	5095.7		0.9	4967.7
	1.0	5088.2		1.0	4960.3
1500	0.5	5014.3	3000	0.5	5020.2
	0.6	5006.6		0.6	5012.9
	0.7	4998.9		0.7	5005.5
	0.8	4991.2		0.8	4998.1
	0.9	4983.5		0.9	4990.8
	1.0	4975.8		1.0	4983.5
2000	0.5	4997.0	3500	0.5	5062.5
	0.6	4989.4		0.6	5055.3
	0.7	4981.8		0.7	5048.0
	0.8	4974.2		0.8	5040.8
	0.9	4966.7		0.9	5033.6
	1.0	4959.1		1.0	5026.4
			4000	0.5	5124.4
				0.6	5117.3
				0.7	5110.2
				0.8	5103.1
				0.9	5096.0
				1.0	5088.9

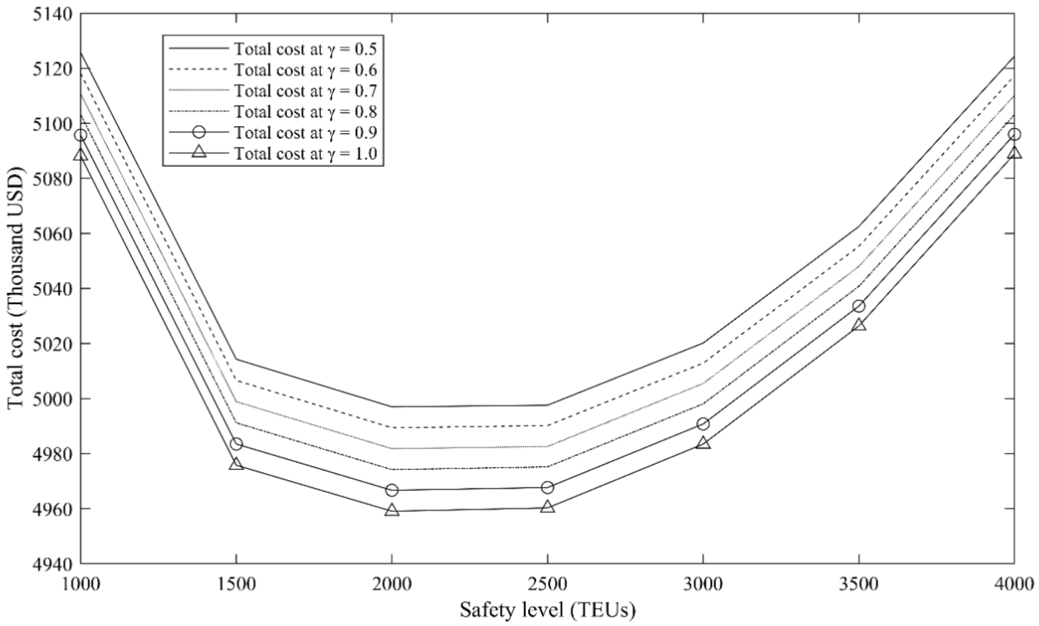

Fig. 5. Total costs with different safety levels and street-turn coefficients.

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APPENDIX 1. PROOF OF THEOREM 1

Proof Equation (8) takes the quadratic function of the control input $u(t)$ having no boundary. The optimal solution can be obtained by taking its derivative concerning u, and setting them to zero,

$$c_o + c_l I_{u(t)} + 2c_h\left(x(t) - \hat{x} - \omega_e(t) + u(t)\right) + J_x = 0 \tag{12}$$

Then, the control input can be given by

$$u^*(t) = A - \frac{J_x}{2c_h} \tag{13}$$

where $A\left(x(t), \omega_e(t)\right) = \dfrac{-c_o - c_l I_{u(t)} + 2c_h\left(\omega_e(t) + \hat{x} - x(t)\right)}{2c_h}$, and $c_h > 0$. Substituting the optimal decision policy (13) into equation (8) leads to,

$$\begin{aligned} J(x(t)) = &-\frac{J_x^2}{4c_h} + \left(-\frac{c_o + c_l I_{u(t)}}{2c_h} - \left(1 + \beta_{x(t)}\right)x(t) + \hat{x} + \gamma\omega_i(t)\right)J_x + \frac{1}{2}\varepsilon^2(t)J_{xx} \\ &+ J_t + A\left(c_o + c_l I_{u(t)}\right) + c_h\left(x(t) - \omega_e(t) - \hat{x} + A\right)^2 + \alpha c_r \omega_i(t) \end{aligned} \tag{14}$$

Next, it is noted that $J(x)$ is the objective function to be minimized. The non-linear differential equation can be solved by considering a quadratic concave function candidate,

$$\begin{aligned} &J(x(t)) = Q(t)x^2 + R(t)x + M(t) \\ &J_t = \dot{Q}x^2 + \dot{R}x + \dot{M},\ \ J_x = 2Qx + R,\ \ J_{xx} = 2Q \end{aligned} \tag{15}$$

where $Q(t)\,(>0)$, $R(t)$ and $M(t)$ can be determined for the minimum value. Substituting (15) into (14) yields

$$
\begin{aligned}
0 = & \left[\dot{Q} - \frac{Q^2(t)}{c_h} - \left(1 + 2\left(1 + \beta_{x(t)}\right)\right) Q(t) + c_h \right] x^2(t) \\
& + \left[\dot{R} - \left(2 + \frac{Q(t)}{c_h} + \beta_{x(t)} \right) R(t) + 2Q(t) \left(\gamma \omega_i(t) - \frac{c_o + c_l I_{u(t)}}{2c_h} + \hat{x} \right) + 2c_h \left(A - \omega_e(t) - \hat{x} \right) \right] x(t) \\
& + \dot{M} - M(t) - \frac{R^2(t)}{4c_h} + R(t) \left(\gamma \omega_i(t) - \frac{c_o + c_l I_{u(t)}}{2c_h} + \hat{x} \right) + \varepsilon^2(t) Q(t) + A\left(c_o + c_l I_{u(t)}\right) \\
& + c_h \left(A - \omega_e(t) - \hat{x} \right)^2 + \alpha c_r \omega_i(t)
\end{aligned} \tag{16}
$$

Solving equation (16) gives

$$
\begin{cases}
\dot{Q} = \dfrac{Q^2(t)}{c_h} + \left(1 + 2\left(1 + \beta_{x(t)}\right)\right) Q(t) - c_h \\
\dot{R} = \left(2 + \dfrac{Q(t)}{c_h} + \beta_{x(t)} \right) R(t) - 2Q(t) \left(\gamma \omega_i(t) - \dfrac{c_o + c_l I_{u(t)}}{2c_h} + \hat{x} \right) - 2c_h \left(A - \omega_e(t) - \hat{x} \right) \\
\dot{M} = M(t) + \dfrac{R^2(t)}{4c_h} - R(t) \left(\gamma \omega_i(t) - \dfrac{c_o + c_l I_{u(t)}}{2c_h} + \hat{x} \right) - \varepsilon^2(t) Q(t) - A\left(c_o + c_l I_{u(t)}\right) \\
\quad - c_h \left(A - \omega_e(t) - \hat{x} \right)^2 - \alpha c_r \omega_i(t)
\end{cases} \tag{17}
$$

The dynamical system (17) represents a hierarchical system of equations. The time evolution of the function $R(t)$ contains the function $Q(t)$, while the function $M(t)$ includes two functions $Q(t)$ and $R(t)$. The non-linear systems can be solved for different cases of $x(t)$ with the following terminal conditions: $Q(t) = 0$, $R(t) = B$, and $M(t) = 0$, where B is constant. Solving equation (17) yields

$$
\begin{cases}
Q(t) = \dfrac{c_h \left(3 + 2\beta_{x(t)} + S\right)\left(e^{(T - t + C_1)S} - 1\right)}{2\left[1 - \left(e^{(T - t + C_1)S}\right) \dfrac{\left(3 + 2\beta_{x(t)}\right) + S}{\left(3 + 2\beta_{x(t)}\right) - S} \right]} \\
R(t) = \dfrac{1}{P} \left(\dfrac{PB - 2Q(t)N - 2c_h \left(A - \omega_e(t) - \hat{x} \right)}{e^{P(T - t + C_2)}} + 2Q(t)N + 2c_h \left(A - \omega_e(t) - \hat{x} \right) \right) \\
M(t) = \dfrac{\dfrac{R^2(t)}{4c_h} - R(t)N - \varepsilon^2(t) Q(t) - A\left(c_o + c_l I_{u(t)}\right) - c_h \left(A - \omega_e(t) - \hat{x} \right)^2 - \alpha c_r \omega_i(t)}{e^{T - t + C_3}} \\
\quad - \dfrac{R^2(t)}{4c_h} + R(t)N + \varepsilon^2(t) Q(t) + A\left(c_o + c_l I_{u(t)}\right) + c_h \left(A - \omega_e(t) - \hat{x} \right)^2 + \alpha c_r \omega_i(t)
\end{cases} \tag{18}
$$

where C_1, C_2, and C_3 are constants that can be determined from terminal conditions. Finally, employing equations (13), (15), and (18) with no boundary on the control input $u(t)$ yields,

$$u^*(t) = -\frac{\left(3+2\beta_{x(t)}+S\right)\left(e^{(T-t+C_1)S}-1\right)}{2\left[1-\left(e^{(T-t+C_1)S}\right)\frac{\left(3+2\beta_{x(t)}\right)+S}{\left(3+2\beta_{x(t)}\right)-S}\right]}x(t)$$
$$-\frac{1}{2c_hP}\left(\frac{PB-2Q(t)N-2c_h\left(A-\omega_e(t)-\hat{x}\right)}{e^{P(T-t+C_2)}}+2Q(t)N+2c_h\left(A-\omega_e(t)-\hat{x}\right)\right) \quad (19)$$
$$+\frac{-c_o-c_lI_{u(t)}+2c_h\left(\omega_e(t)+\hat{x}-x(t)\right)}{2c_h}$$

The proof of Theorem 1 is completed.

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